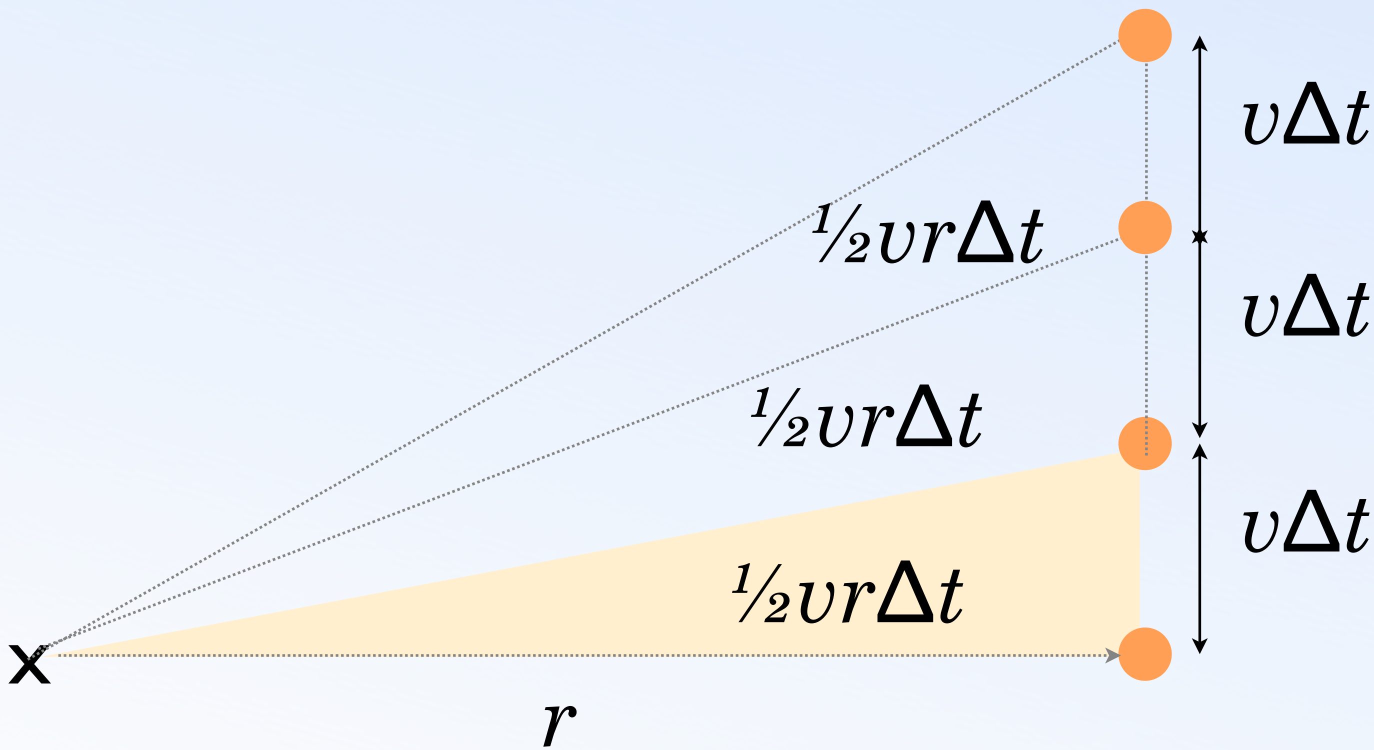
Rotations

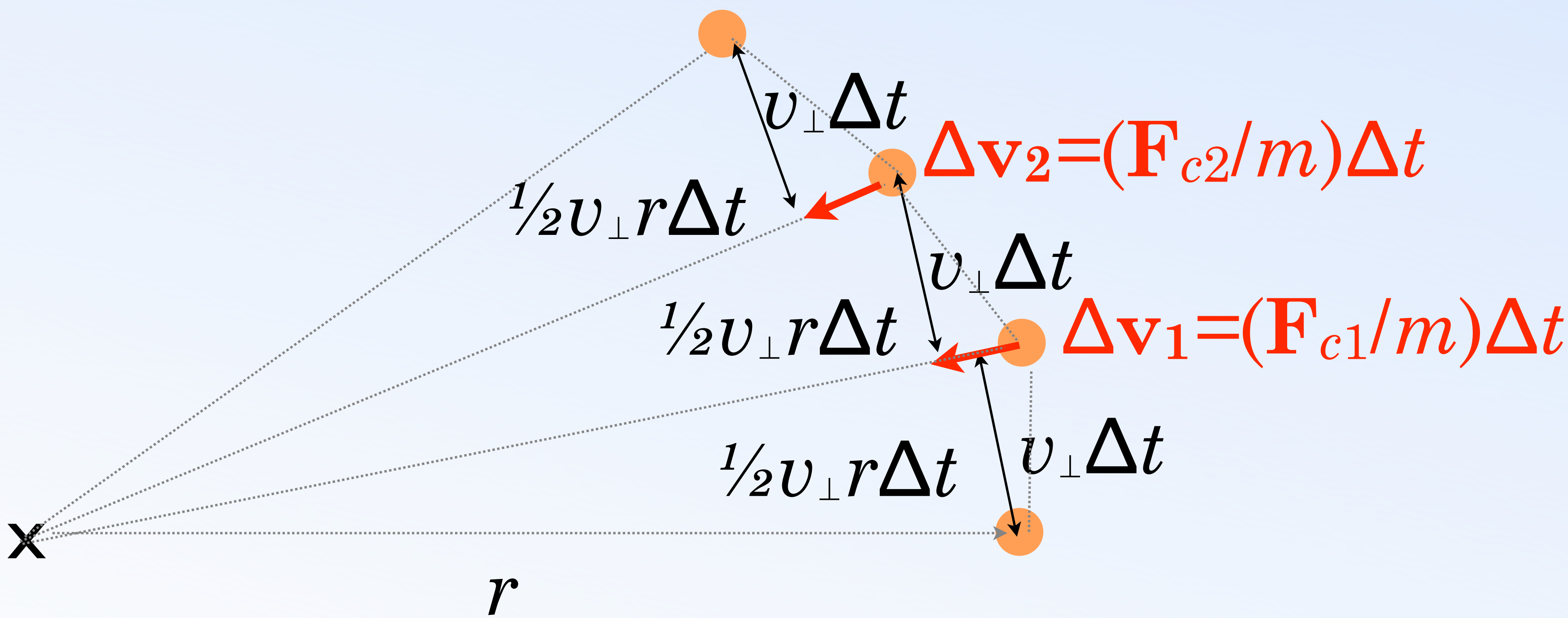
Linear Motion

- Kepler's 2nd Law
- Equal areas are swept out in equal times.



Central Force

- Kepler's 2nd Law
- Equal areas are swept out in equal times.



$v_\perp$ is the component of velocity perpendicular to **r.**

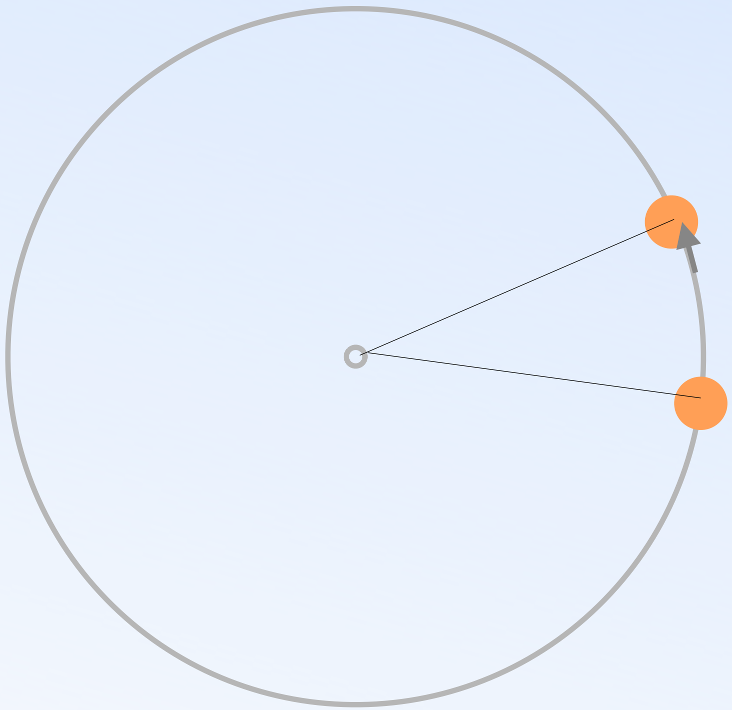
Angular Momentum

$$L = mv_{\perp}r$$

- When there are only central forces, L doesn’t change.
- $L = 2$m(Area) swept out.
- Angular momentum is conserved

Angular momentum

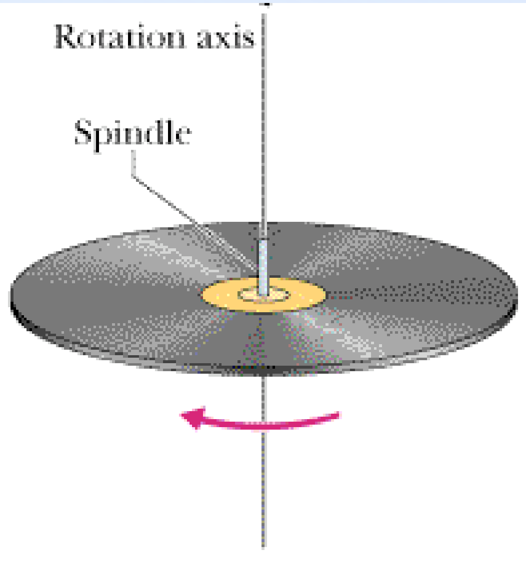
- In terms of angular variables.

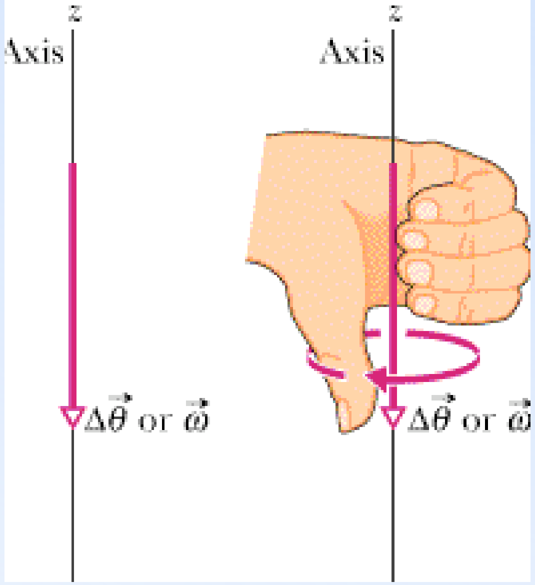


$$\omega = \frac{d\theta}{dt}$$

$$L = m\omega r^2$$

Vector nature of angular variables



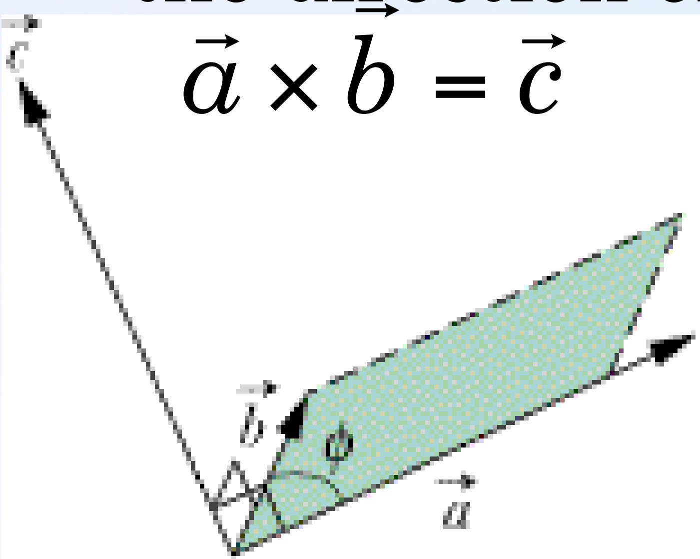


- use the “right hand rule” to assign a direction to angular rotations.

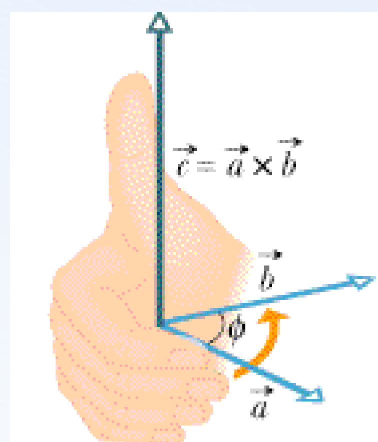
Cross Product

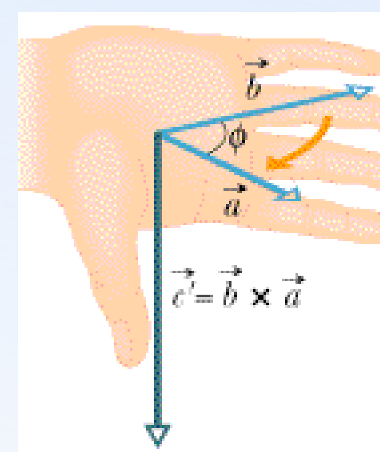
- Direction is given by “right hand rule”
- Imagine turning a screw from **a** to **b**.
- The direction a normal r.h. screw moves is the direction of **c**.

$$\vec{a} \times \vec{b} = \vec{c}$$



area of parallelogram $= ab \sin\phi$



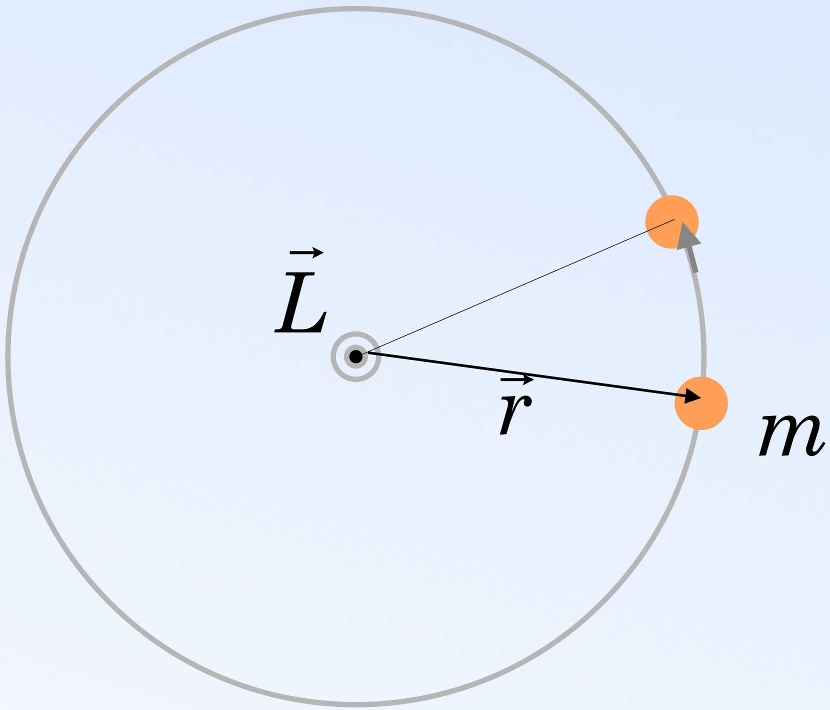


Angular Momentum

- Defines as a vector (or cross) product of **r** and **p.**

$$\vec{p} = m\vec{v}$$

$$\vec{L} = \vec{r} \times \vec{p}$$



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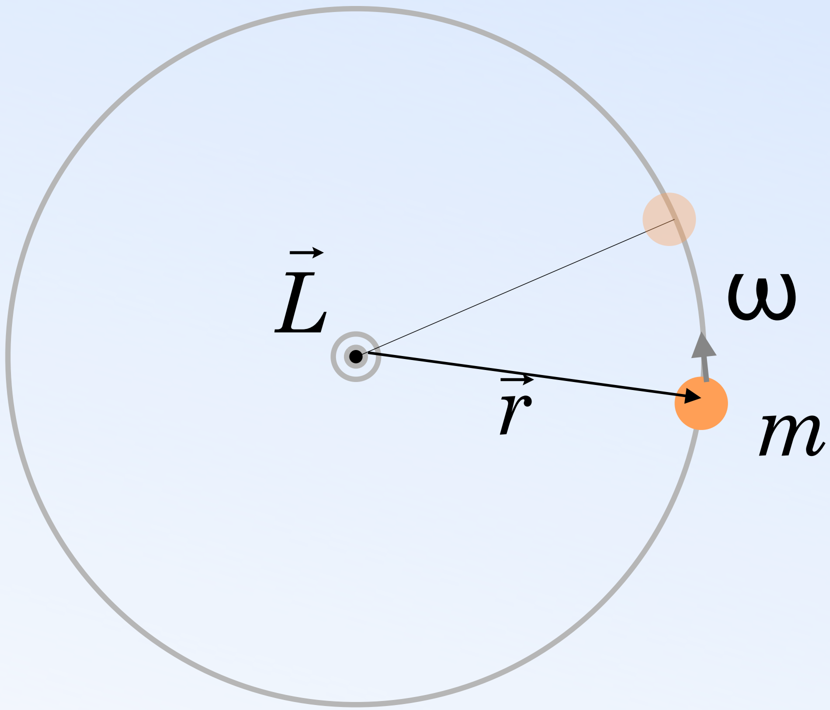
Kinetic Energy

$$K = \frac{1}{2}mv^2 = \left(\frac{1}{2}mr^2\right)\omega^2$$

"Moment of Inertia"

$$I = mr^2$$

$$K = \frac{1}{2}I\omega^2$$



Analogous to $K = \frac{1}{2}mv^2$

Rigid Body

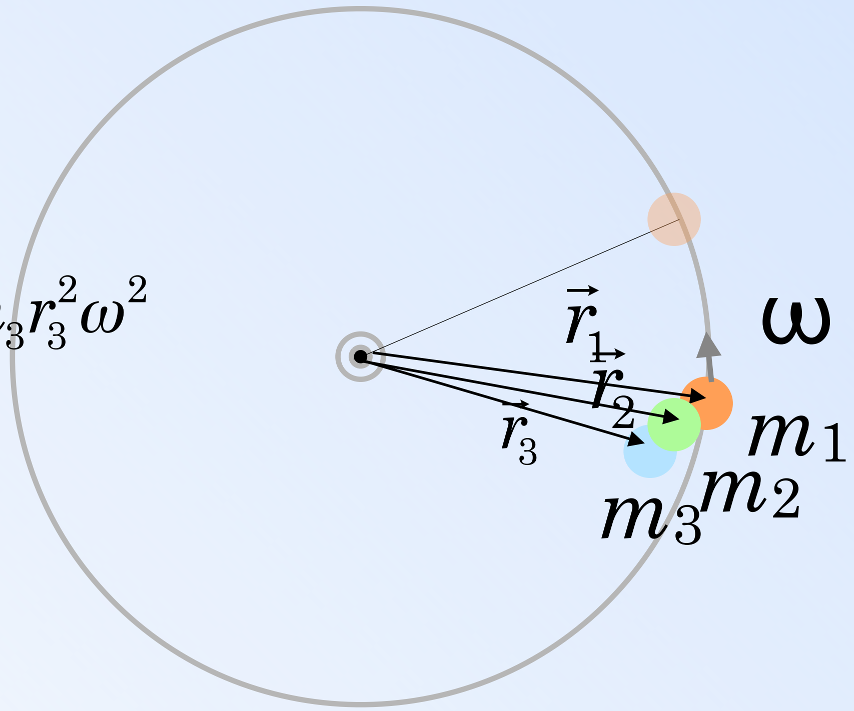
Everything's all stuck together.

ω is the same for all the *m*'s

$$K = \tfrac{1}{2} m_1 v^2 + \tfrac{1}{2} m_2 v^2 + \tfrac{1}{2} m_3 v^2$$

$$K = \tfrac{1}{2} m_1 r_1^2 \omega^2 + \tfrac{1}{2} m_2 r_2^2 \omega^2 + \tfrac{1}{2} m_3 r_3^2 \omega^2$$

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$



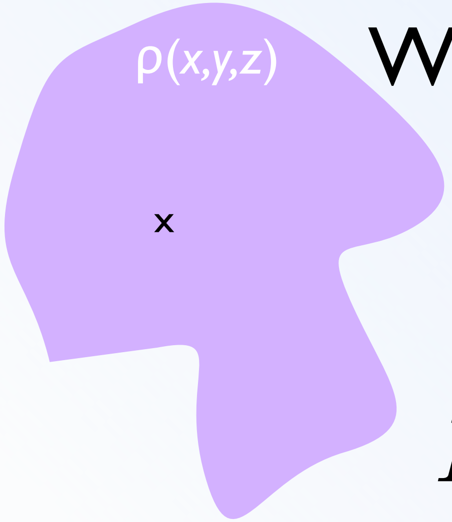
$$K = \tfrac{1}{2} I \omega^2$$

$$\vec{L} = I\vec{\omega}$$

Moment of Inertia

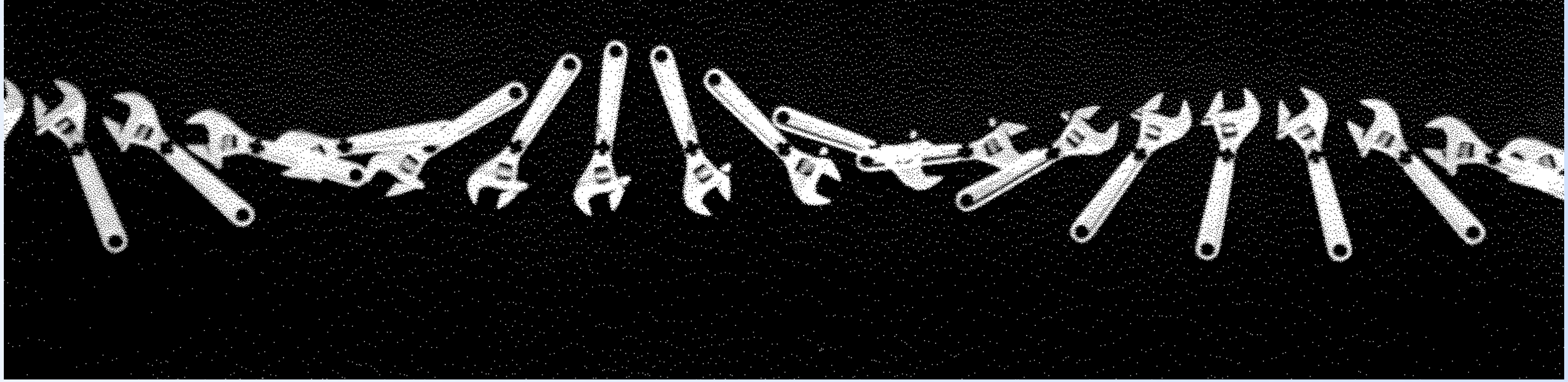
In general

$$I = \int r^2 dm = \iiint r^2 \rho(x,y,z)\,dxdydz$$

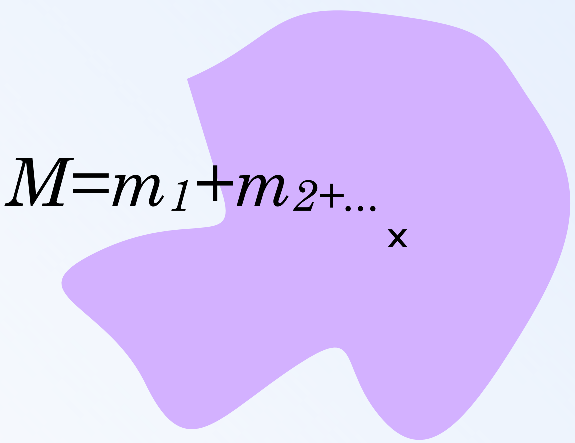


Where ρ(*x*,*y*,*z*) is the mass density.

I depends on the reference point x.



Centre of Mass



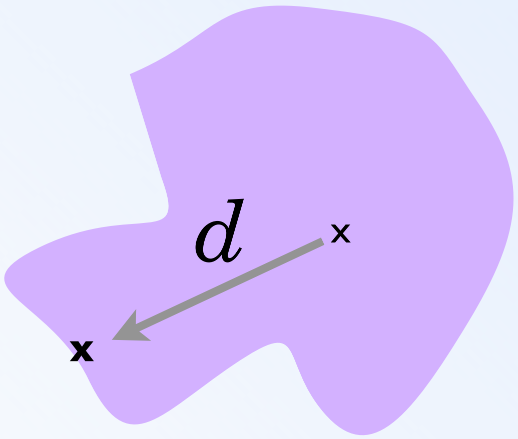
$$x_{\mathrm{cm}} = \frac{1}{M}\sum_i m_i x_i$$

$$y_{\mathrm{cm}} = \frac{1}{M}\sum_i m_i y_i$$

$$z_{\mathrm{cm}} = \frac{1}{M}\sum_i m_i z_i$$

A natural reference point for *I* is the centre of mass. That’s the points the object would rotate about in free space.

Parallel Axis Theorem



$$I = I_{\mathrm{cm}} + Md^2$$