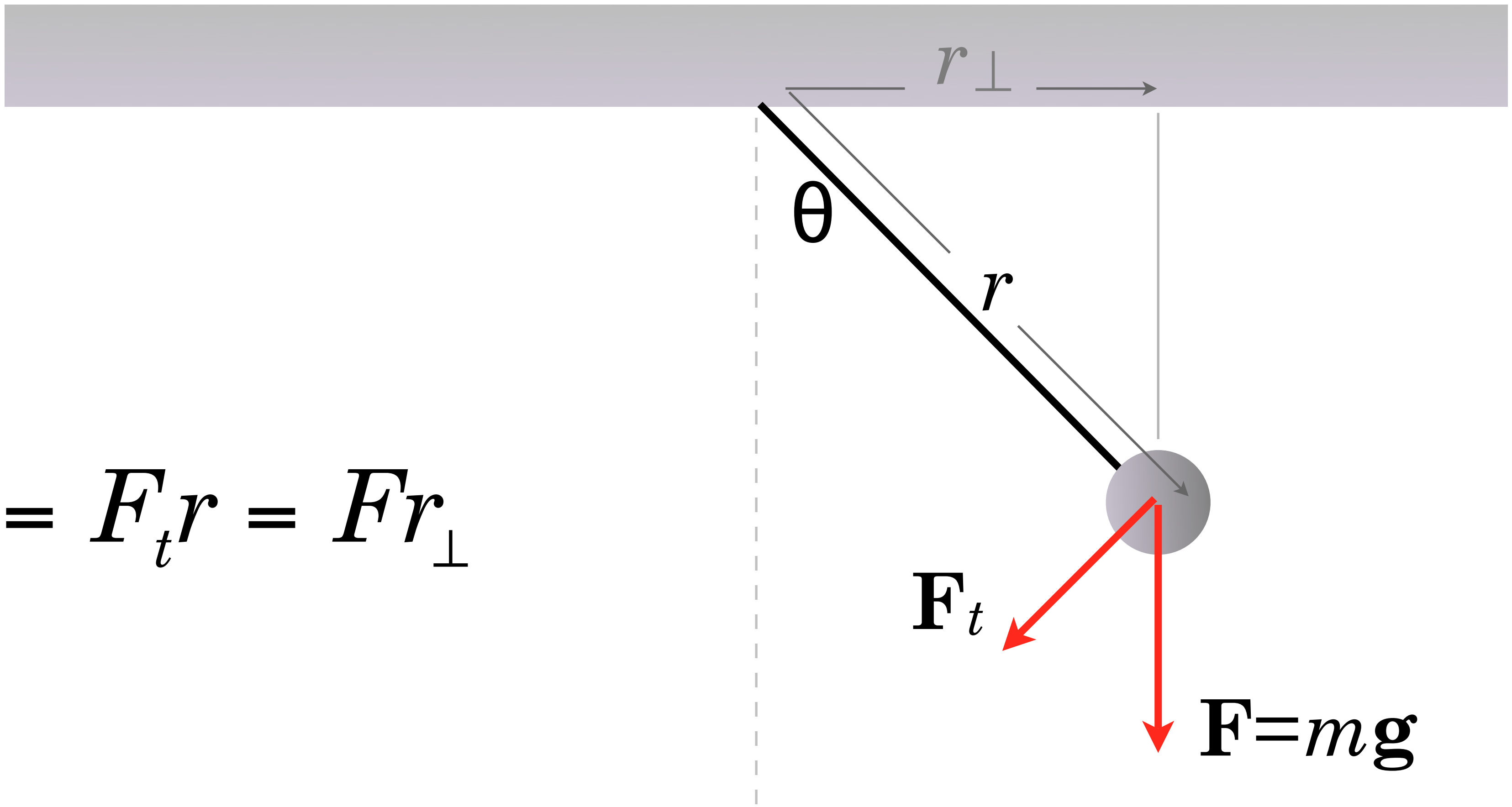
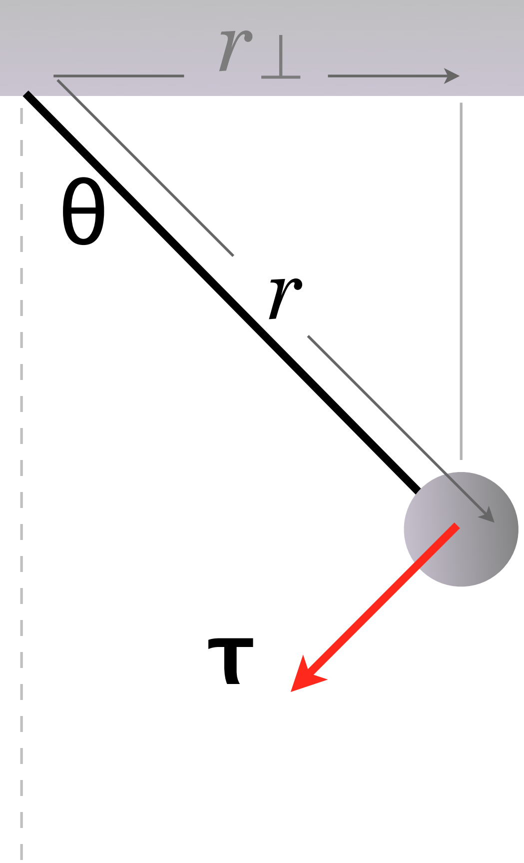
Torque

Pendulum



$$\tau = F_t r = F r_\perp$$

Pendulum



$$F_t = ma$$

$$\frac{\tau}{r} = \frac{I}{r^2}\alpha r$$

$$\tau = I\alpha$$

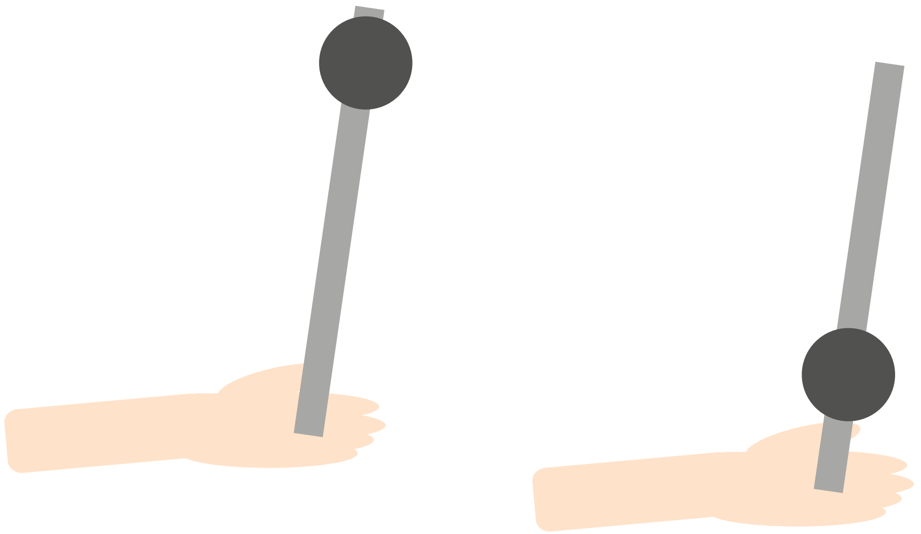
$$F_t = \frac{\tau}{r}$$

$$a = \alpha r$$

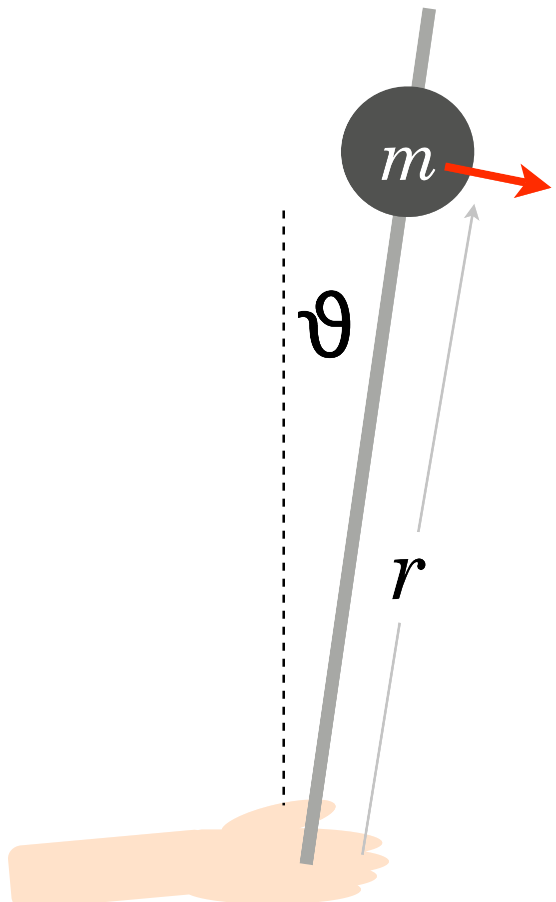
$$m = \frac{I}{r^2}$$

Balancing Stick

- Which is easier to balance?



Balancing Stick



$\tau = mgr \sin \vartheta$

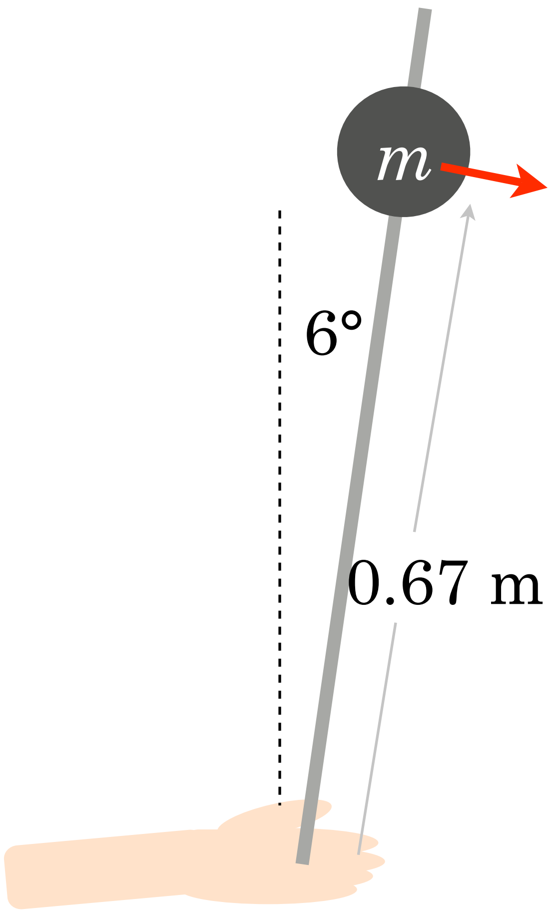
$I = mr^2$

$\alpha = \tau / I$

$\alpha = mgr \sin \vartheta / mr^2$

$\alpha = g \sin \vartheta / r$

Balancing Stick

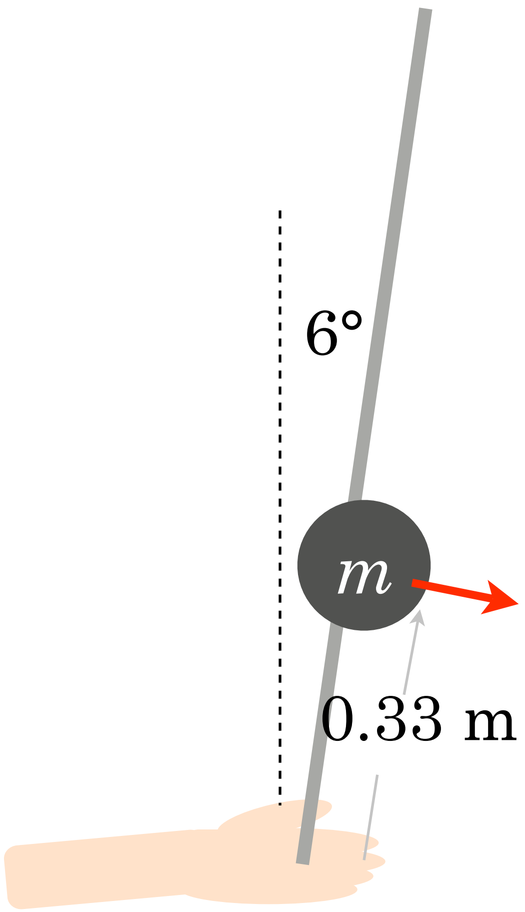


$\alpha = g \sin\vartheta / r$

$\alpha = (9.8\ \text{m/s}^2)(0.1)/(0.67\ \text{m})$

$\alpha = 1.4\ \text{rad/s}^2$

Balancing Stick



$\alpha = g \sin\vartheta / r$

$\alpha = (9.8\ \text{m/s}^2)(0.1)/(0.33\ \text{m})$

$\alpha = 2.8\ \text{rad/s}^2$

- Mass near bottom:

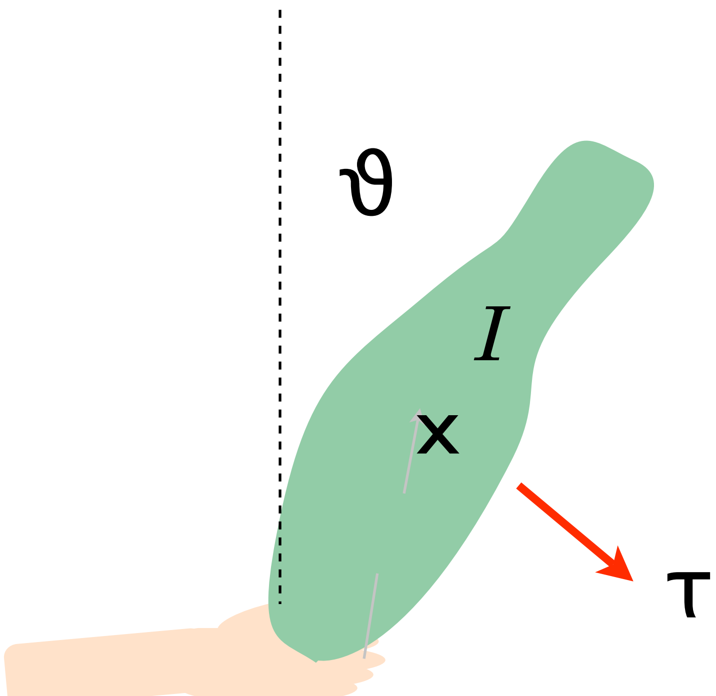
  $\alpha = 2.8\ \text{rad/s}^2$

- Mass near top:

  $\alpha = 1.4\ \text{rad/s}^2$

- The stick falls more slowly when the mass is near the TOP.

Any rigid object...



- Torque from gravity acts on centre of mass
- All particles in the object move at same ω
- Internal forces on particles occur in action-reaction pairs.
- Can show:

  $I = \alpha\ \tau$ for any rigid body.

Analogies

Pure Translation (x axis)		Pure Rotation (Symmetry about a Fixed Rotation Axis)	
Position component	x	Rotational position component	θ
Velocity component	$v_x = dx/dt$	Rotational velocity component	$\omega = d\theta/dt$
Acceleration	$a_x = dv_x/dt$	Rotational acceleration component	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's Second Law	$F_x^{\text{net}} = ma_x$	Newton's Second Law	$\tau^{\text{net}} = I\alpha$
Work	$W = \int F_x\, dx$	Work	$W = \int \tau\, d\theta$
Kinetic energy	$K = \frac{1}{2}mv_x^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power	$P = F_x v_x$	Power	$P = \tau\omega$
Work-kinetic energy theorem	$W = \Delta K$	Work-kinetic energy theorem	$W^{\text{rot}} = \Delta K^{\text{rot}}$