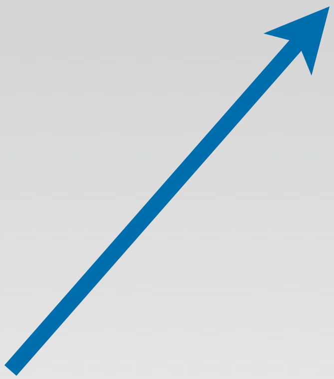
$\vec{V}$ectors

or how to get there

Properties of vectors

- When are they equal?
- How do you add them?
- How do you get the negative?
- How do you subtract them?
- 3 dimensions
- What's a unit vector?

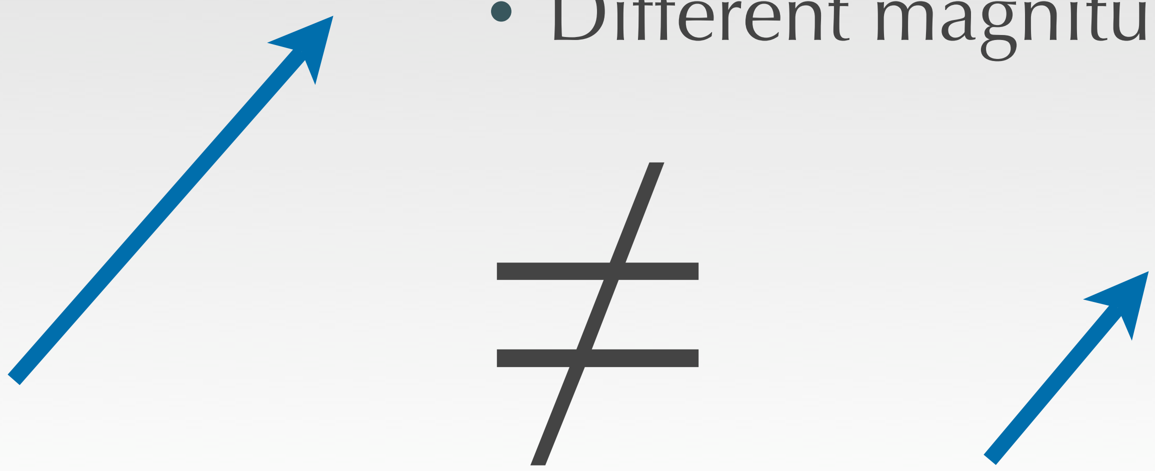
When are vectors equal?



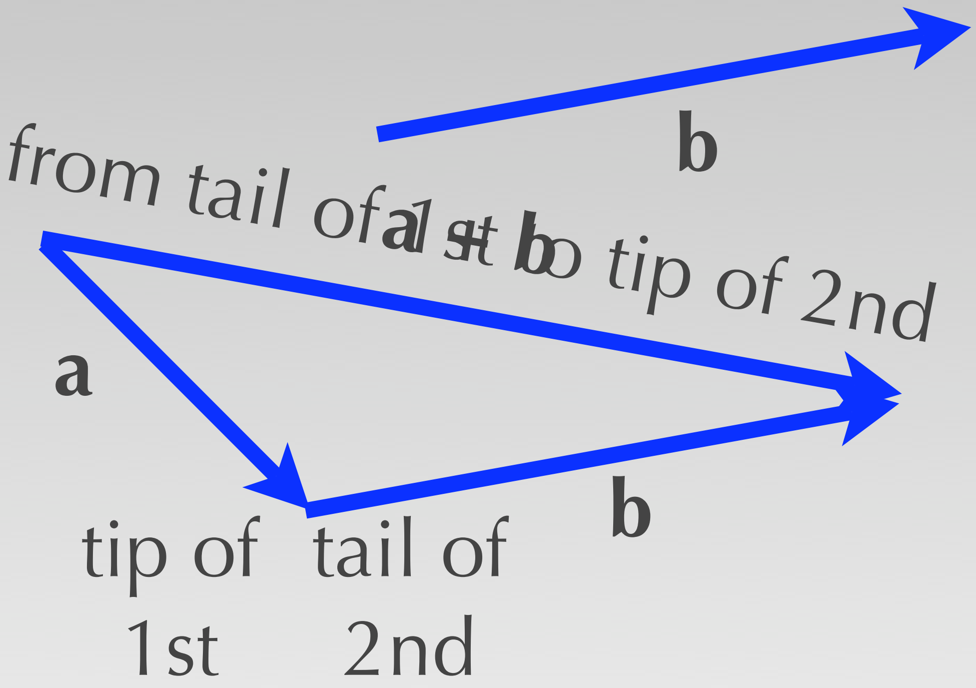
- Equal Magnitude
- Same Direction
- They may be in different places

When are vectors not equal?

- Different Direction
- Different magnitude

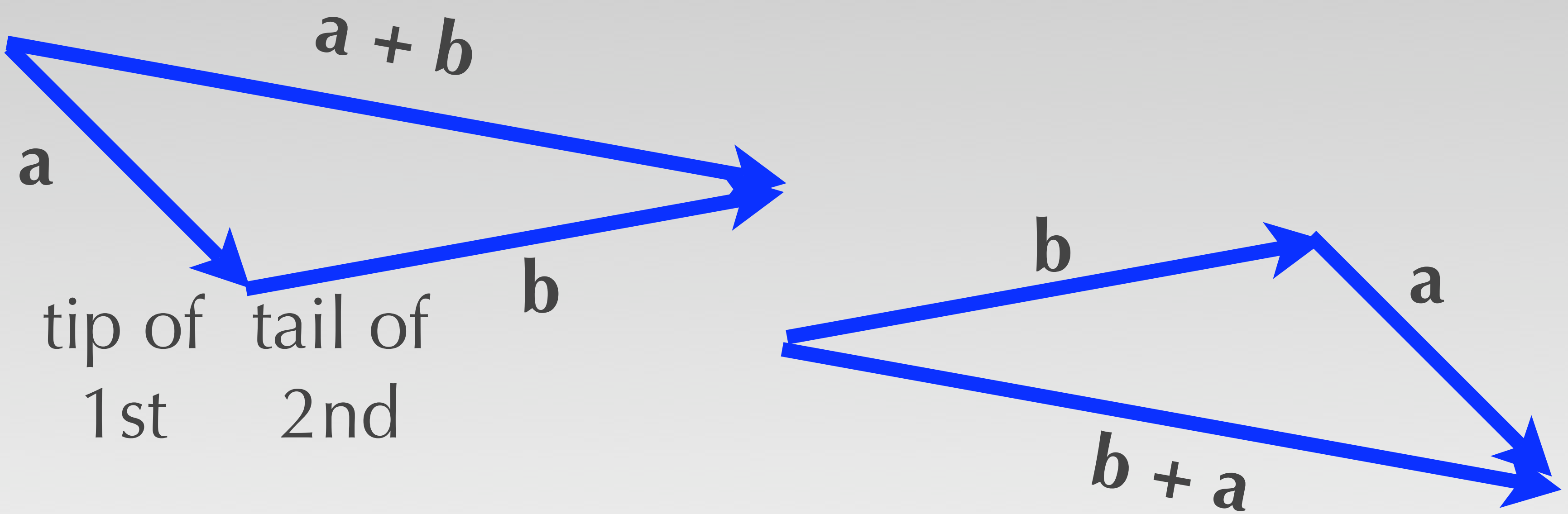


Adding Vectors



- The "protype" vector is displacement
- Think about the net result of two displacements

Commutativity



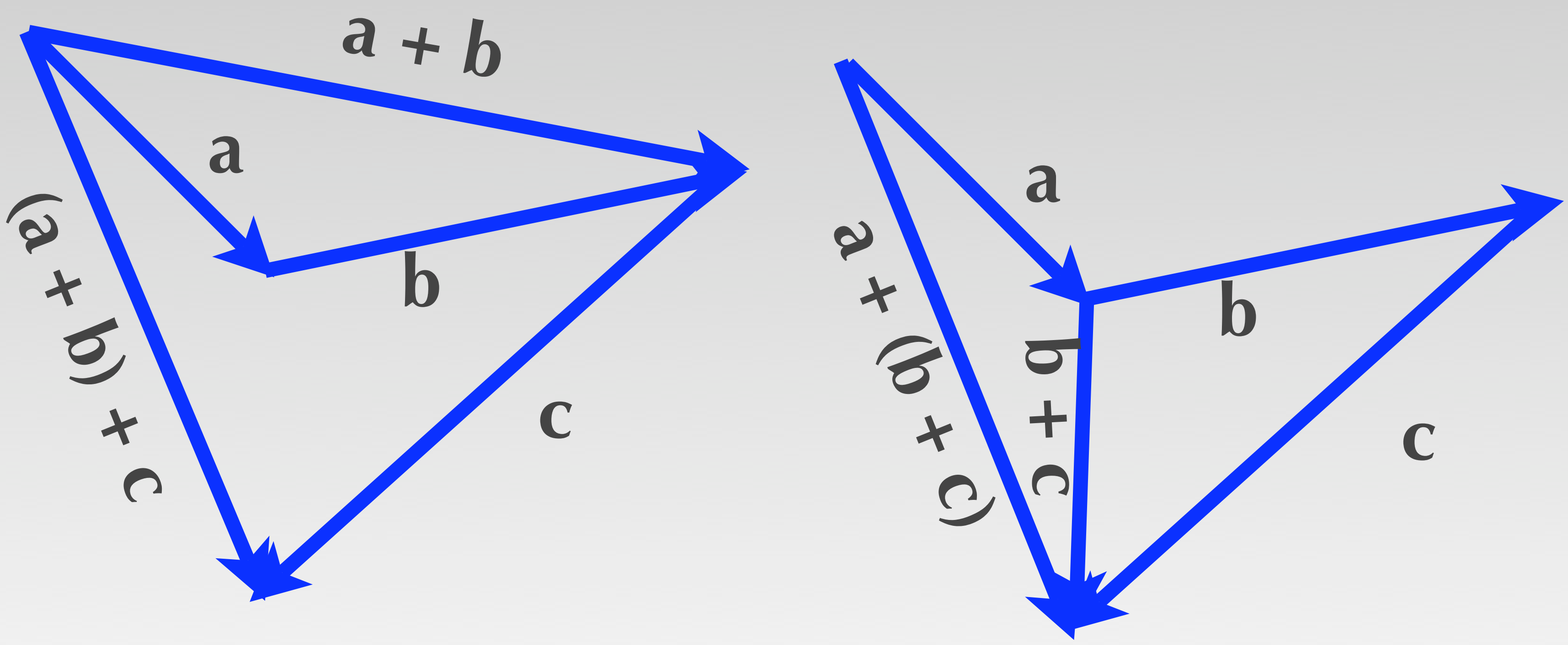
- **b + a** has the same length as **a + b**
- **b + a** points in the same direction as **a + b**
- Therefore: **b + a = a + b**

Commutativity

- Vector addition is commutative.
- There are quantities which have both direction and magnitude, but don't add commutatively. ($\boldsymbol{a}+\boldsymbol{b} \neq \boldsymbol{b}+\boldsymbol{a}$)
- These quantities are NOT vectors.
- (stay tuned)

Associativity

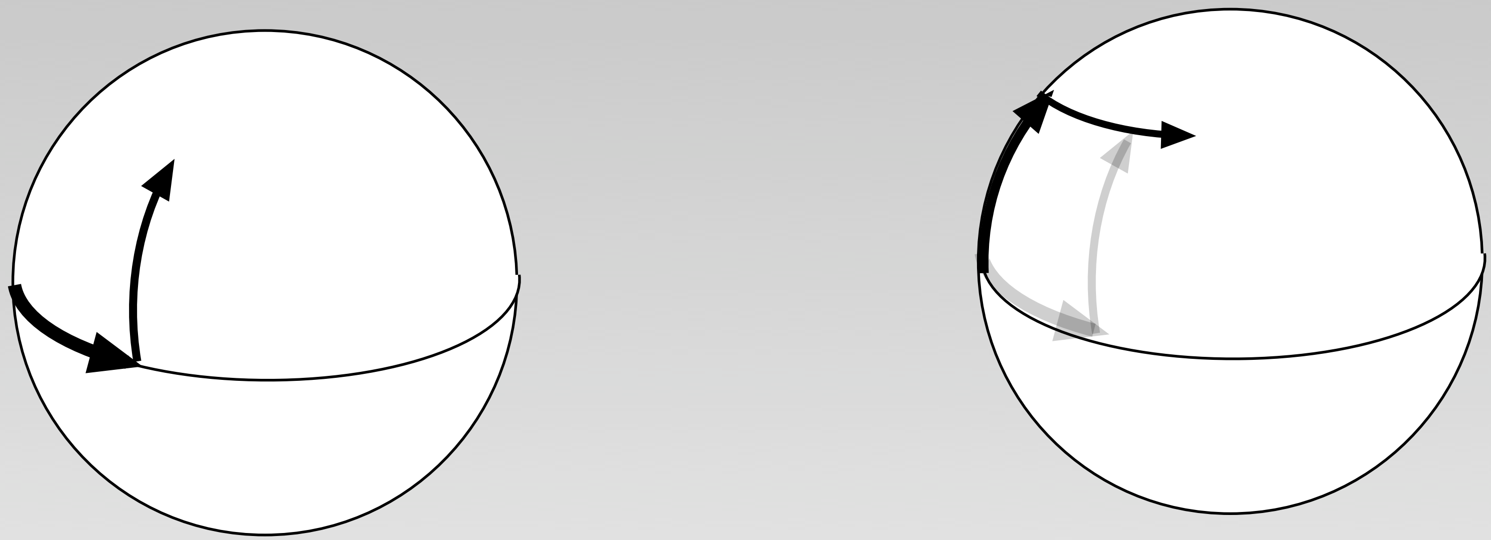
a + b + c



When 3 or more vectors are added,
it doesn't matter what order you add them.

(a + b) + c = a + (b + c)

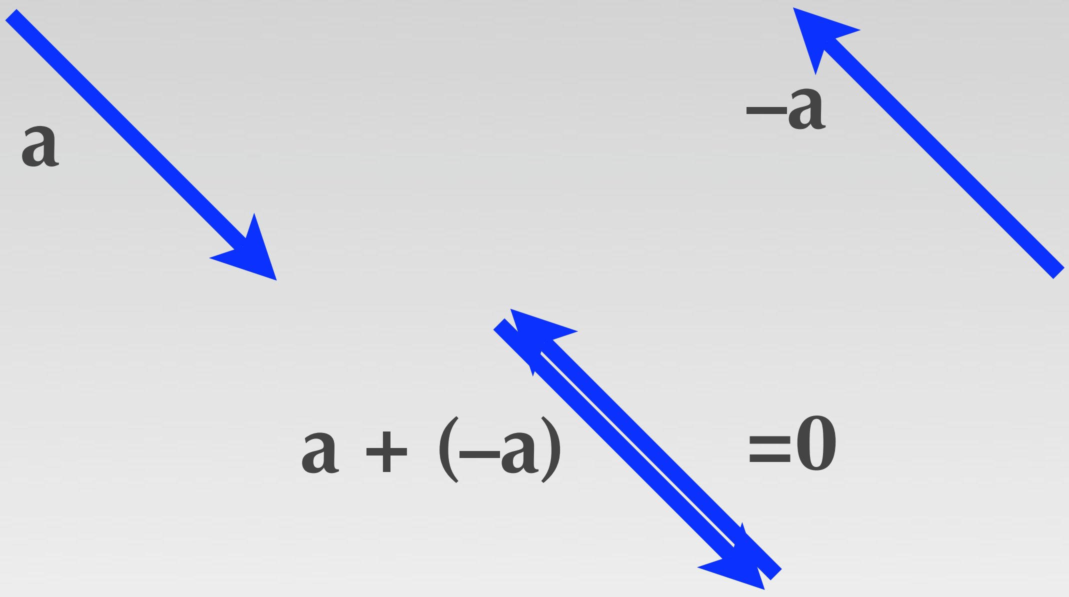
Non-commutativity



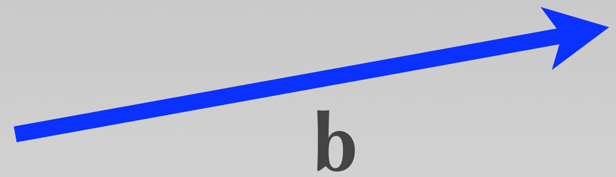
- Displacements on a sphere don’t commute.

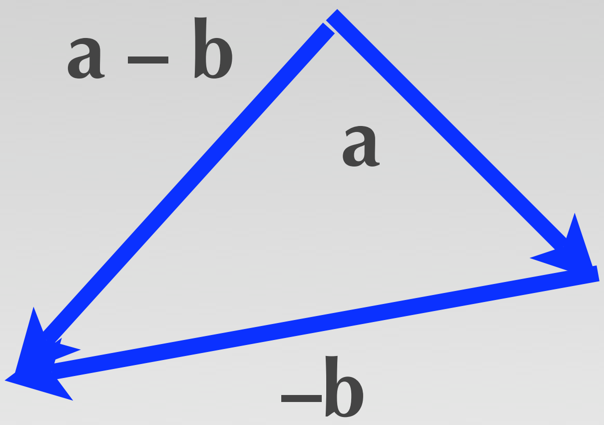
Negative

Same magnitude, opposite direction.



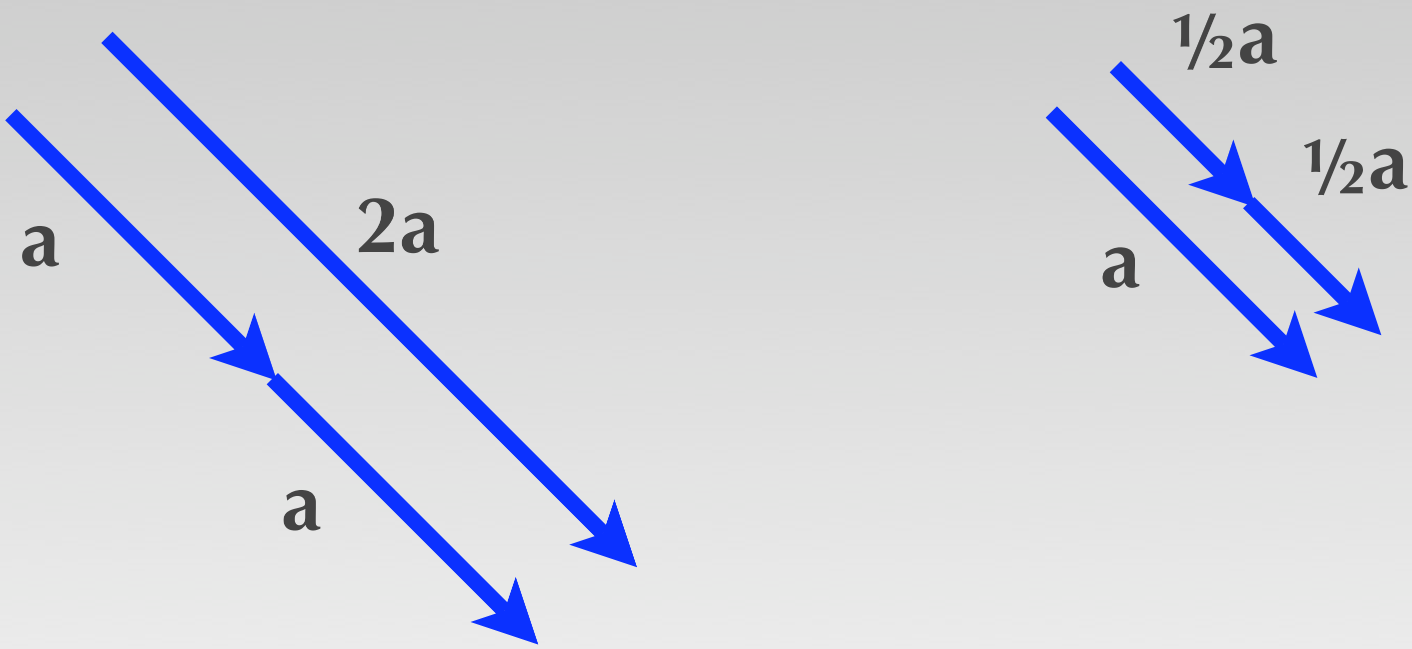
Subtraction





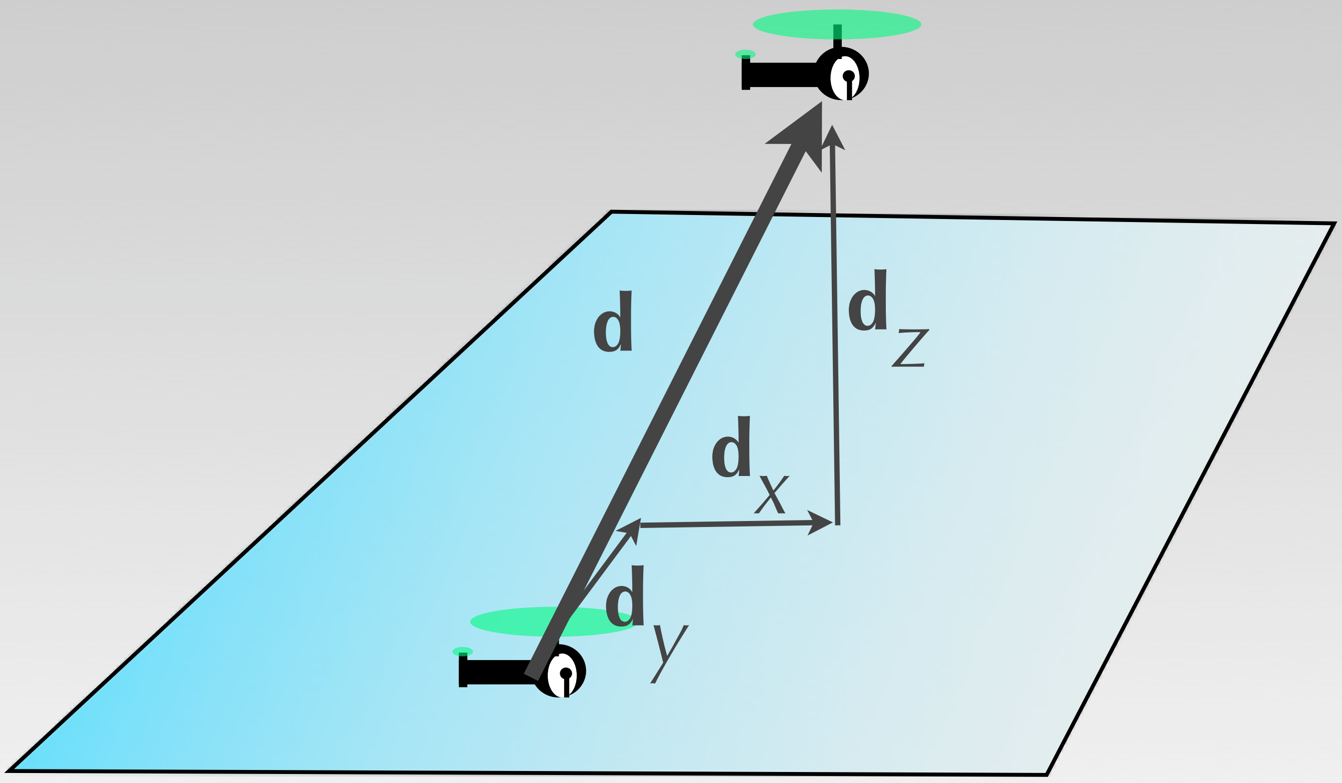
$$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$$

Multiplication by a Scalar

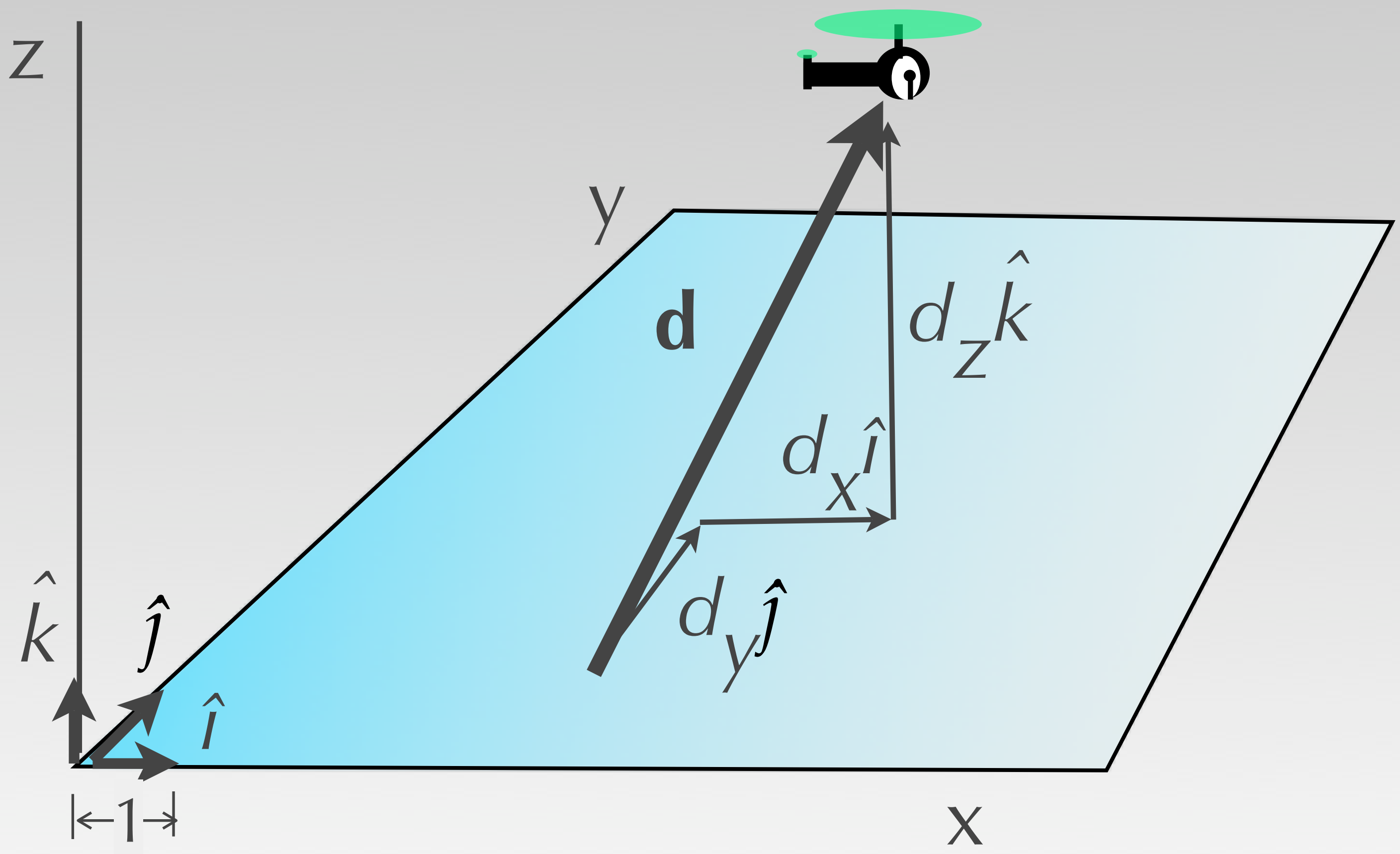


- scales the length but doesn't change direction

Vectors in 3 Dimensions

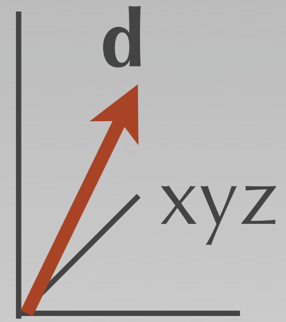


Unit vectors

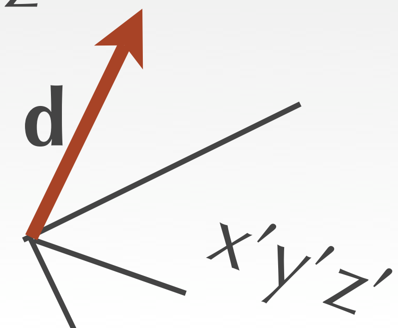


$$\vec{d} = d_x\hat{\imath} + d_y\hat{\jmath} + d_z\hat{k}$$

Vector Components

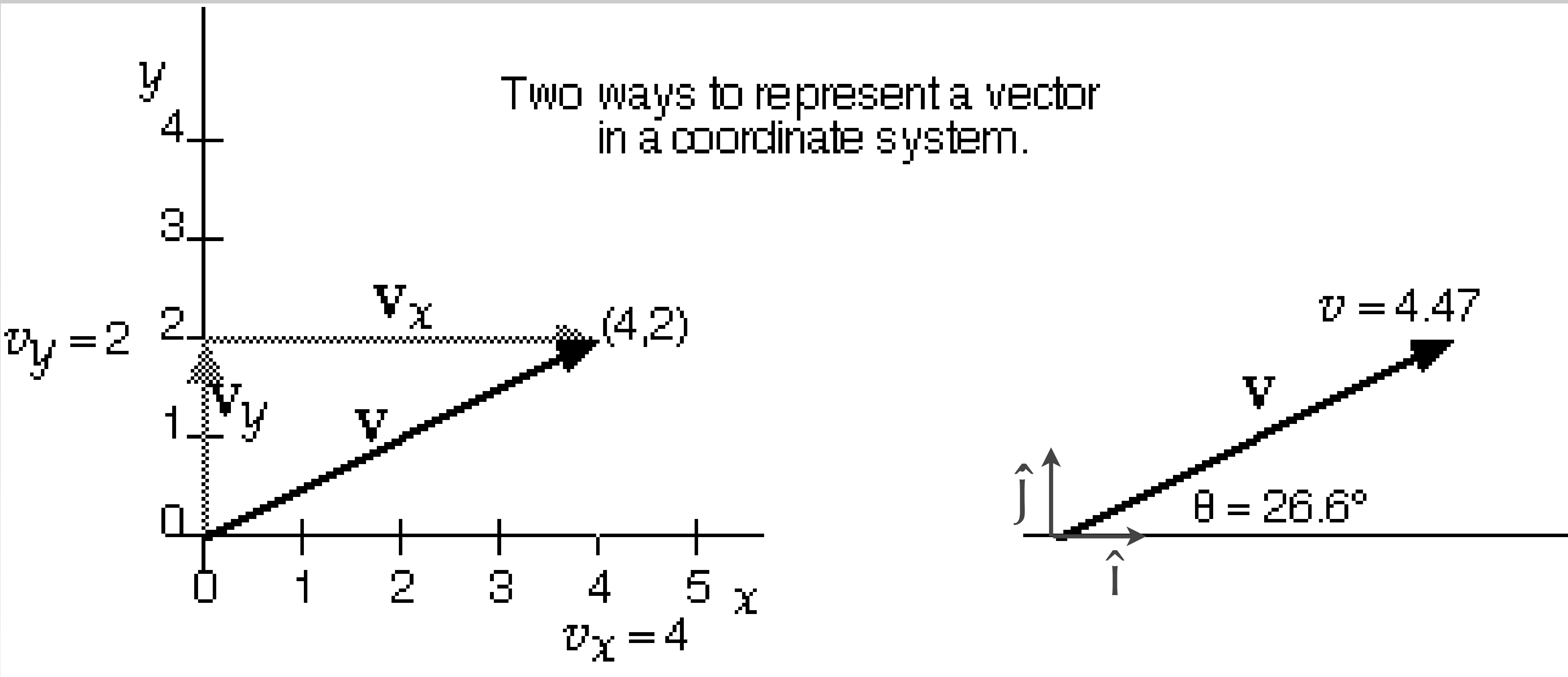


- Given that we know the coordinate system, x,y,z
- the three scalars d_x , d_y and d_z define the vector **d**.
- They are called the <u>vector components</u> of **d** in the xyz coordinate system.
- If the coordinate system changes to x′y′z′ but **d** stays the same, the numbers d_x , d_y and d_z change to $d_{x'}$, $d_{y'}$, and $d_{z'}$.



Calculating with Vectors

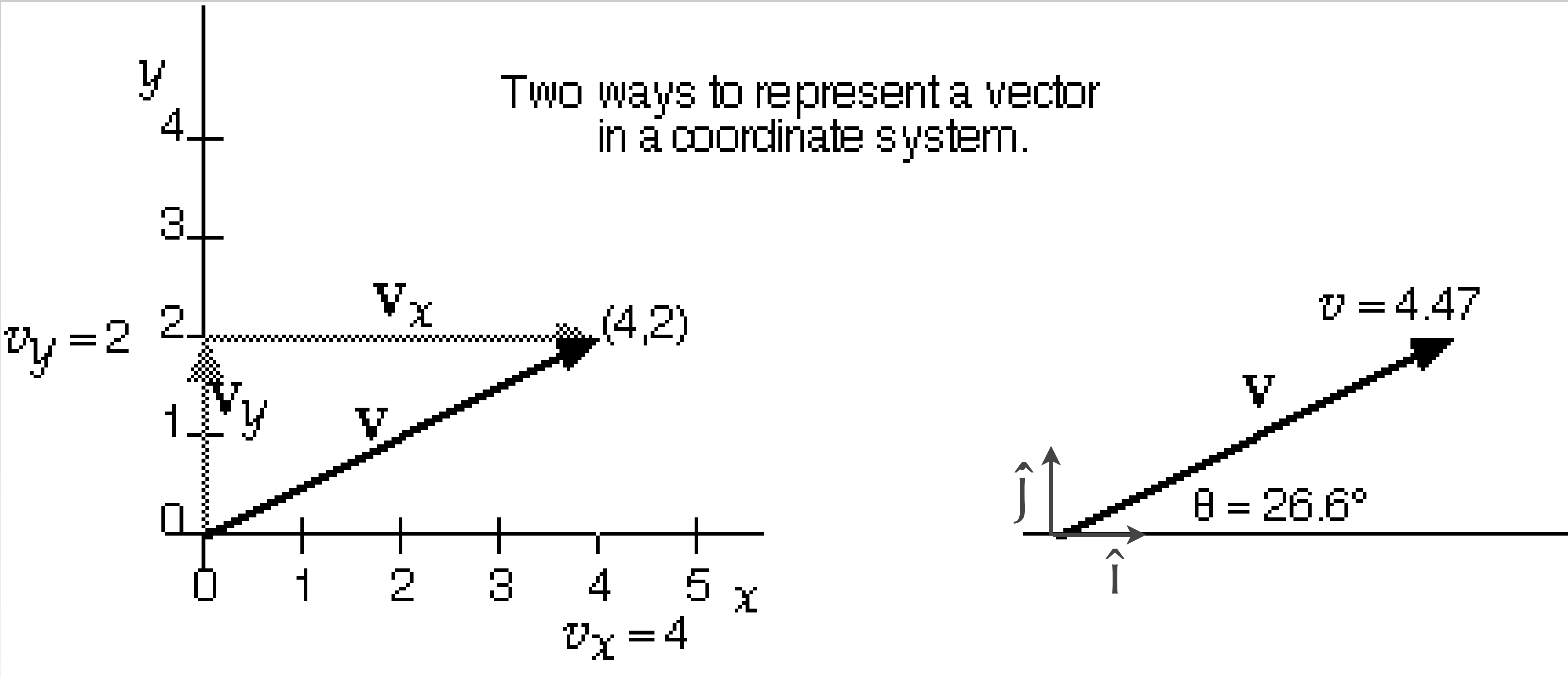
- Cartesian vs. Polar coordinates



$$\mathbf{v} = \mathbf{v}_x + \mathbf{v}_y = v_x\hat{\imath} + v_y\hat{\jmath}$$

Calculating with Vectors

Magnitude and angle in terms of components

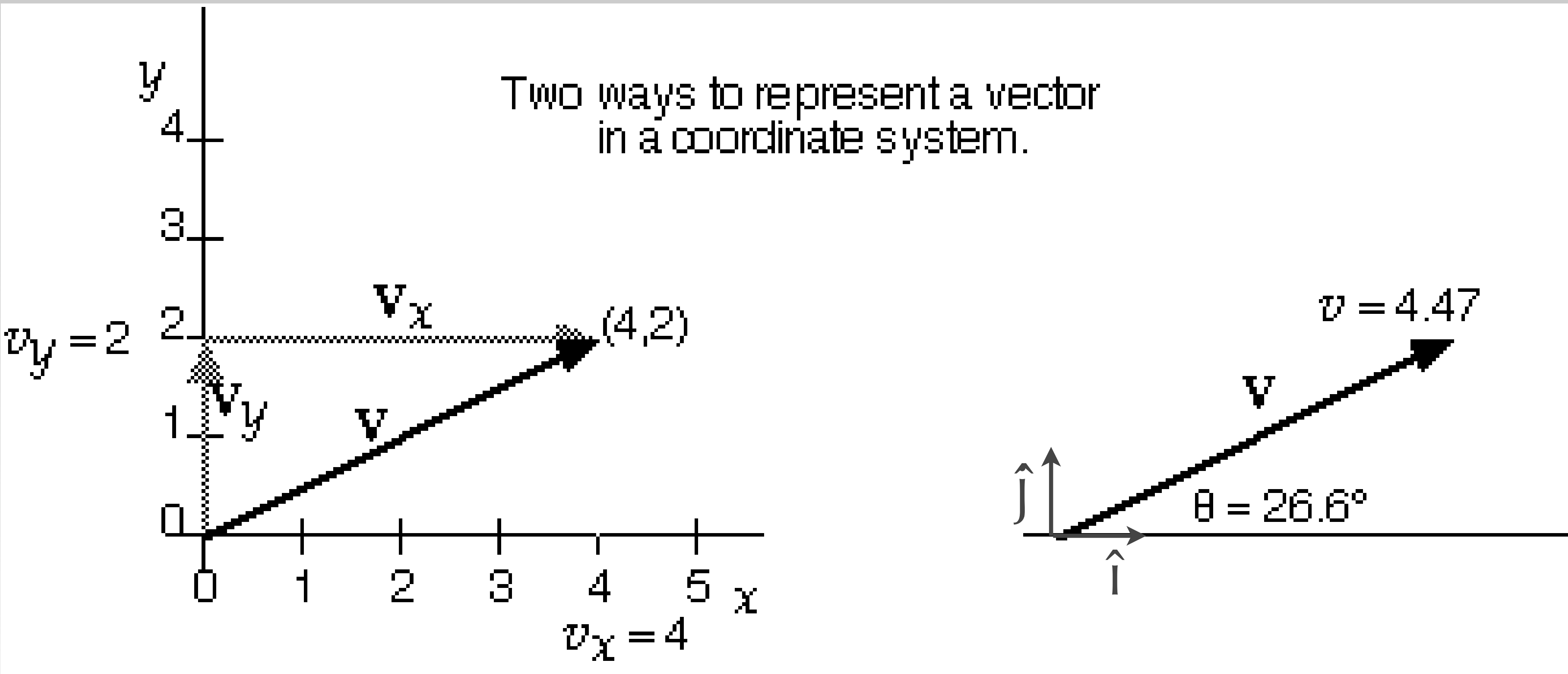


$$v^2 = v_x^2 + v_y^2$$

$$\theta = \tan^{-1}(v_y/v_x)$$

Calculating with Vectors

Components in terms of Magnitude and angle



$$v_x = v \cos \theta$$
$$v_y = v \sin \theta$$

Vector Strategies

1. Draw an accurate scale drawing. You can multiply by scalars, add and subtract graphically, taking care to be as exact as possible. Measure the magnitudes and directions of the resulting vectors.

2. Use trigonometry.

   - Pythogoras works for right triangles

   - Sine law and cosine law for other triangles

3. Use a rectangular coordinate system

   - Add or subtract components of vectors to find the components of the sum or the difference.