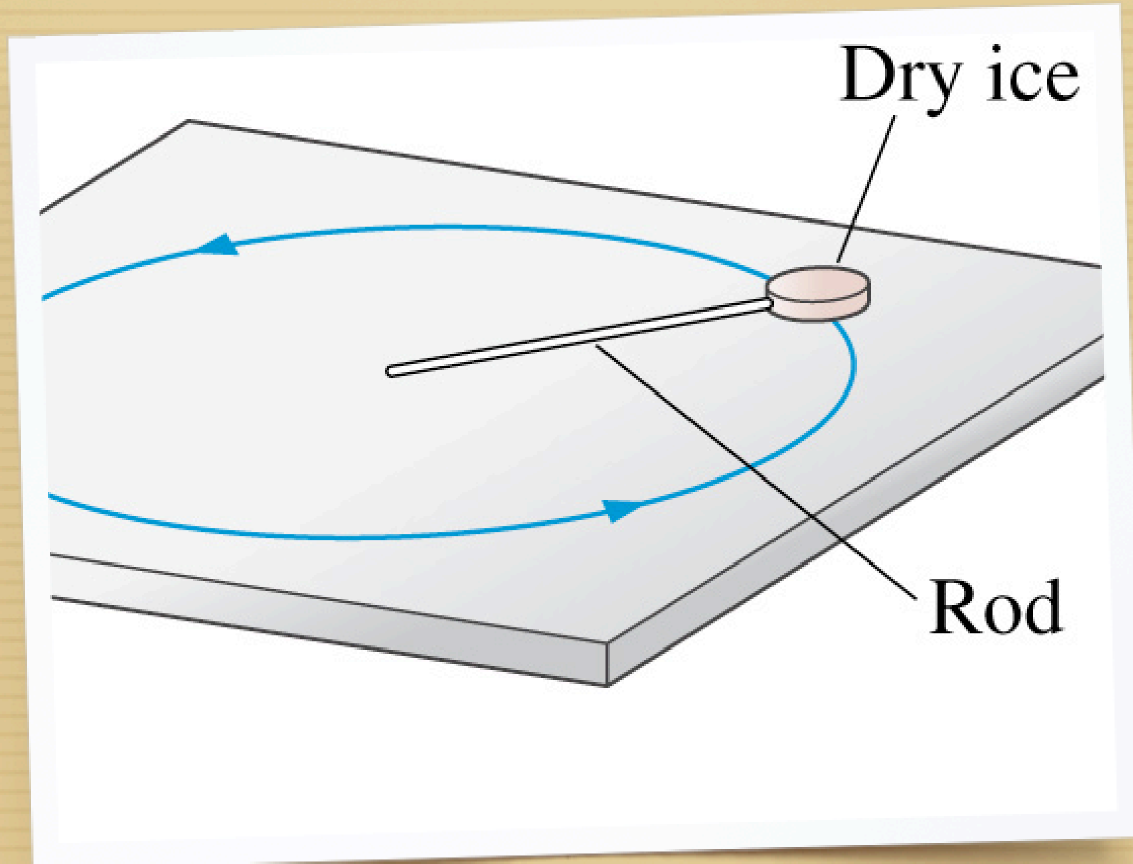
Work & Energy

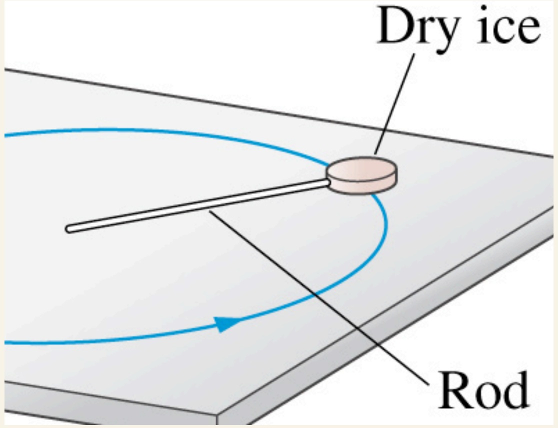
Stop to Think 9.6



A dry ice puck revolves in a circle on the end of a lightweight rigid rod that turns on frictionless bearings. A cushion of CO_2 gas allows the puck to glide across the surface without friction. As the puck sublimates, does its speed increase, decrease or stay the same?

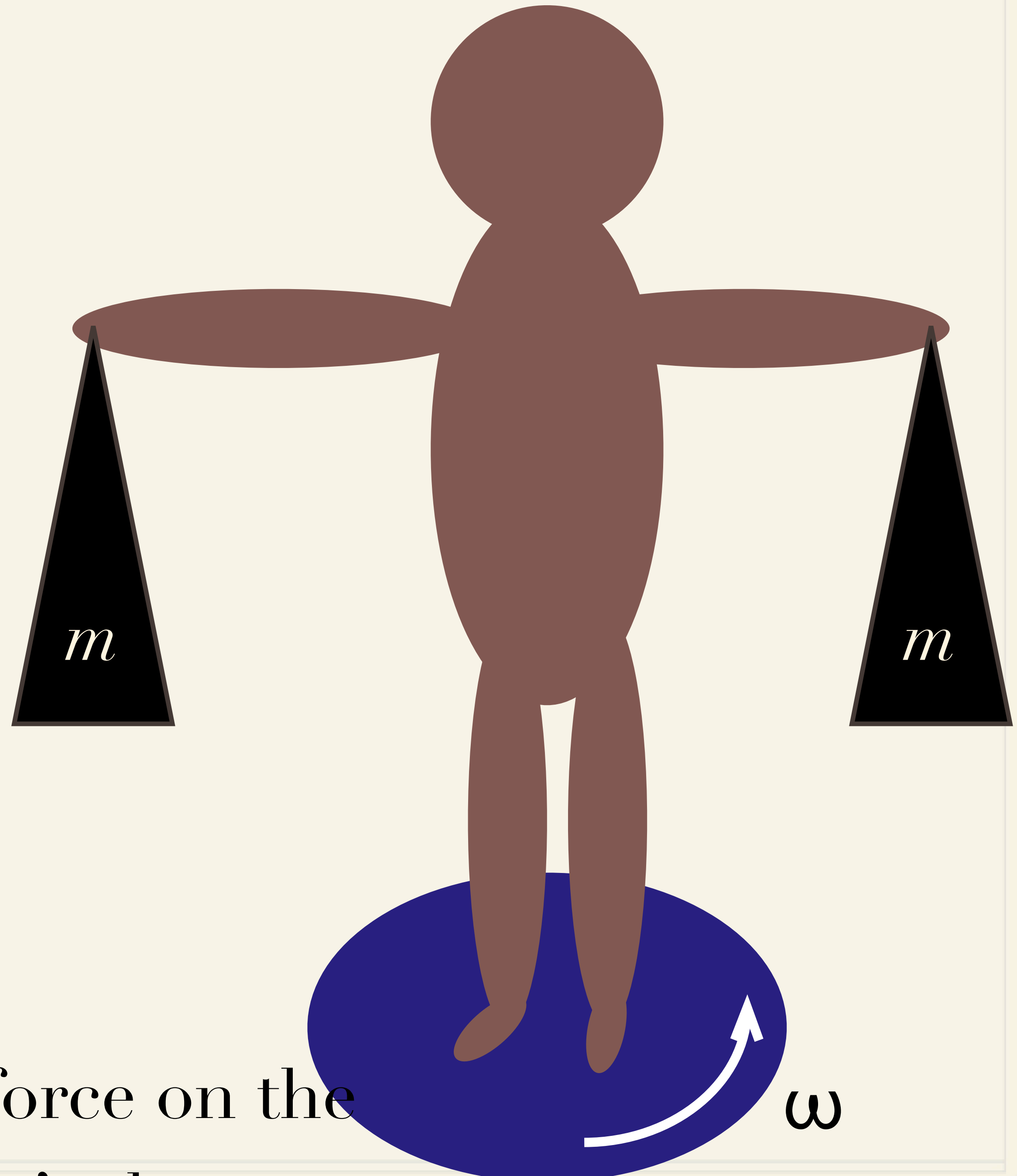
The book says......

- “There are no tangential forces on the puck, so its angular momentum, $L=mrv$, is conserved. The tangential velocity has to increase as m decreases to keep L constant.”
- What do YOU think?



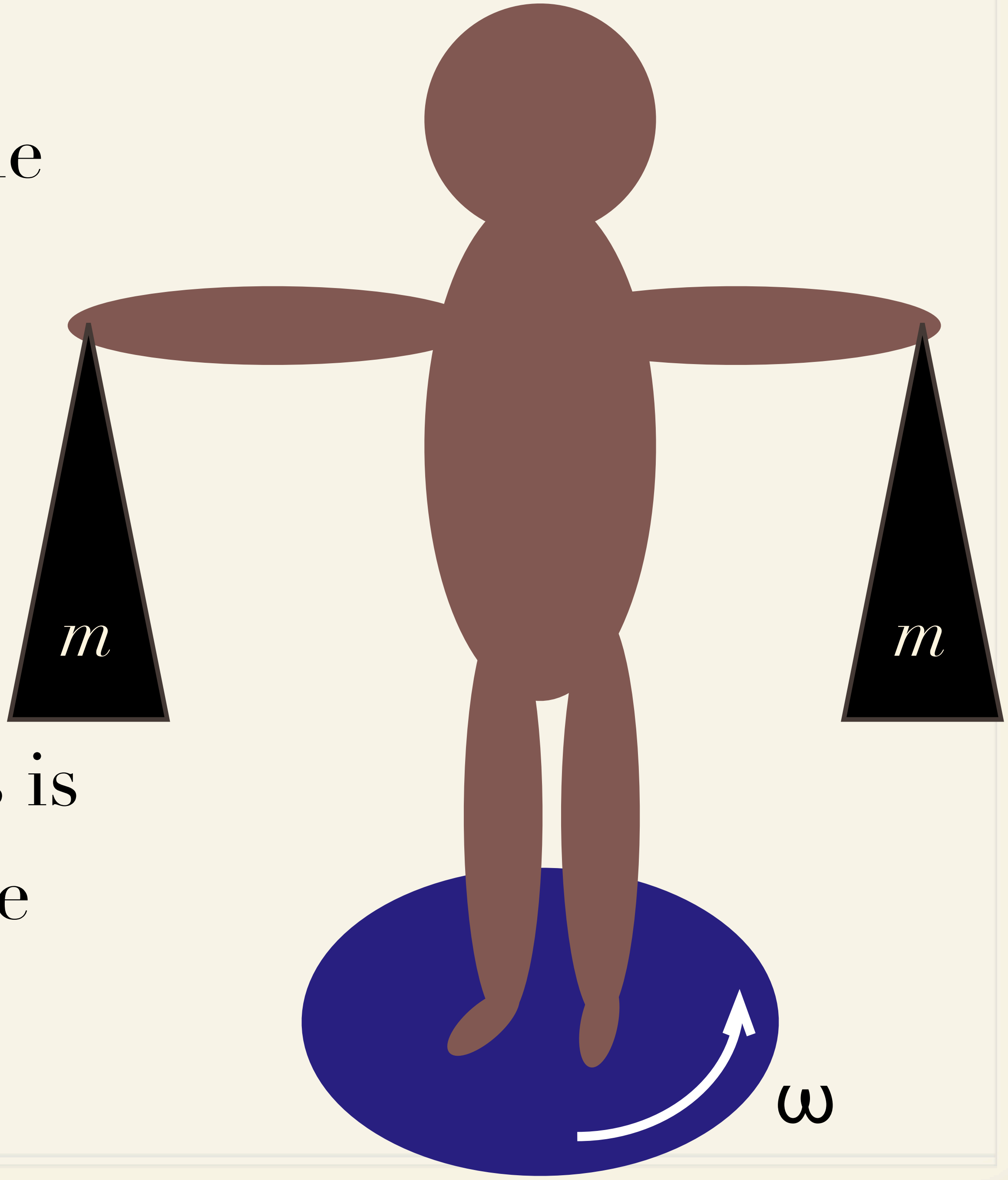
Think about this....

- A person holding two masses at arms length is rotating on a platform.
- What happens to his angular velocity if he drops the masses?



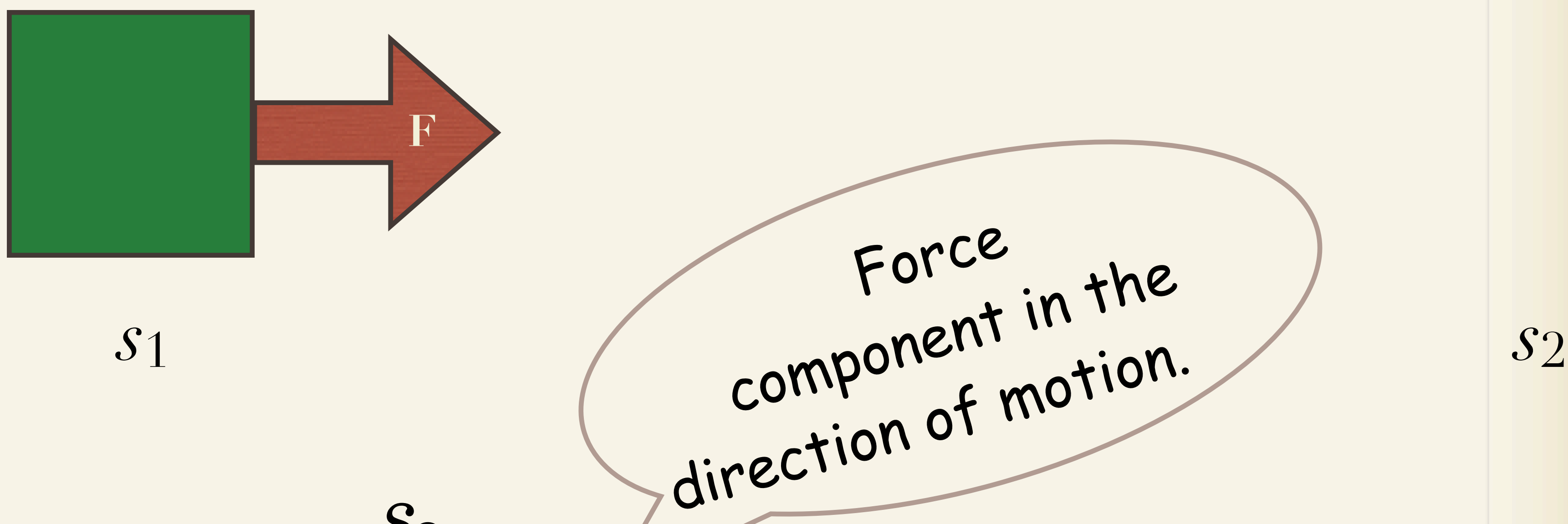
There is no tangential force on the person, his motion won't change.

- When he drops the masses they continue to have angular momentum.
- The angular momentum of the man and the masses is conserved—until the masses hit the ground.



- The dry ice puck is the similar to the rotating man dropping the weights.
- When the CO_2 sublimates, each molecule has angular momentum.
- The angular momentum of the solid puck alone is not conserved because it is not the whole system.
- The book is wrong.

Work — definition



$$W = \int_{s_1}^{s_2} F_{\parallel} ds$$

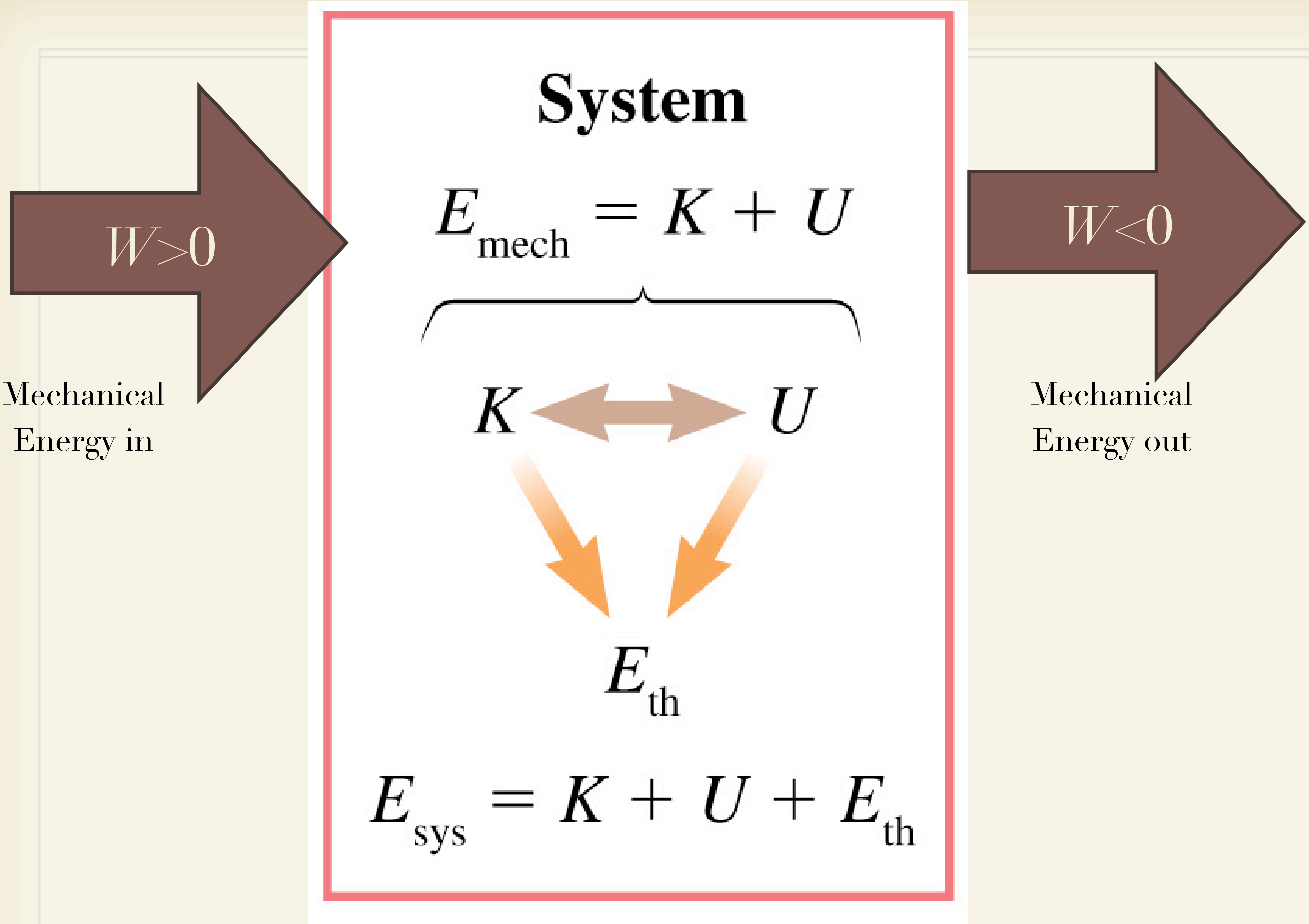
Work—compare to Impulse

$$\vec{J} = \int_{t_1}^{t_2} \vec{F} dt$$

vector

$$W = \int_{s_1}^{s_2} F ds$$

scalar

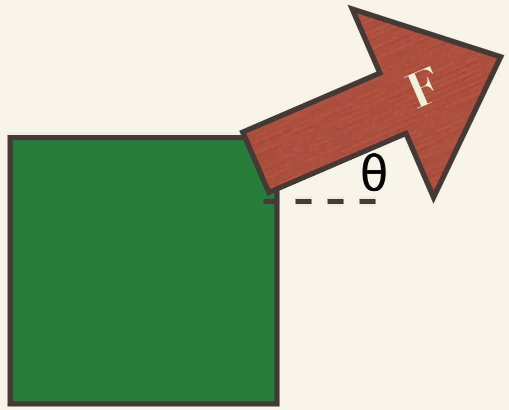

System
$E_{mech} = K + U$
K
U
E_{th}
$E_{sys} = K + U + E_{th}$
$W > 0$
Mechanical
Energy in
$W < 0$
Mechanical
Energy out

Examples

- Piston
- Wind-up clock
- Battery clock
  - putting in a battery isn’t mechanical work.
- Hot cup of coffee
  - Heat flow is not mechanical work

Work—definition

When force is not in the direction of motion.



s_1

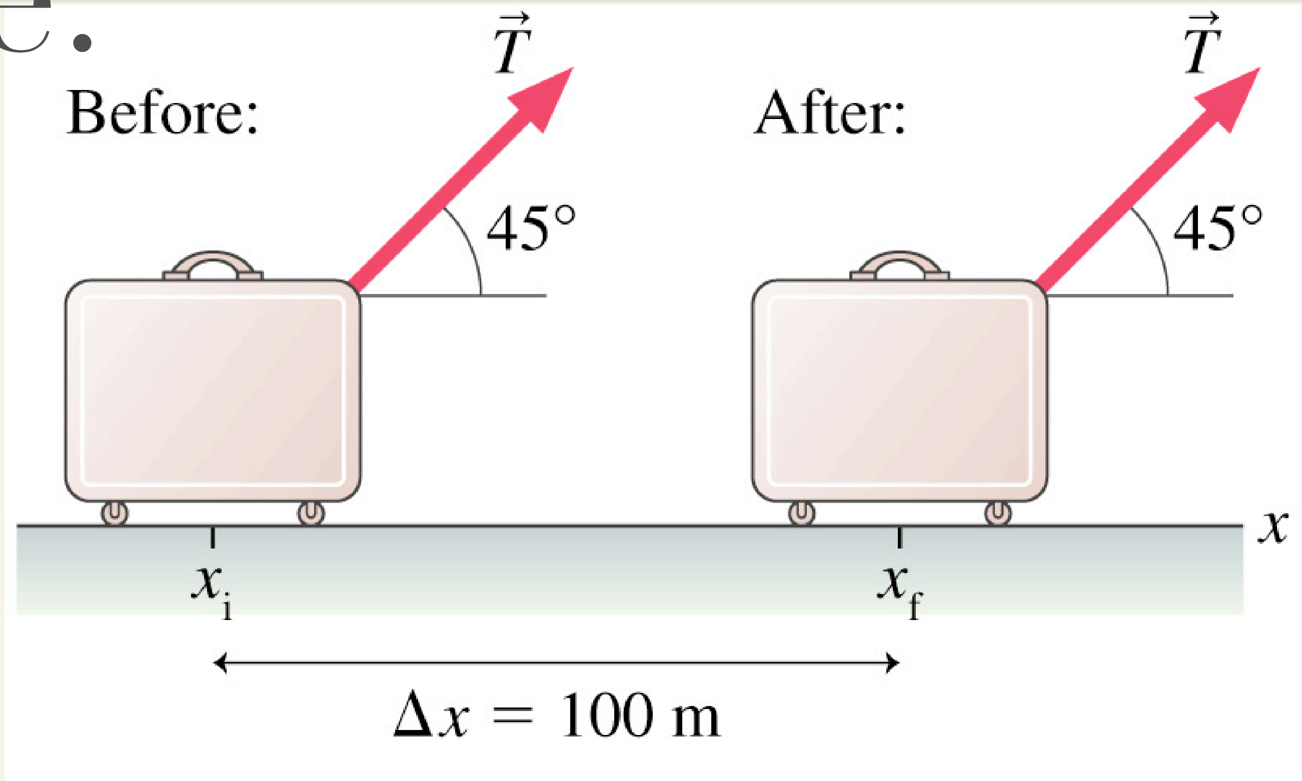
s_2

Force component in the direction of motion.

$$W = \int_{s_1}^{s_2} F_{\parallel} ds$$

$F_{||} = F \cos\theta$

Example:

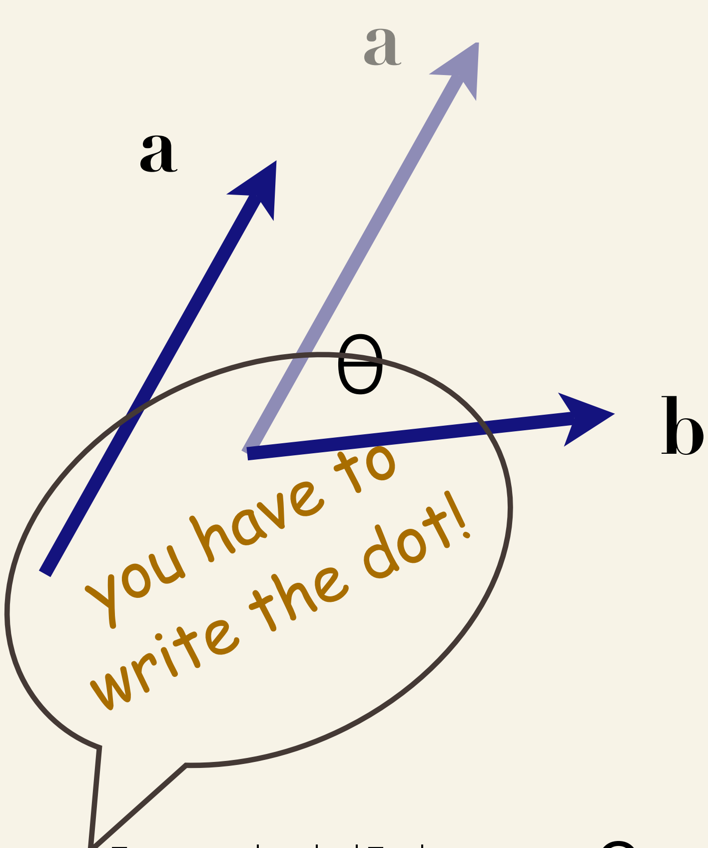


$$W = T\Delta x \cos\theta$$

$$= (20\ \text{N})(100\ \text{m})\cos 45^\circ$$

$$= 1410\ \text{J}$$

Dot Product

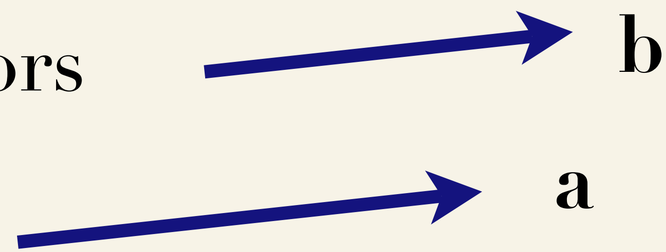


$$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}|\ |\mathbf{b}| \cos \theta$$

The result is a scalar

Dot Product

parallel vectors



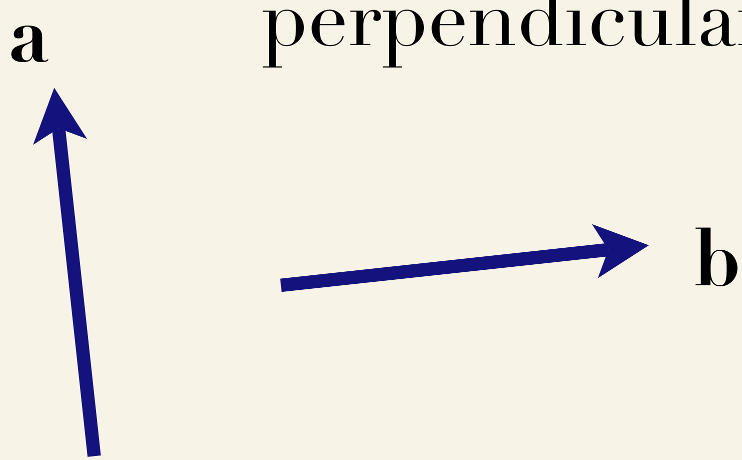
$\theta = 0$

$$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}|\ |\mathbf{b}| \cos\theta = |\mathbf{a}|\ |\mathbf{b}|$$

or ab

Dot Product

perpendicular vectors



$\theta = 90^\circ = \pi/2$

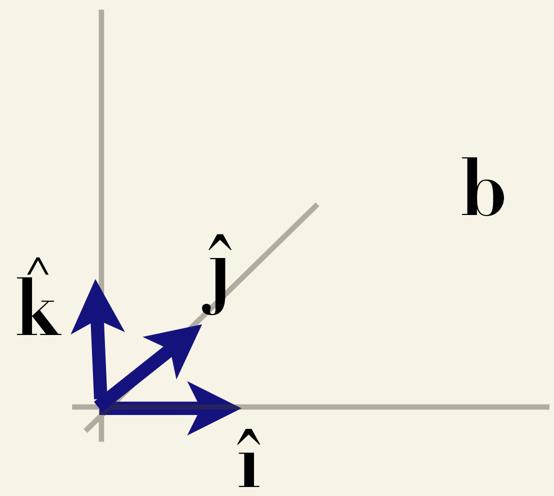
$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta = 0$

Dot product

When is the dot product negative?

Dot Product

unit vectors



$\hat{\mathbf{i}} \bullet \hat{\mathbf{j}} = 0$

$\hat{\mathbf{i}} \bullet \hat{\mathbf{k}} = 0$

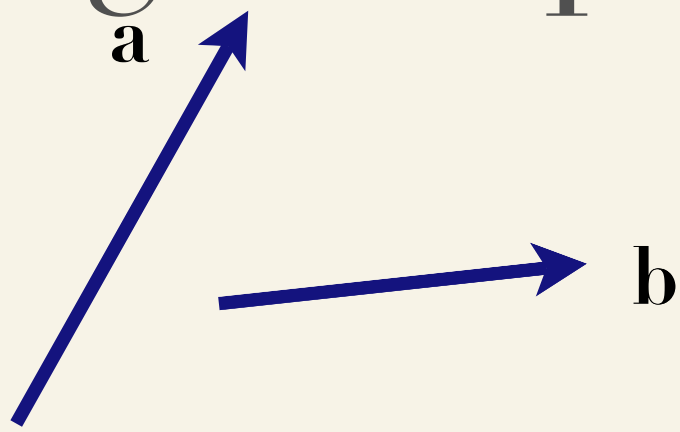
etc

$\hat{\mathbf{i}} \bullet \hat{\mathbf{i}} = 1$

$\hat{\mathbf{j}} \bullet \hat{\mathbf{j}} = 1$

$\hat{\mathbf{k}} \bullet \hat{\mathbf{k}} = 1$

Dot products using components

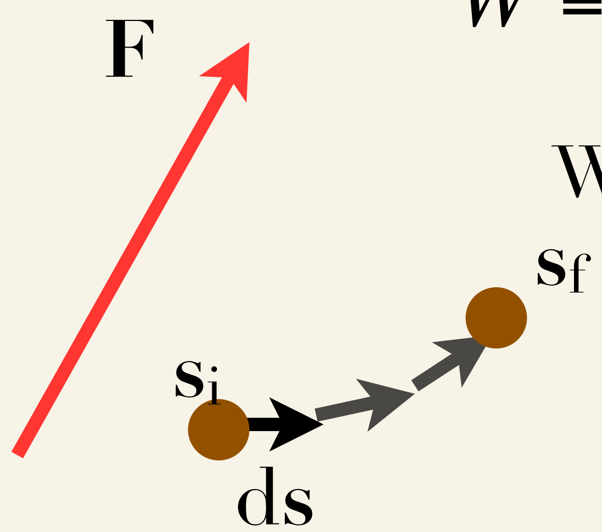


$$\mathbf{a} = a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}} + a_z\,\hat{\mathbf{k}} \qquad \mathbf{b} = b_x\,\hat{\mathbf{i}} + b_y\,\hat{\mathbf{j}} + b_z\,\hat{\mathbf{k}}$$

$$\mathbf{a} \bullet \mathbf{b} = a_x\,b_x + a_y\,b_y + a_z\,b_z$$

Work again

$$W = \int_{\mathbf{s}_i}^{\mathbf{s}_f} \mathbf{F} \bullet d\mathbf{s} \quad \text{or} \quad \int_{\vec{s}_i}^{\vec{s}_f} \vec{F} \bullet d\vec{s}$$



We can write this in component form:

$$\mathbf{F} = F_x \hat{\mathbf{i}} + F_y \hat{\mathbf{j}} + F_z \hat{\mathbf{k}}$$

$$\mathbf{s} = x\ \hat{\mathbf{i}} + y\ \hat{\mathbf{j}} + z\ \hat{\mathbf{k}}$$

$$d\mathbf{s} = dx\ \hat{\mathbf{i}} + dy\ \hat{\mathbf{j}} + dz\ \hat{\mathbf{k}}$$

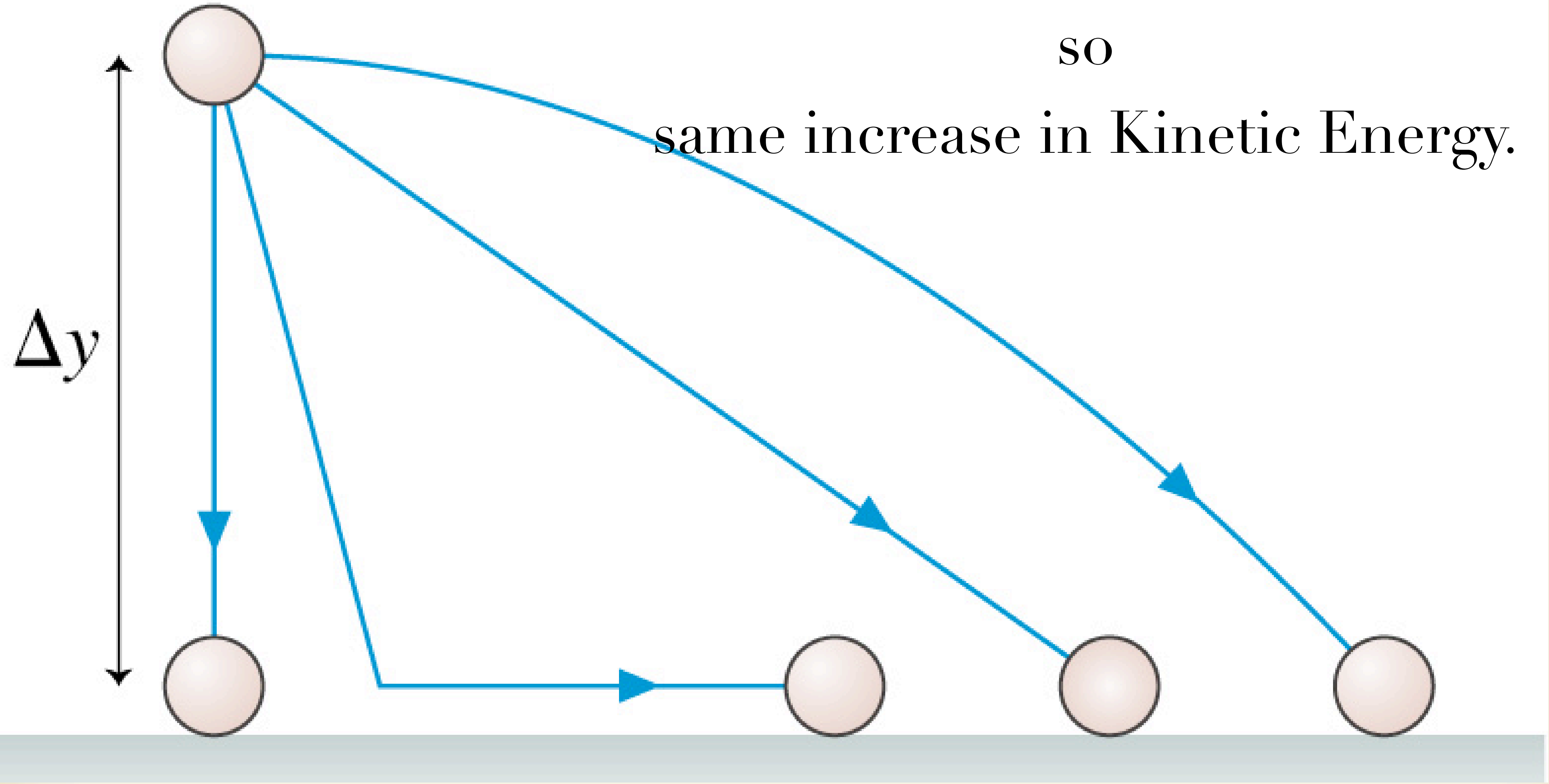
$$W = \int_{\mathbf{s}_i}^{\mathbf{s}_f} \mathbf{F} \bullet d\mathbf{s} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_x dy + \int_{z_i}^{z_f} F_z dz$$

Conservative Forces

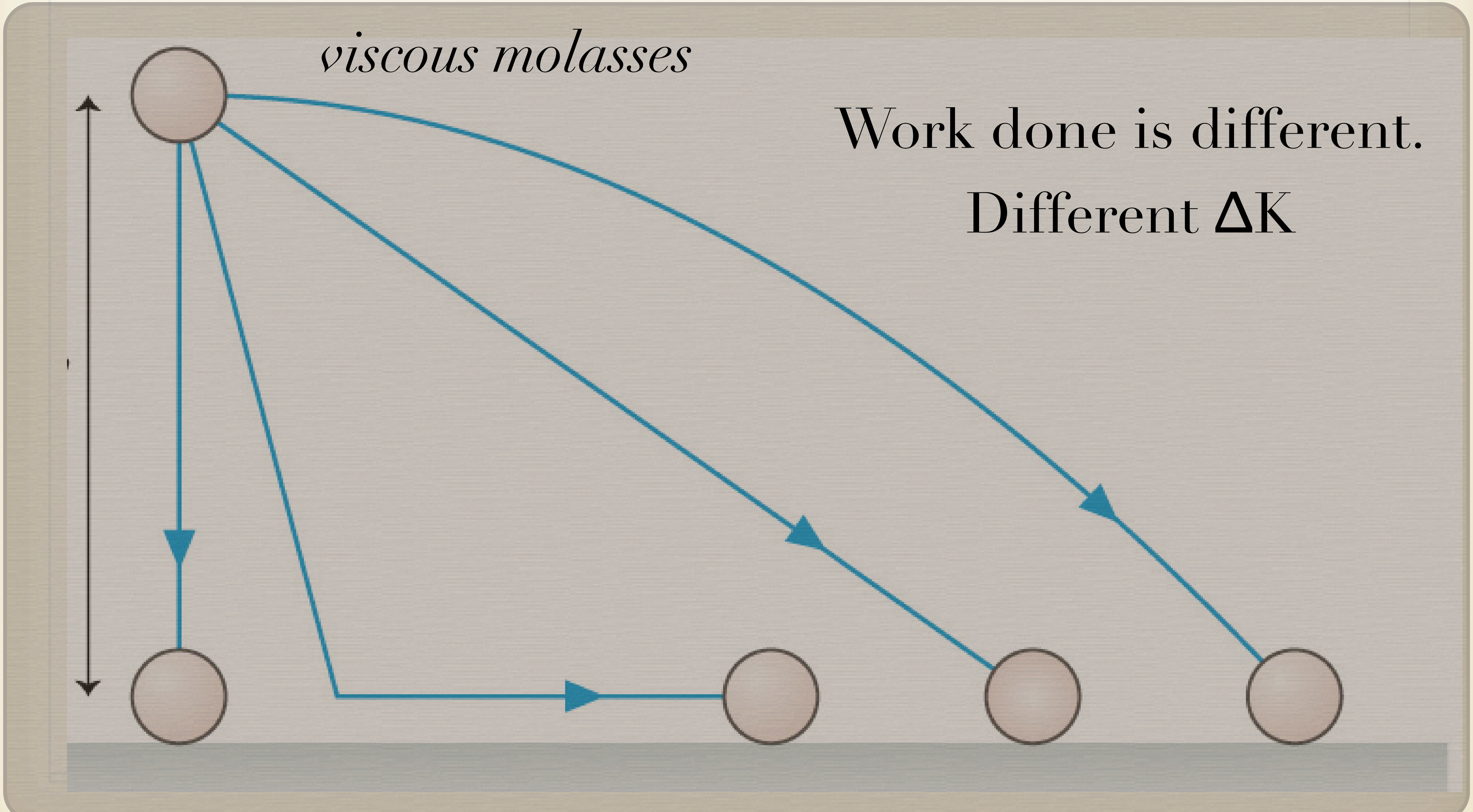
Gravity does the same work.

so

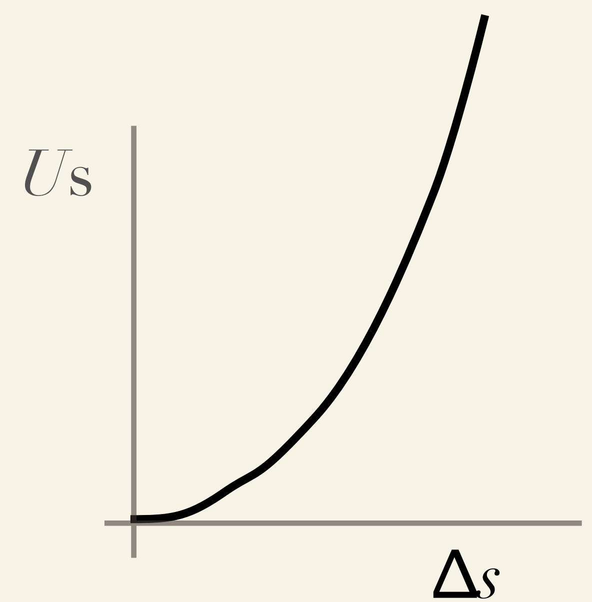
same increase in Kinetic Energy.



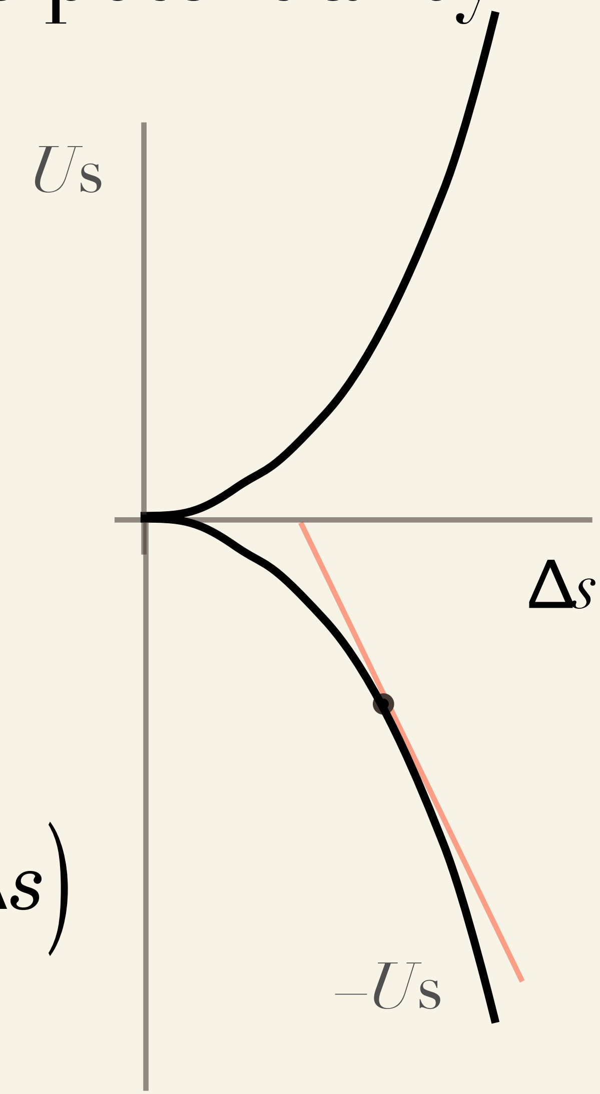
Non-conservative Forces



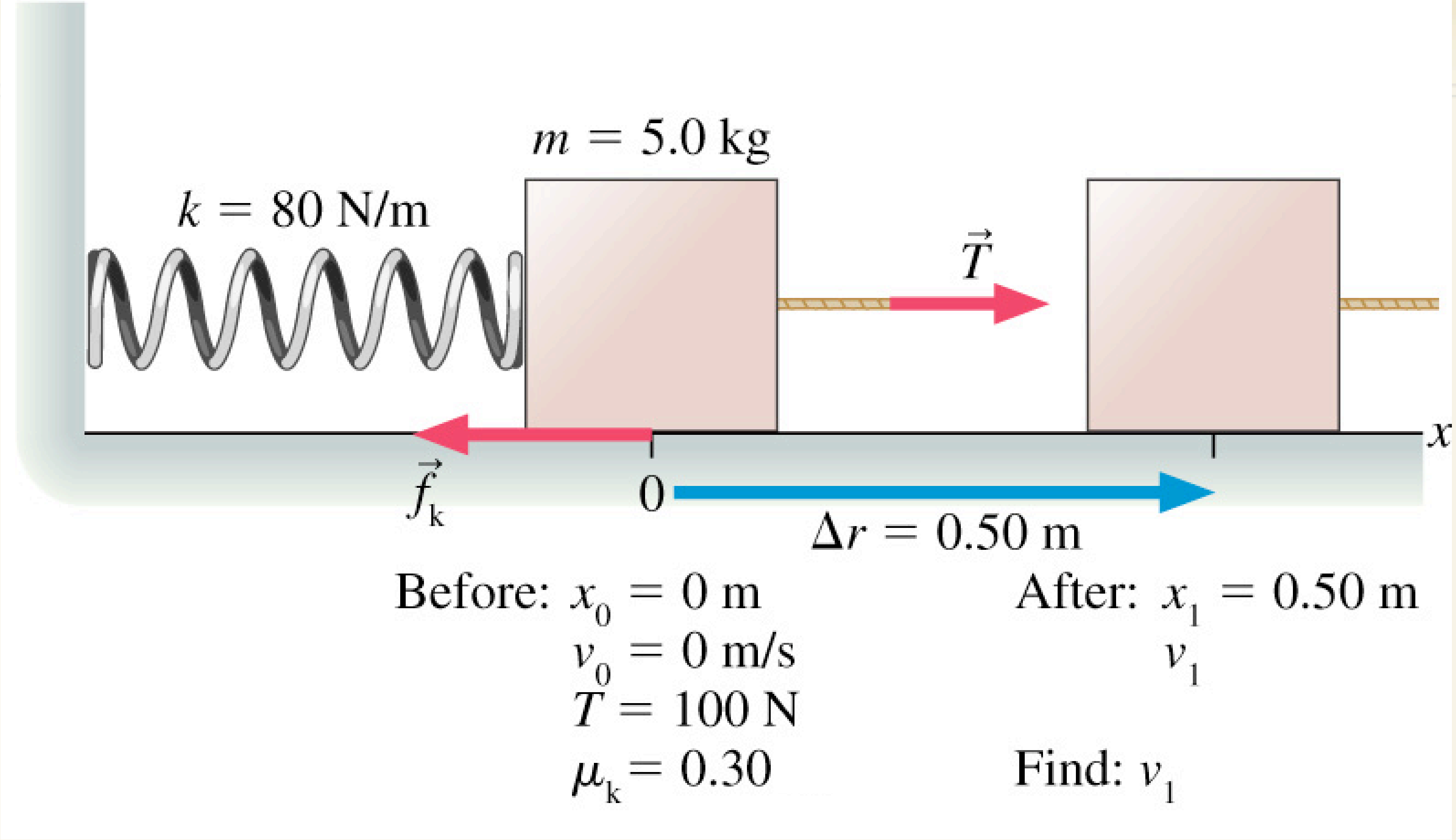
- Only if a force is conservative then potential energy can be defined for it.
- $U\text{g} = mgy$
- $U\text{s} = (^1/_2)\,(k\Delta s)^2$



- The force is got from the potential by differentiating $-U$.



$$F_s = -\frac{dU_s}{d(\Delta s)} = -\frac{d}{d(\Delta s)}\left[\frac{1}{2}k\left(\Delta s\right)^2\right] = -k\left(\Delta s\right)$$



Rope pulls box tied to the wall with a spring. What is the speed of the box after 0.5 m?

To find the answer, look in your textbook.