Classical Mechanics Lecture 7

Today's Concepts:

Work & Kinetic Energy

Notices

Midterm Exam –Friday Feb 8–

will cover stuff we do until today.

10 multiple choice + 2 problems, 2 or 3 parts each

Bring pencil for the multiple choice bubbles

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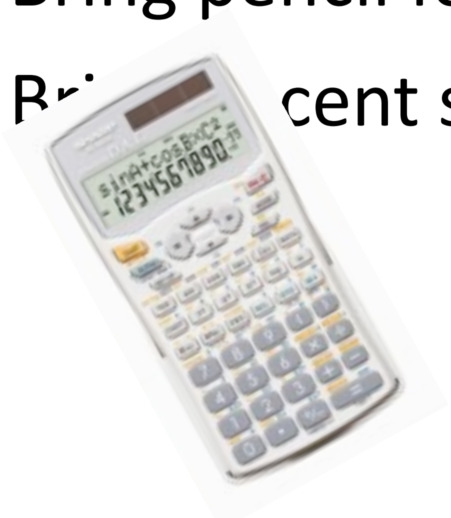
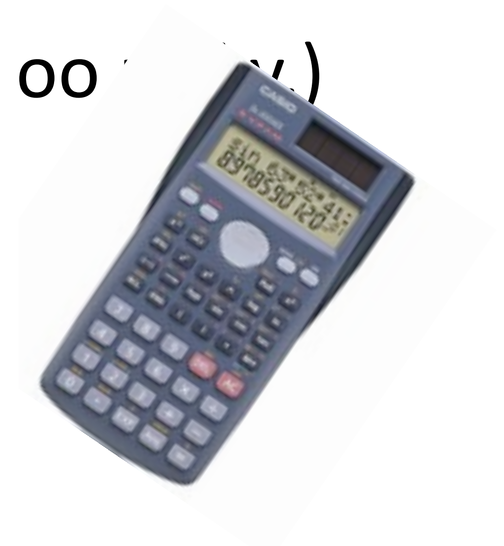




Stuff to study:
https://wiki.sfu.ca/public/index.php/Physics_self_help

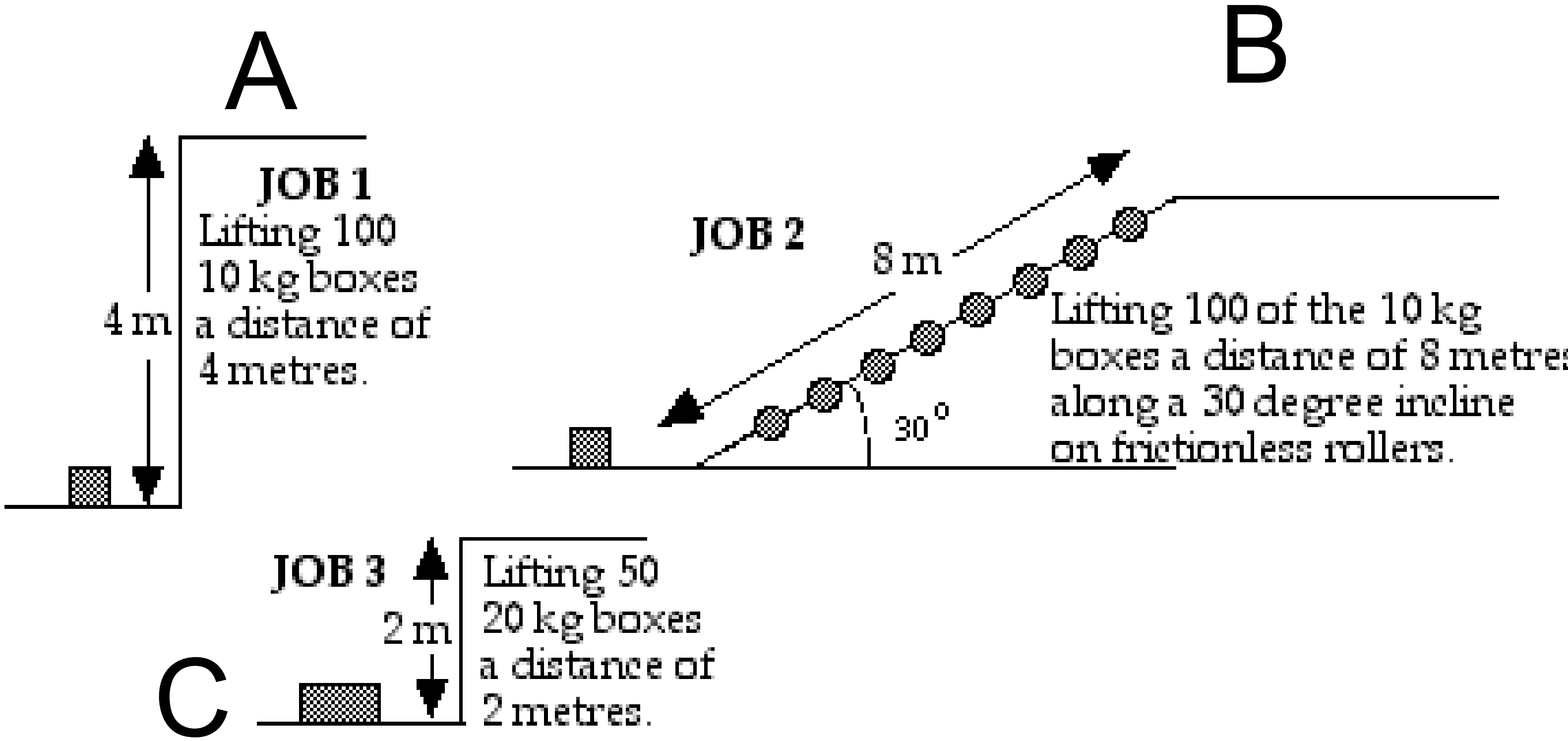
Which Job?

All three jobs pay the same amount of money. Which one would you choose for your crew?

A. Job 1

B. Job 2

C. Job 3



Stuff you asked about:

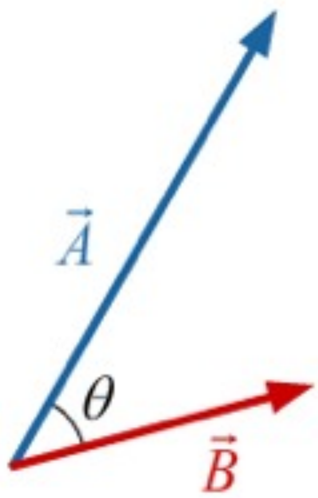
When a third grader was asked to cite Newton's first law, she said, "Bodies in motion remain in motion, and bodies at rest stay in bed unless their mothers call them to get up."

I don't really understand the last part of the prelecture, which is about the work done by gravity far from the earth.

integrals?? How do you integrate W without its function?

Physics quote of the day: Anything that doesn't matter has no mass.

The Dot Product



Dot Product

$$\boxed{\vec{A}\cdot\vec{B} \equiv AB\cos\theta}$$

$\vec{A}\cdot\vec{B} = AB$ (Parallel Vectors)

$\vec{A}\cdot\vec{B} = 0$ (Perpendicular Vectors)

$\vec{A}\cdot\vec{B} = -AB$ (Anti-Parallel Vectors)

Work-Kinetic Energy Theorem

The **net** work done by the **net** force F as it acts on an object that moves between positions r_1 and r_2 is equal to the change in the object's kinetic

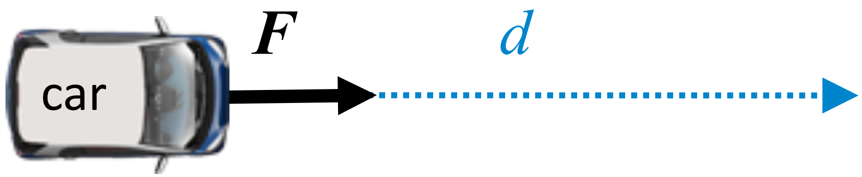
$$W = \Delta K$$

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{l}$$

$$K = \frac{1}{2} m v^2$$

Work-Kinetic Energy Theorem: 1-D Example

If the force is constant and the directions aren't changing then this is very simple to evaluate:



$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{l} = \vec{F} \cdot \vec{d}$$

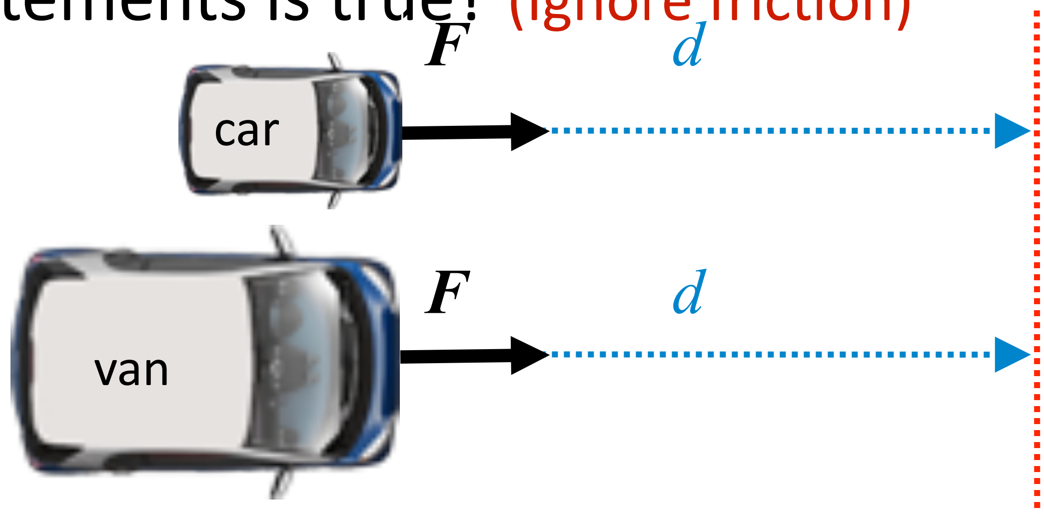
$$\left(\int_{\vec{r}_1}^{\vec{r}_2} d\vec{l} = \vec{d} \right)$$

In this case $= Fd$ since $\cos(0)=1$

This is probably what you remember from High School.
(The notation may be confusing though.)

Clicker Question

A lighter car and a heavier van, each initially at rest, are pushed with the same constant force F. After both vehicles travel a distance d, which of the following statements is true? (Ignore friction)

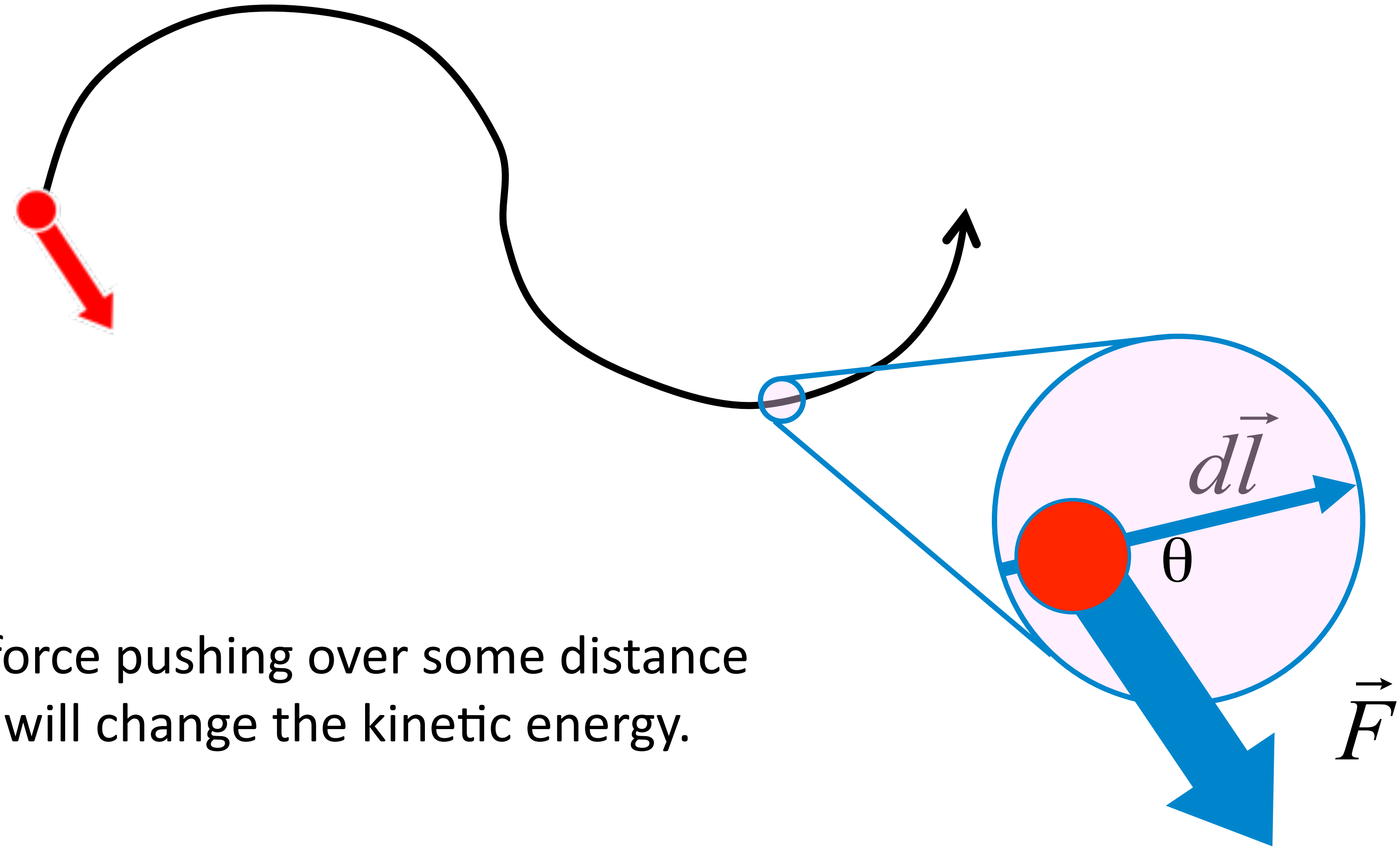


A) They will have the same velocity
B) They will have the same kinetic energy
C) They will have the same momentum

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{l} = \Delta K$$

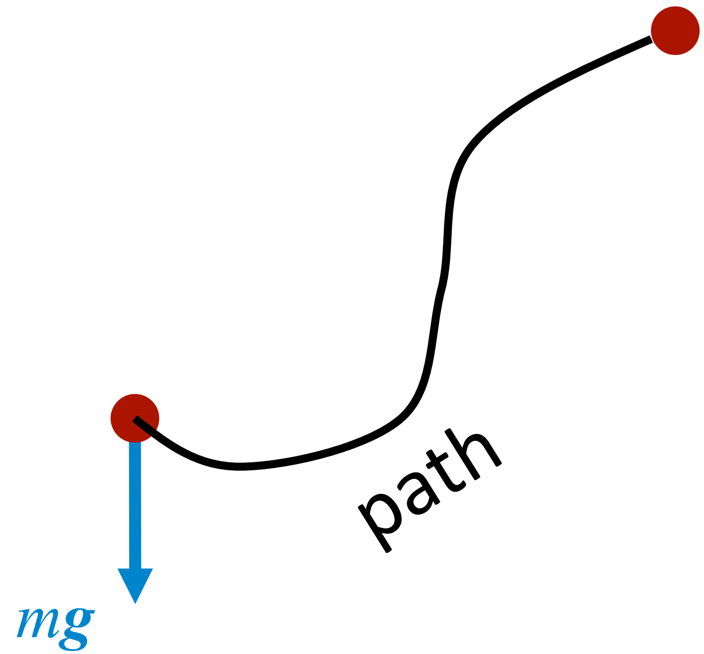
Derivation – not so important

<u>Concept – very important</u>



A force pushing over some distance will change the kinetic energy.

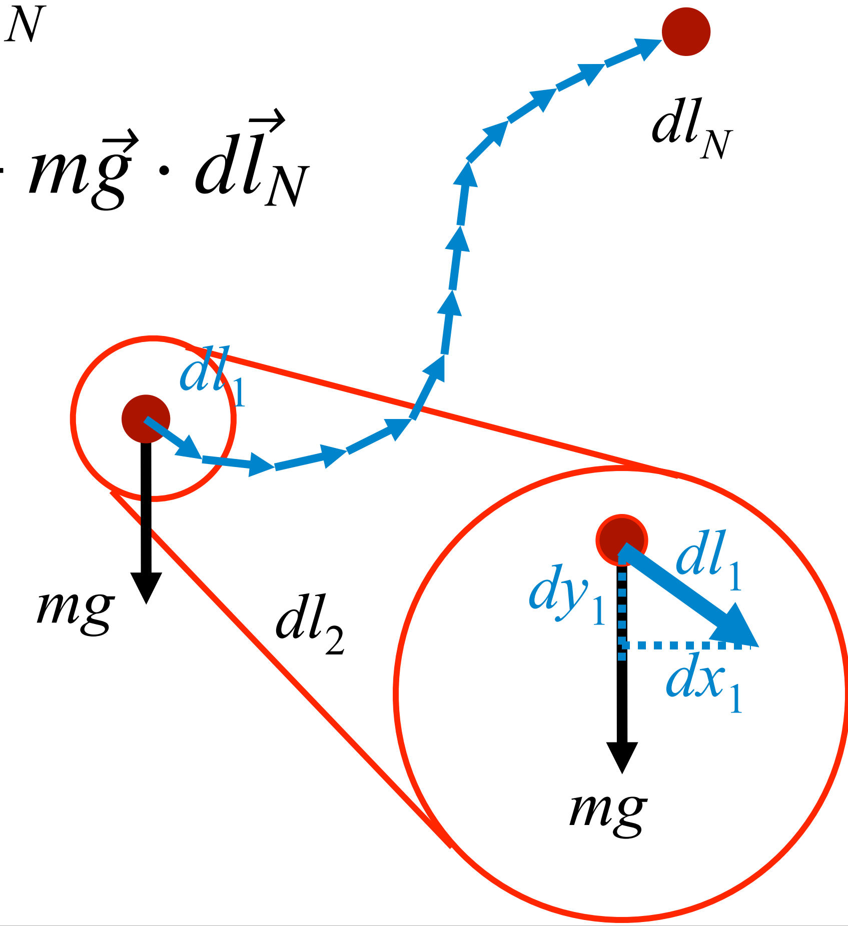
Work done by gravity near the Earth's surface



Work done by gravity near the Earth's surface

$$W_{TOT} = W_1 + W_2 + \ldots + W_N$$

$$= m\vec{g} \cdot d\vec{l}_1 + m\vec{g} \cdot d\vec{l}_1 + \cdots + m\vec{g} \cdot d\vec{l}_N$$



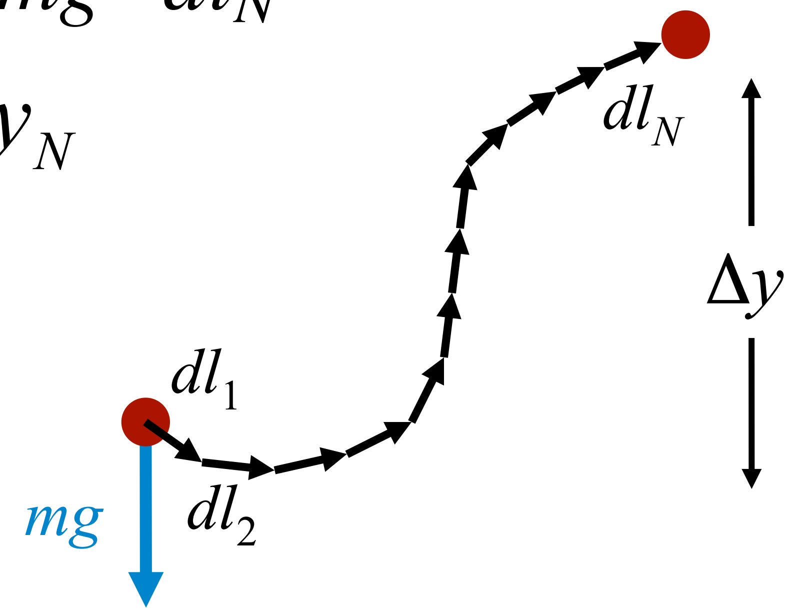
Work done by gravity near the Earth's surface

$$W_{TOT} = W_1 + W_2 + \ldots + W_N$$

$$= m\vec{g} \cdot d\vec{l}_1 + m\vec{g} \cdot d\vec{l}_2 + \cdots + m\vec{g} \cdot d\vec{l}_N$$

$$= -mgdy_1 - mgdy_2 \ldots - mgdy_N$$

$$= -mg\Delta y$$



$$W_g = -mg\Delta y$$

Work-Kinetic Energy Theorem

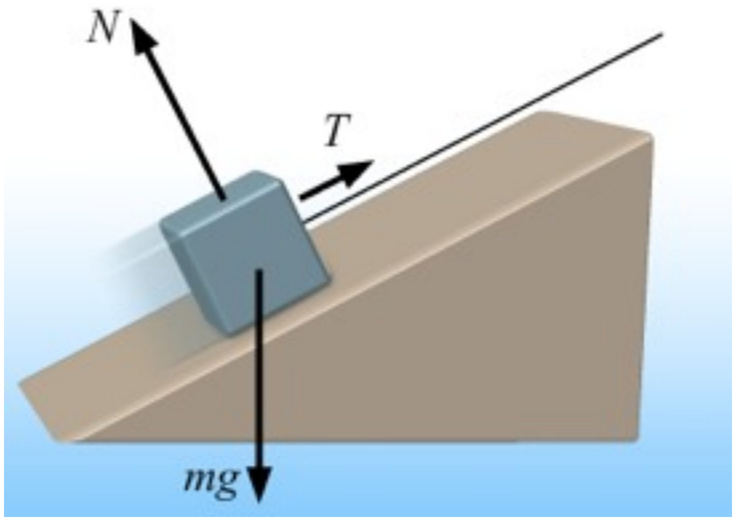
If there are several forces acting then W is the work done by the net (total) force:

$$W_{NET} = \Delta K$$

$$= W_1 + W_2 + \ldots$$

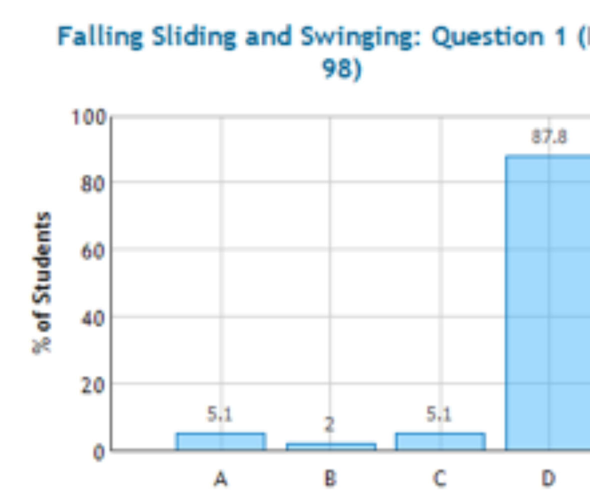
You can just add up the work done by each force

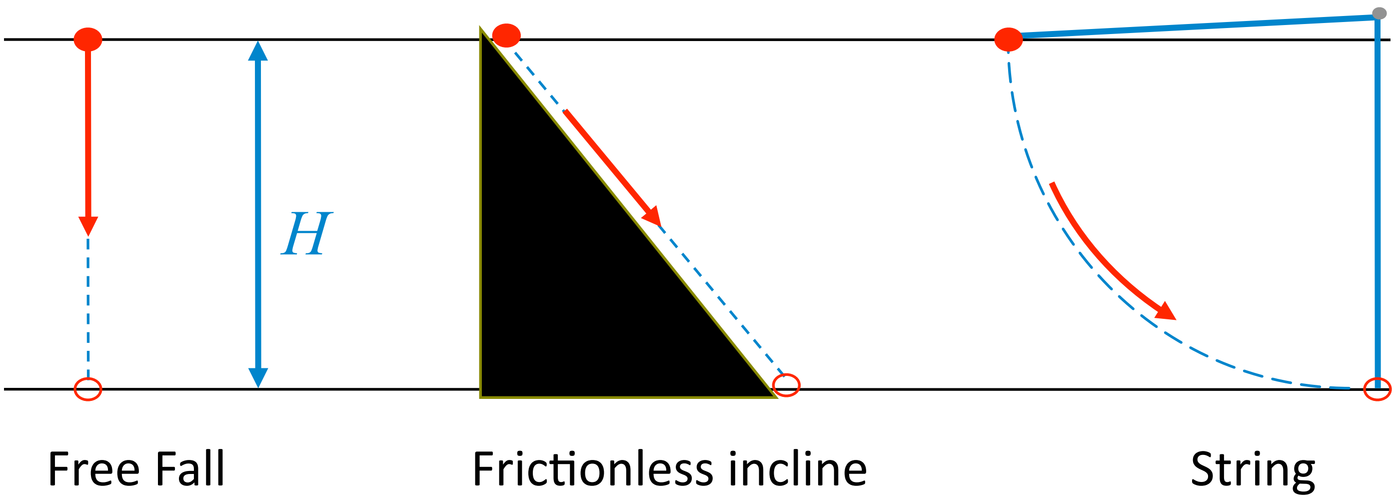
$$W_{NET} = W_{TOT}$$



CheckPoint

Three objects having the same mass begin at the same height, and all move down the same vertical distance H. One falls straight down, one slides down a frictionless inclined plane, and one swings on the end of a string.





In which case does the object have the biggest net work done on it by all forces during its motion?

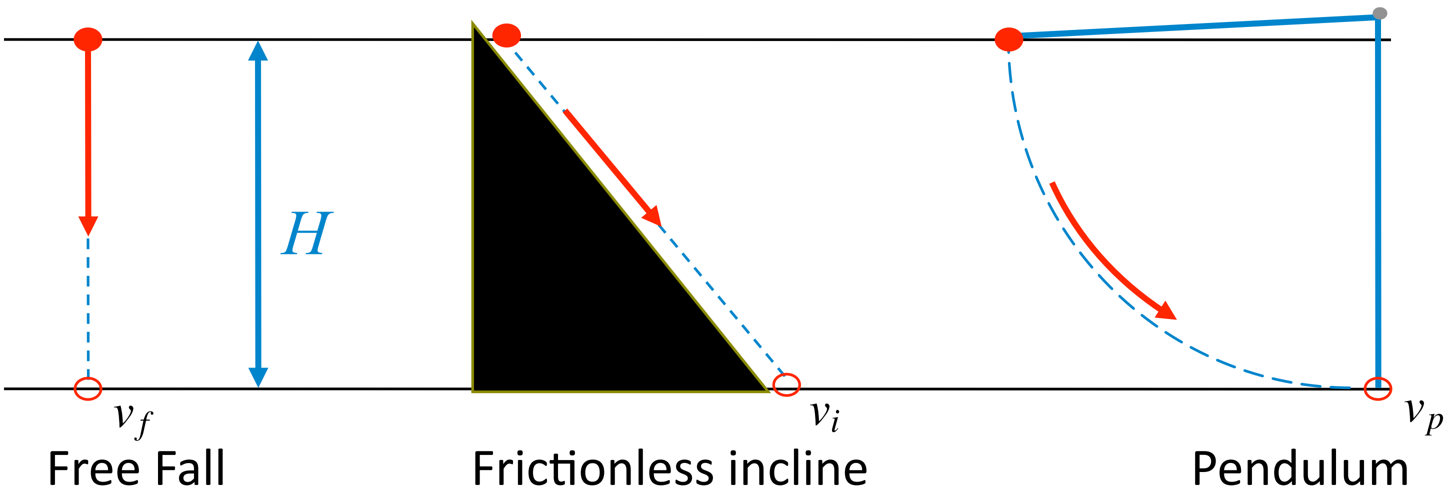
A) Free Fall B) Incline C) String D) All the same

~88% got this right…

Clicker Question

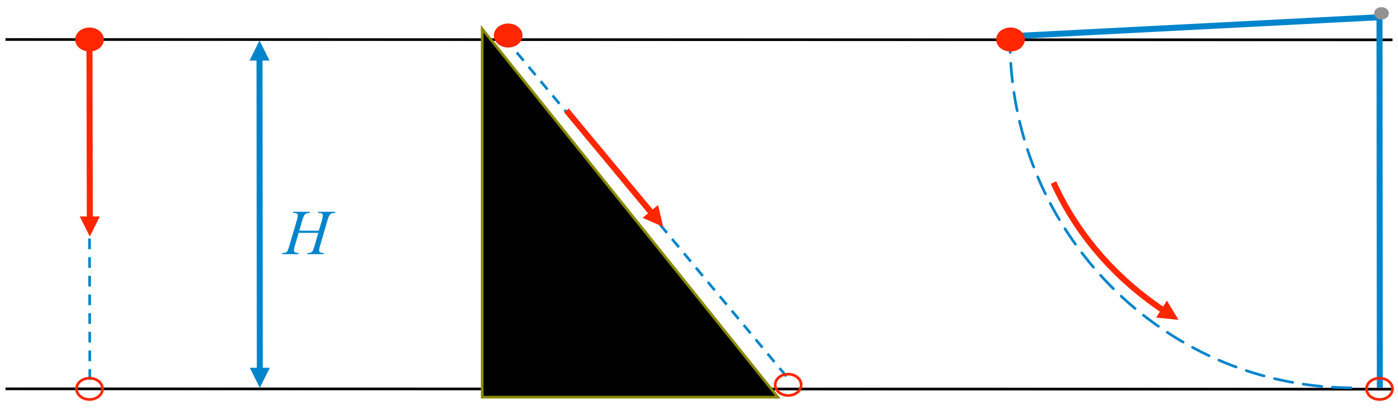
Three objects having the same mass begin at the same height, and all move down the same vertical distance H. One falls straight down, one slides down a frictionless inclined plane, and one swings on the end of a string.

What is the relationship between their speeds when they reach the bottom?



A) $v_f > v_i > v_p$ B) $v_f > v_p > v_i$ C) $v_f = v_p = v_i$

CheckPoint / Clicker Question



A) $v_f > v_i > v_p$ B) $v_f > v_p > v_i$ C) $\boxed{v_f = v_p = v_i}$

Only gravity will do work: $W_g = \Delta K$

$$\left\{ \begin{array}{l} W_g = mgH \\ \Delta K = {}^1/_2\, m{v_2}^2 \end{array} \right\} \quad v = \sqrt{2gH}$$

$$\Rightarrow \boxed{v_f = v_i = v_p = \sqrt{2gH}}$$

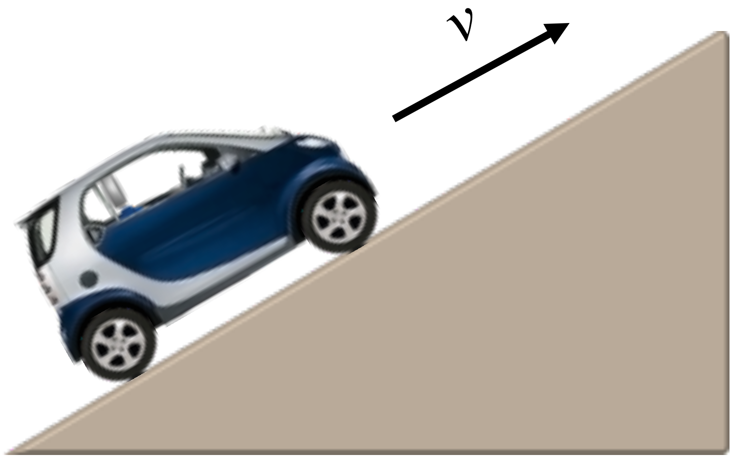
CheckPoint

A car drives up a hill with constant speed. Which statement best describes the total work W_{TOT} done on the car by all forces as it moves up the hill?

A) $W_{TOT} = 0$

B) $W_{TOT} > 0$

C) $W_{TOT} < 0$

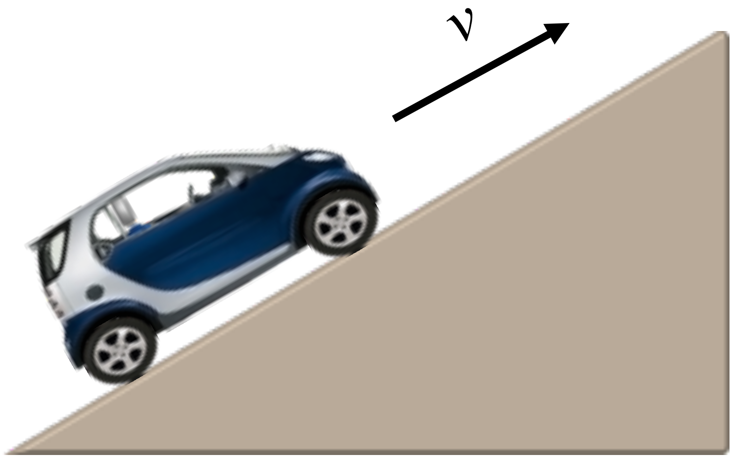


Less than 45% got this right…

Clicker Question

A car drives up a hill with constant speed. How does the kinetic energy of the car change as it moves up the hill?

A) It increases
B) It stays the same
C) It decreases



A car drives up a hill with constant speed. Which statement best describes the total work W_{TOT} done on the car by all forces as it moves up the hill?

A) $W_{TOT} = 0$

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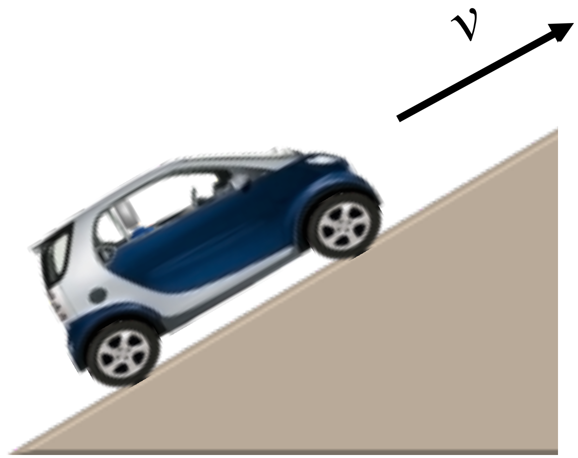
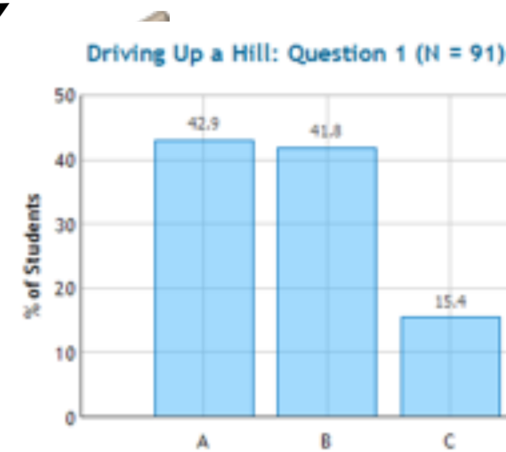




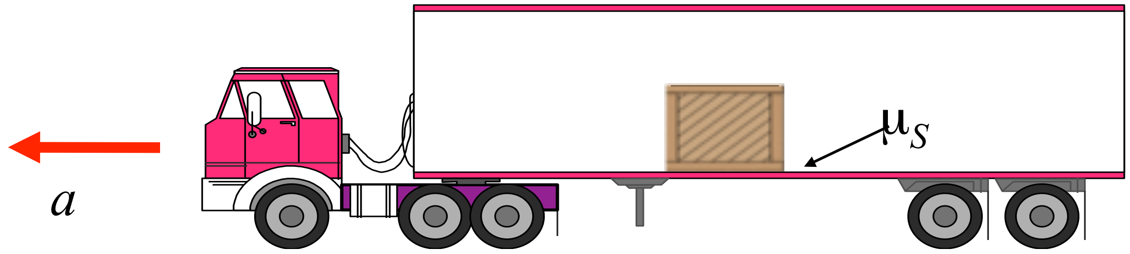
A) The change in kinetic energy is zero because the velocity is constant. By the work-kinetic energy theorem, we know that work must also be zero.

B) The force of friction between the car's wheels and the ramp exert a force up the ramp over a particular distance. W=Fd, so the work is positive.

C) gravity is downward and car moves up the hill.

CheckPoint

A box sits on the horizontal bed of a moving truck. Static friction between the box and the truck keeps the box from sliding around as the truck drives.



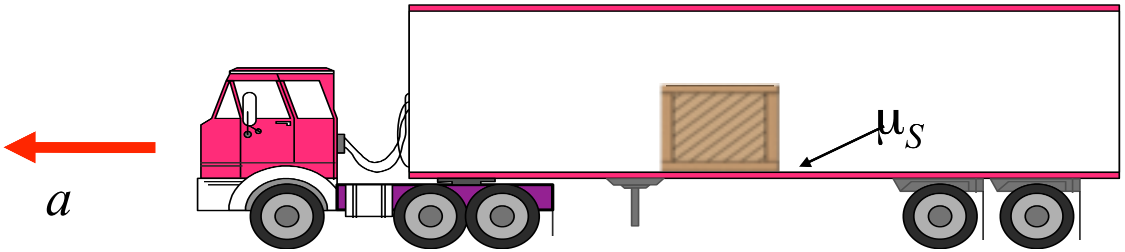
The work done on the box by the static frictional force as the truck moves a distance D is:

A) Positive B) Negative C) Zero D) Depends on speed

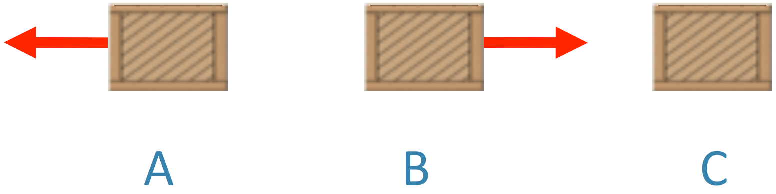
About 50% got this right…

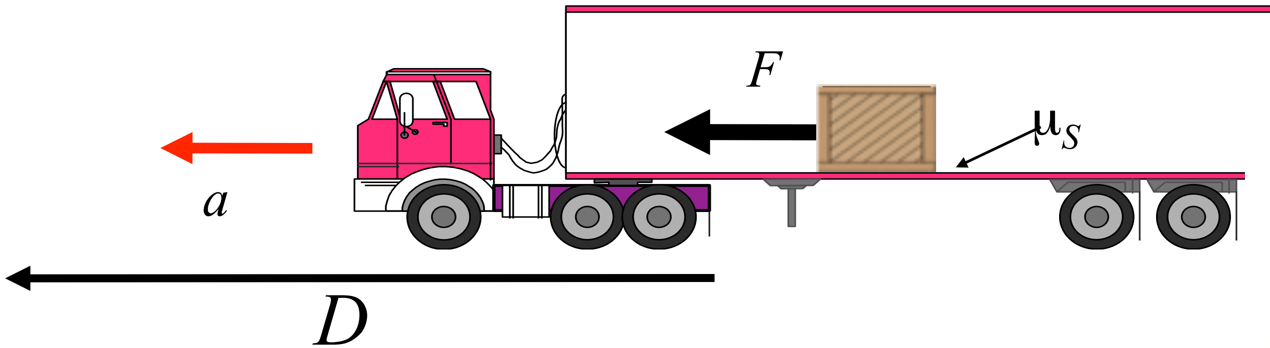
From Before

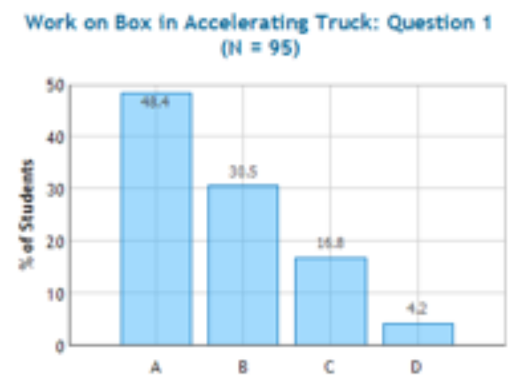
A box sits on the horizontal bed of a moving truck. Static friction between the box and the truck keeps the box from sliding around as the truck drives.



If the truck moves with constant accelerating to the left as shown, which of the following diagrams best describes the static frictional force acting on the box:







The work done on the box by the static frictional force as the truck moves a distance D is:

A) Positive B) Negative C) Zero D) Depends on speed

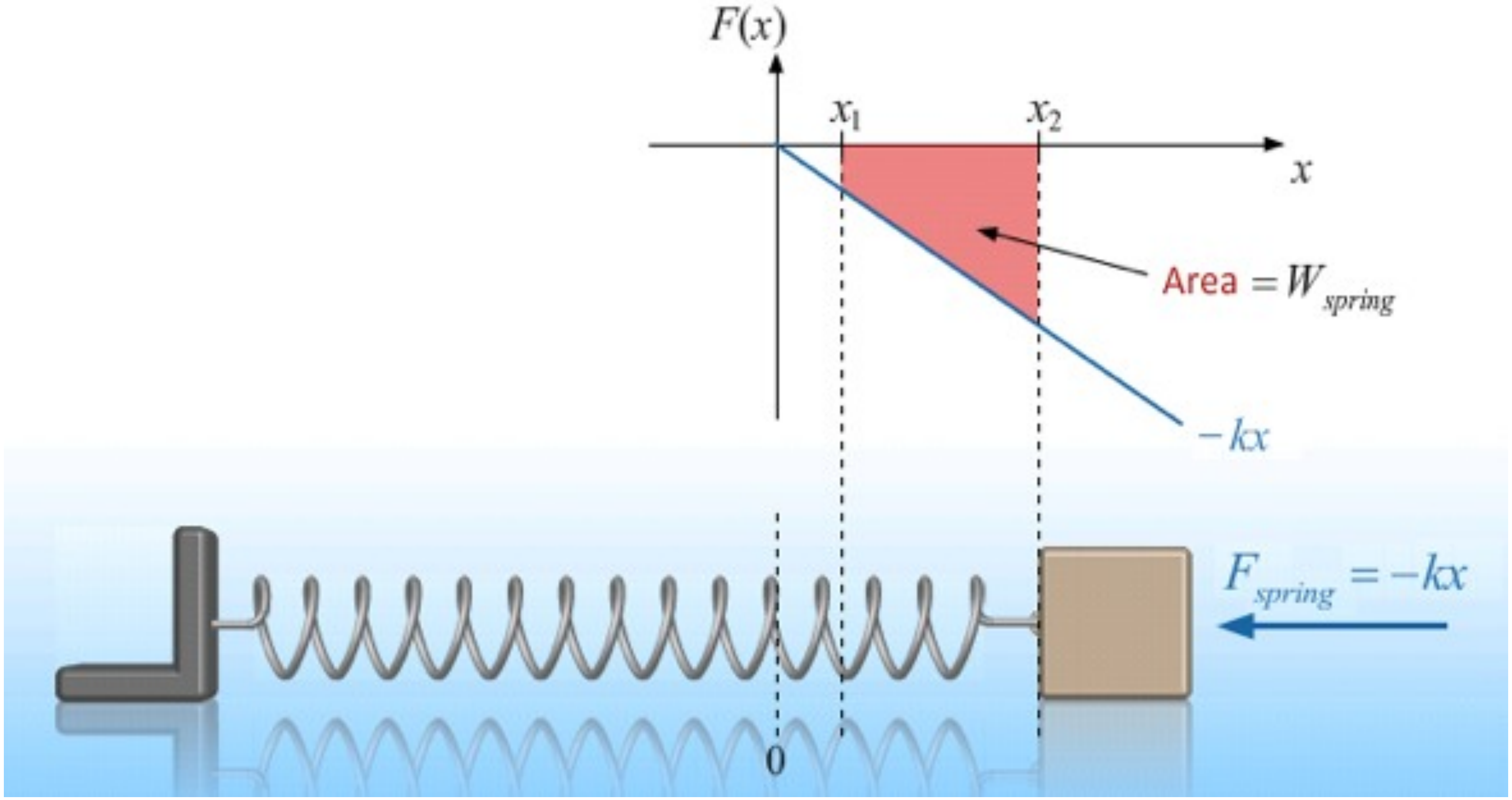
A) because the direction of the static frictional force and the direction the box travels are all same.

B) Since the friction force always has opposite direction from the movement direction, the work done is negative

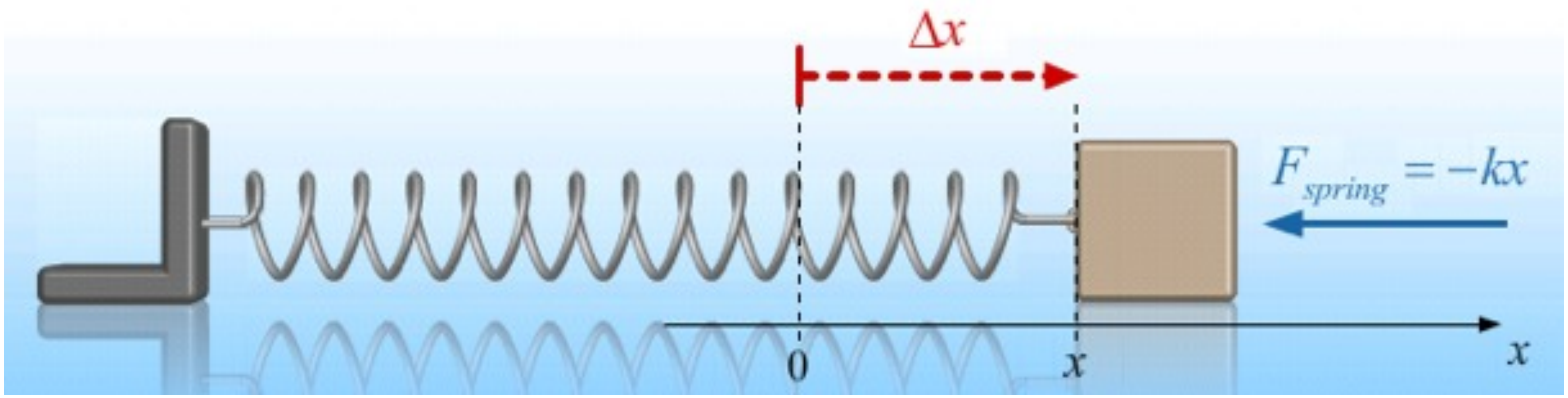
C) Friction keeps the box from sliding, thus D=0, and work done is 0.

Work done by a Spring

$$W_{1\to 2} = -\frac{1}{2}k(x_2^2 - x_1^2)$$



I am confused about the positive work and negative work and also the positive and negative forces for the spring problems.



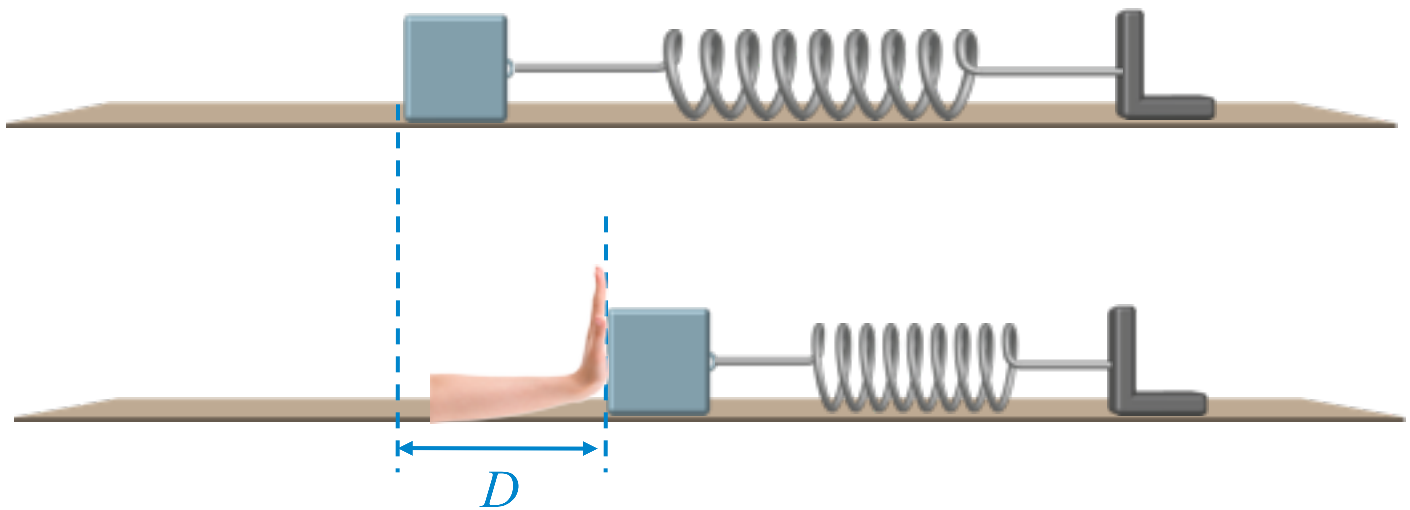
Use the formula to get the magnitude of the work

$$W_{1\to 2} = -\frac{1}{2}k(x_2^2 - x_1^2)$$

Use a picture to get the sign (look at directions)

In this example the spring does –ve work since F and Δx are in opposite direction. The axes don't matter.

A box attached at rest to a spring at its equilibrium length. You now push the box with your hand so that the spring is compressed a distance D, and you hold the box at rest in this new location.

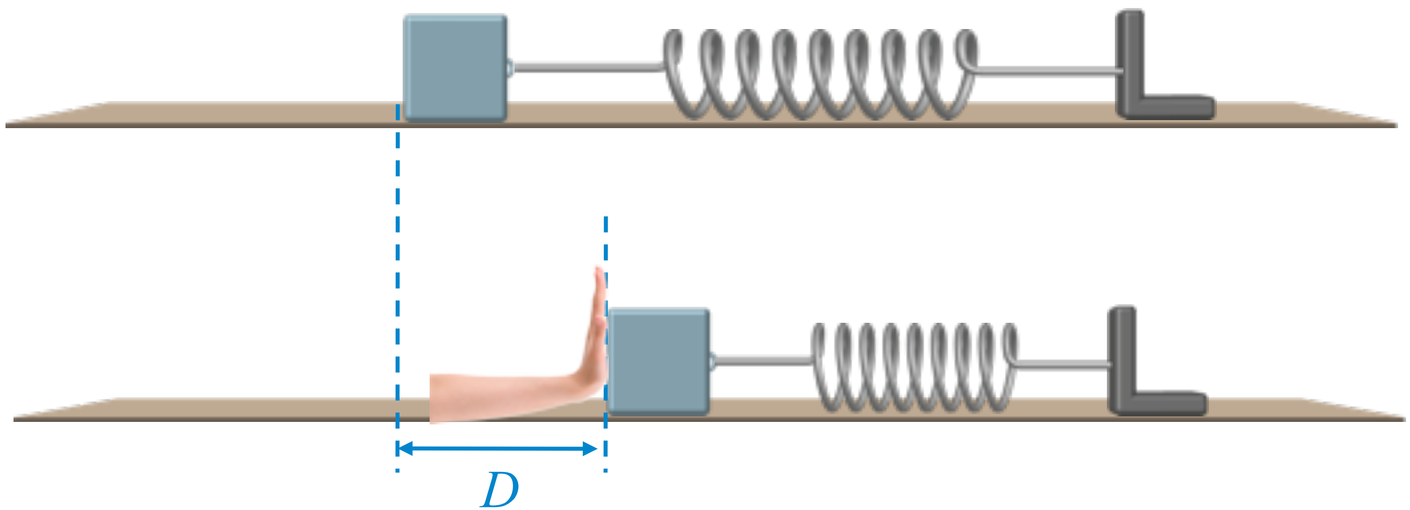


During this motion, the spring does:

A) Positive Work B) Negative Work C) Zero work

Clicker Question

A box attached at rest to a spring at its equilibrium length. You now push the box with your hand so that the spring is compressed a distance D, and you hold the box at rest in this new location.



During this motion, your hand does:

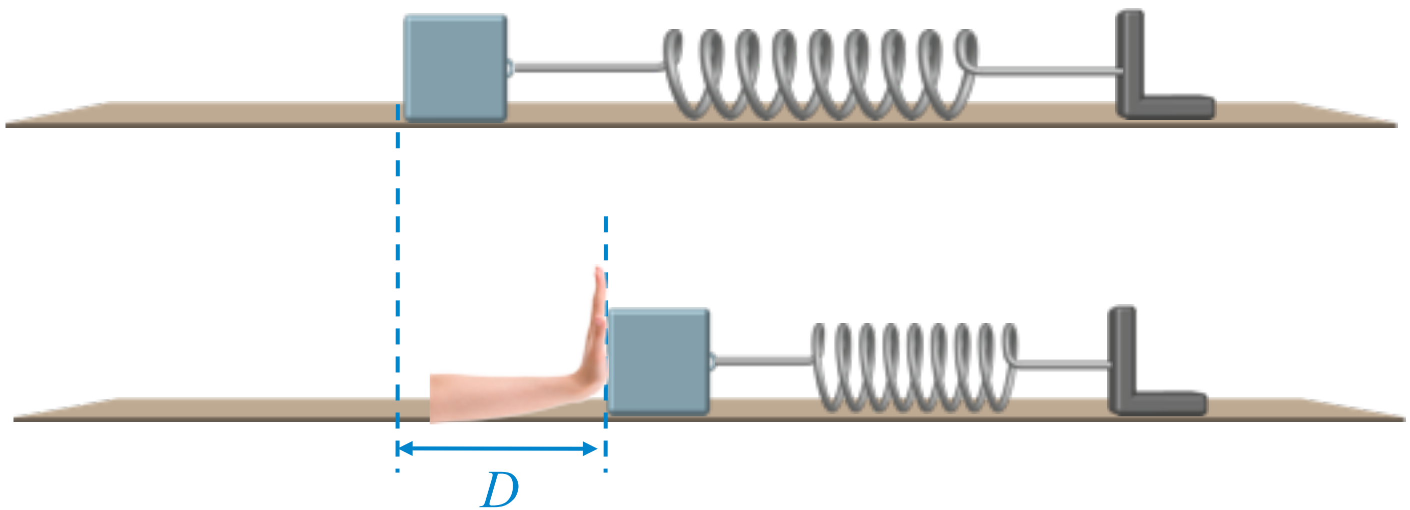
A) Positive Work

B) Negative Work

C) Zero work

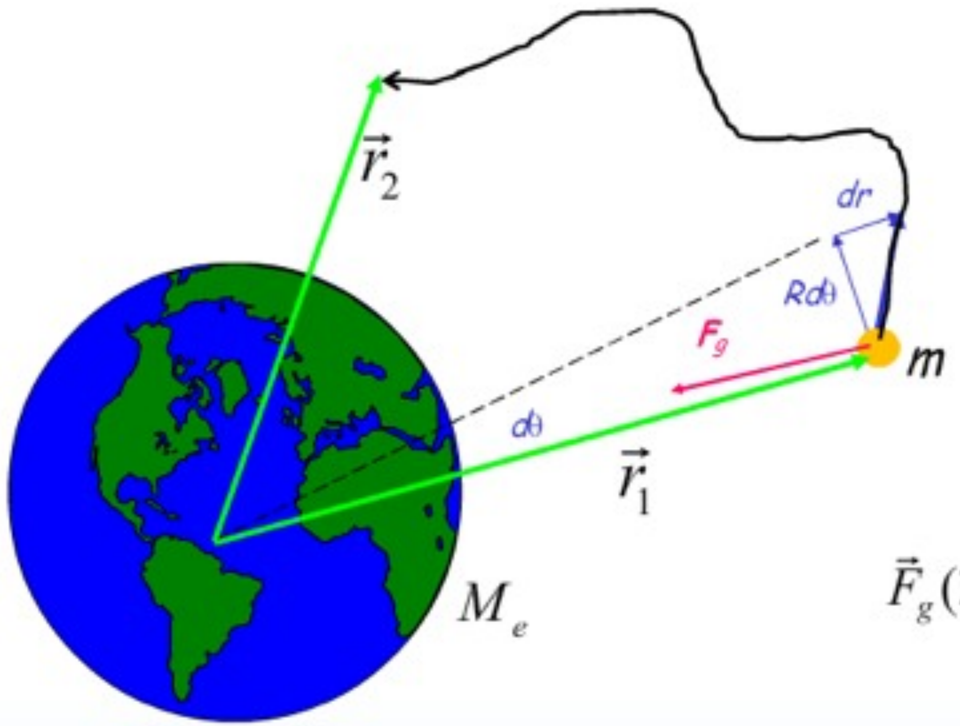
Clicker Question

A box attached at rest to a spring at its equilibrium length. You now push the box with your hand so that the spring is compressed a distance D, and you hold the box at rest in this new location.



During this motion, the total work done on the box is:

A) Positive B) Negative C) Zero



$$dW = -\frac{GM_e m}{r^2}\,dr$$

$$\vec{F}_g(\vec{r}) = -\frac{GM_e m}{r^2}\,\hat{r}$$

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r} = -\int_{r_1}^{r_2} \frac{GM_e m}{r^2}\,dr = \left.\frac{GM_e m}{r}\right|_{r_1}^{r_2} \boxed{= GM_e m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

Clicker Question

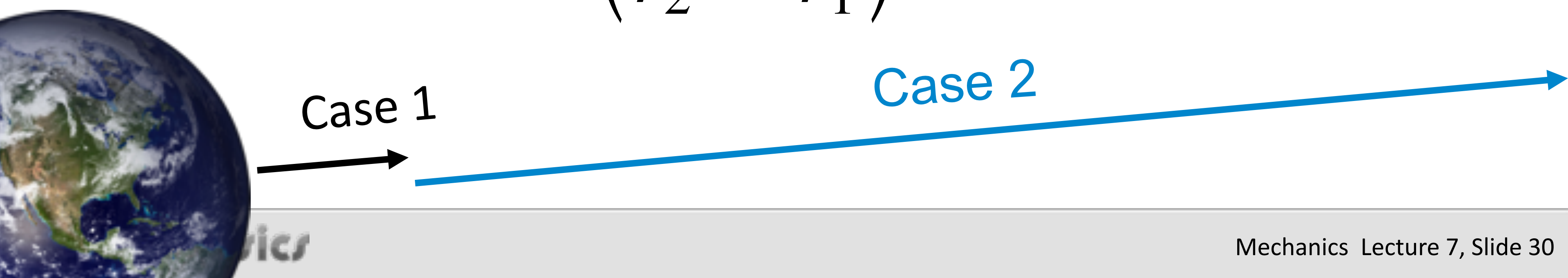
In Case 1 we send an object from the surface of the earth to a height above the earth surface equal to one earth radius.

In Case 2 we start the same object a height of one earth radius above the surface of the earth and we send it infinitely far away.

In which case is the magnitude of the work done by the Earth's gravity on the object biggest?

A) Case 1 B) Case 2 C) They are the same

$$W = GM_e m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

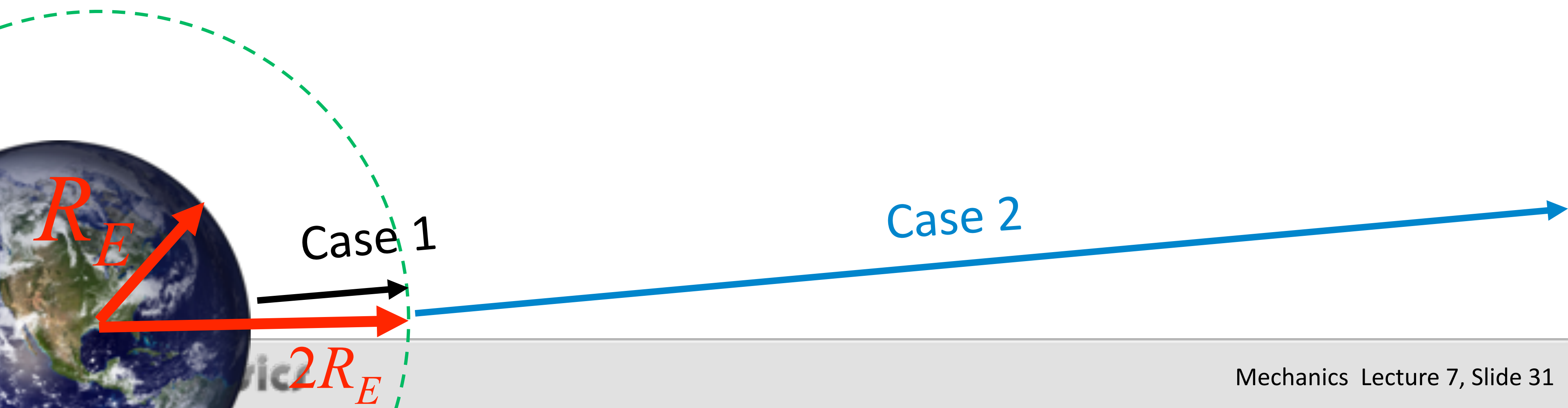


Clicker Question Solution

Case 1: $$W = GM_e m\left(\frac{1}{2R_E} - \frac{1}{R_E}\right) = -\frac{GM_e m}{2R_E}$$

Case 2: $$W = GM_e m\left(\frac{1}{\infty} - \frac{1}{2R_E}\right) = -\frac{GM_e m}{2R_E}$$

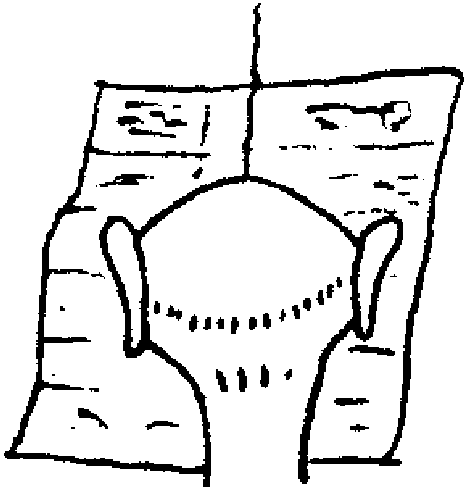
Same!



Warning!

Writing $\dfrac{1}{\infty} = 0$

will make a mathematician's hair stand on end.



Which Job?

All three jobs pay the same amount of money. Which one would you choose for your crew?

A. Job 1

B. Job 2

C. Job 3

$W = g\,(m\,\Delta h)$ $[kg\ m^2/s^2]$

A) W =g (100 x 10 x 4) (4000)

B) W =g (100 x 10 x 8 x sin 30°) (4000)

C) W =g (20 x 50 x 2) (2000)