Classical Mechanics
Lecture 12

Today's Concepts:

a) Elastic Collisions

b) Centre-of-Mass Reference Frame

Your Comments

How do you calculate the final angles in a 2d collision using the CM method?

can you explain how to solve problems using the center of mass frame?

I got so confused about this chapter!!!How should I analyze when is the kinetic energy or the momentum conserved?? Before, during and after the elastic collision, the momentum and the kinetic energy of a system are always conserved?? But how about each individual object? how to analyze them?

Watching the prelectures almost made me flip a table. Can we go over the thingys that have to do with center frames or whatever that's called?

are we taking 2-D collisions or no?

Centre of Mass & Collisions so far:

$$\vec{F}_{net,ext} = M_{tot}\vec{A}_{cm} = \frac{d\vec{P}_{tot}}{dt}$$

The CM behaves just like a point particle

If $\vec{F}_{net,ext} = 0$ then momentum is conserved

$$\vec{P}_{tot} = M_{tot}\vec{V}_{cm}$$

If you are in a reference frame moving along with the CM then the total momentum you measure is 0.

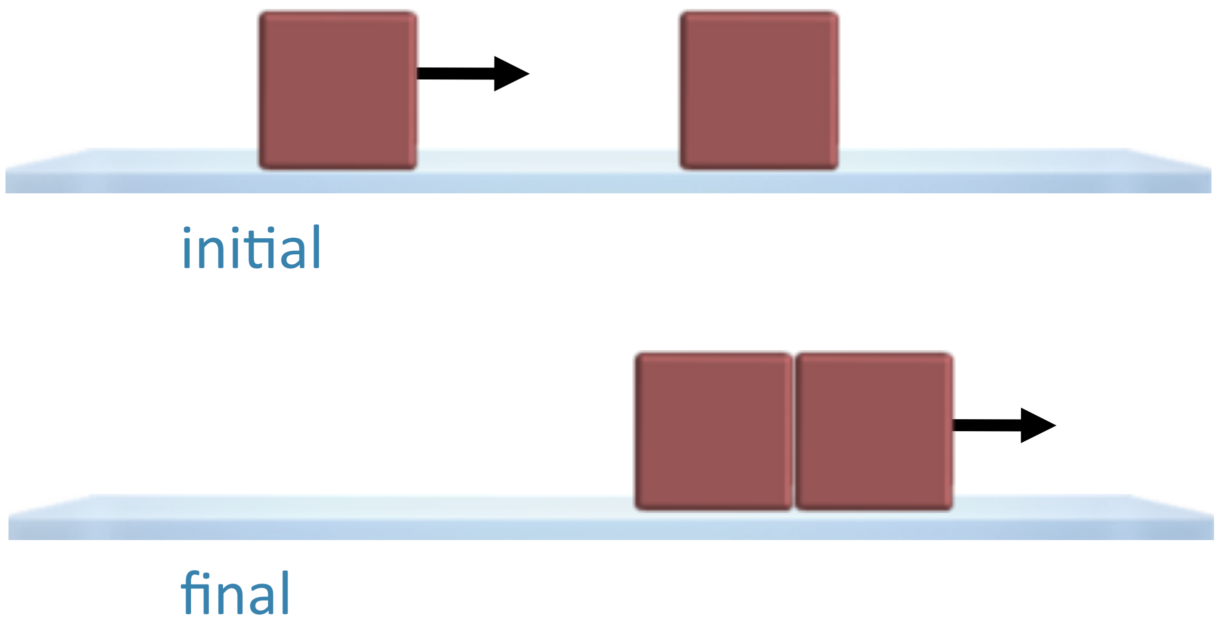
CheckPoint

A box sliding on a frictionless surface collides and sticks to a second identical box which is initially at rest. Compare the initial and final kinetic energies of the system.

A) $K_{initial} > K_{final}$

B) $K_{initial} = K_{final}$

C) $K_{initial} < K_{final}$

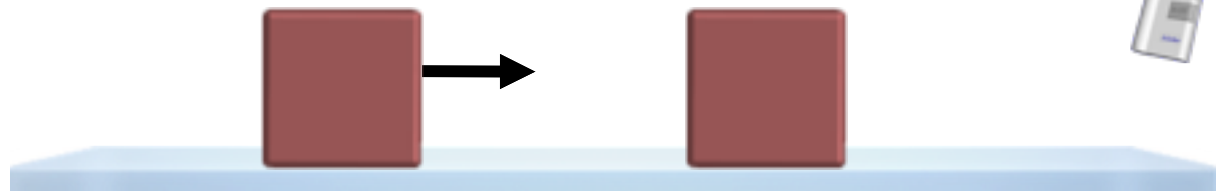


CheckPoint Response

$\boxed{\text{A) } K_{initial} > K_{final}}$

B) $K_{initial} = K_{final}$

C) $K_{initial} < K_{final}$



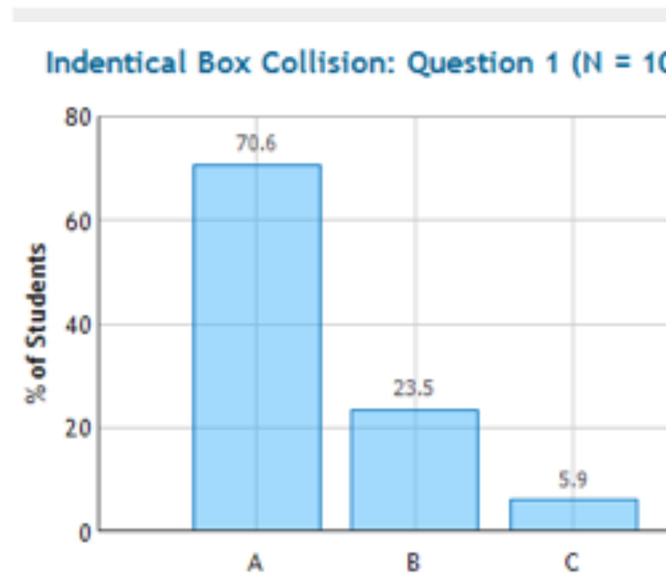
initial

final

A) K(initial) = $^1/_2\, mv^2$, K(final) = $^1/_2(^1/_{2m})m^2v^2 = {}^1/_4 mv^2$
Therefore K(initial) > K(final)

B) The KE is conserved because the surface is frictionless

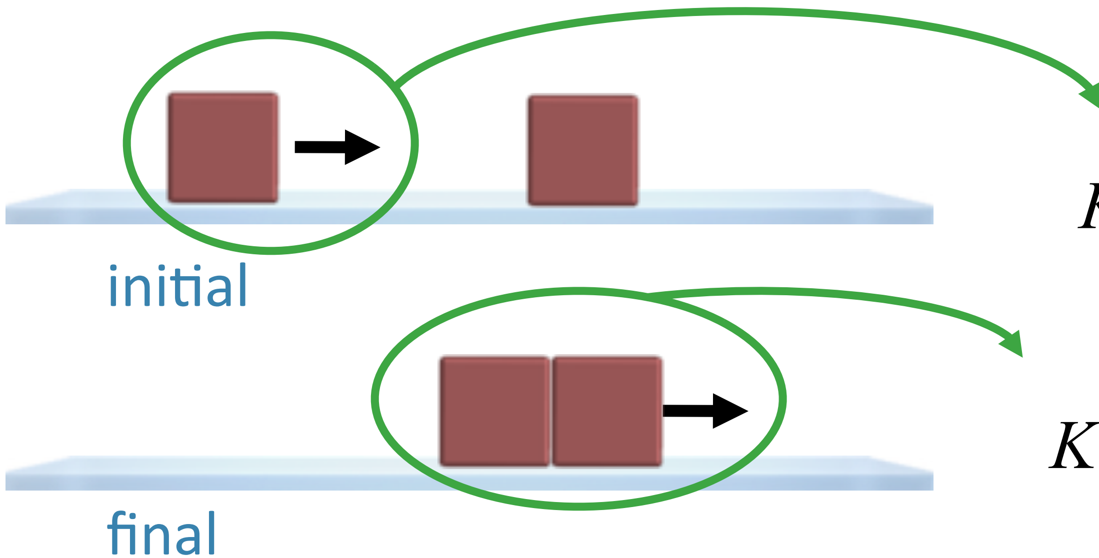
C) the boxes are still boxes in the end



Relationship between Momentum & Kinetic Energy

$$K = \frac{1}{2}mv^2 = \frac{1}{2m}m^2v^2 = \frac{p^2}{2m} \qquad \text{since } \vec{p} = m\vec{v}$$

This is often a handy way to figure out the kinetic energy before and after a collision since p is conserved.



$$K_{Initial} = \frac{p^2}{2M}$$

same p

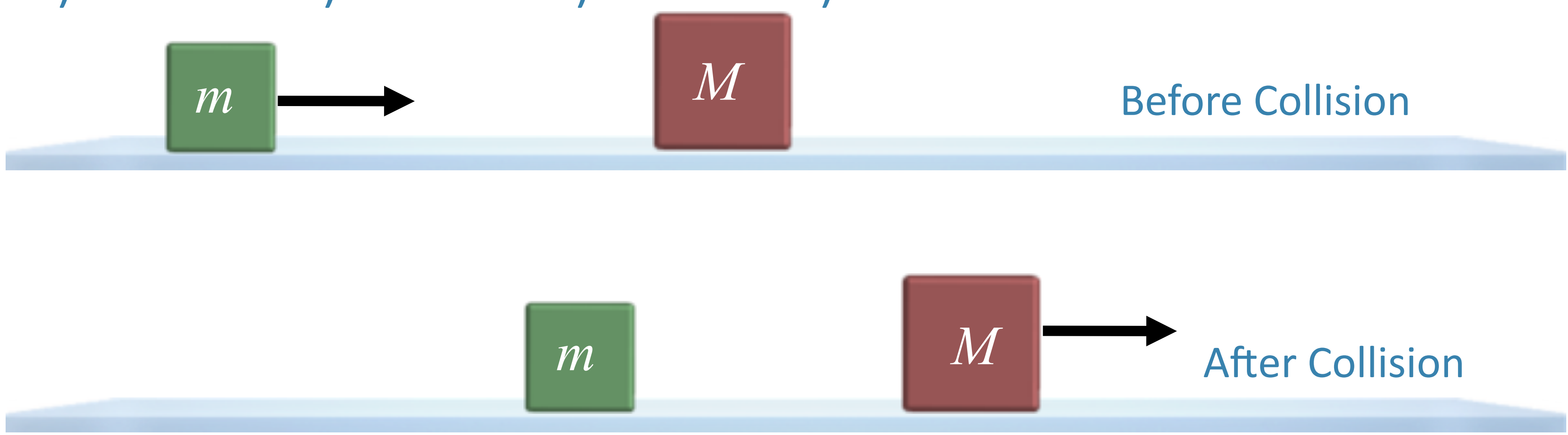
$$K_{final} = \frac{p^2}{2(2M)}$$

CheckPoint

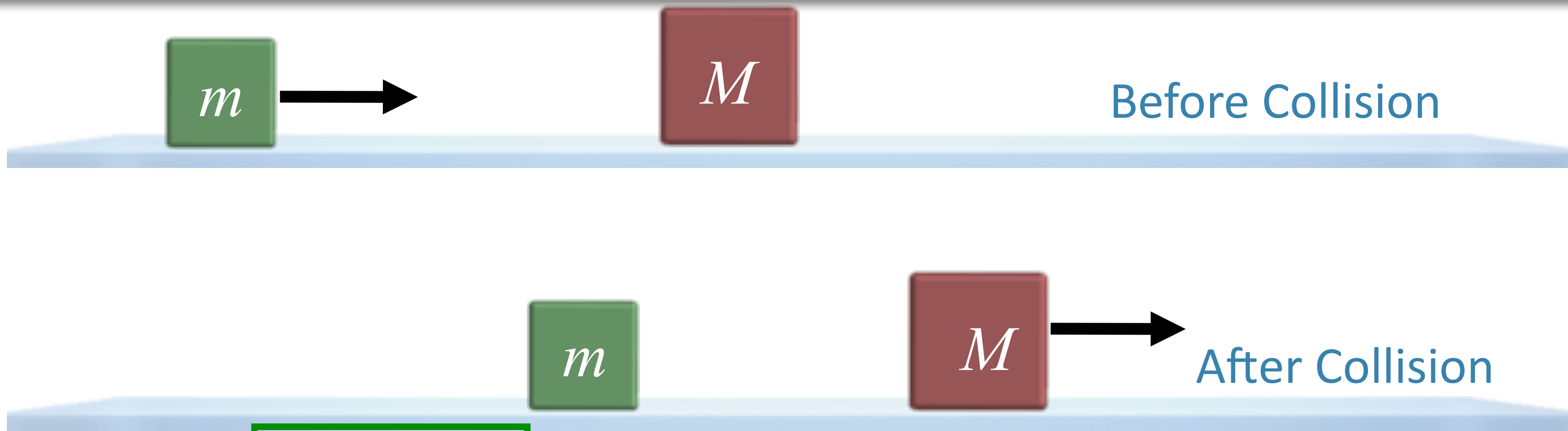
A green block of mass m slides to the right on a frictionless floor and collides elastically with a red block of mass M which is initially at rest. After the collision the green block is at rest and the red block is moving to the right.

How does M compare to m?

A) $m > M$ B) $M = m$ C) $M > m$ D) Need more information

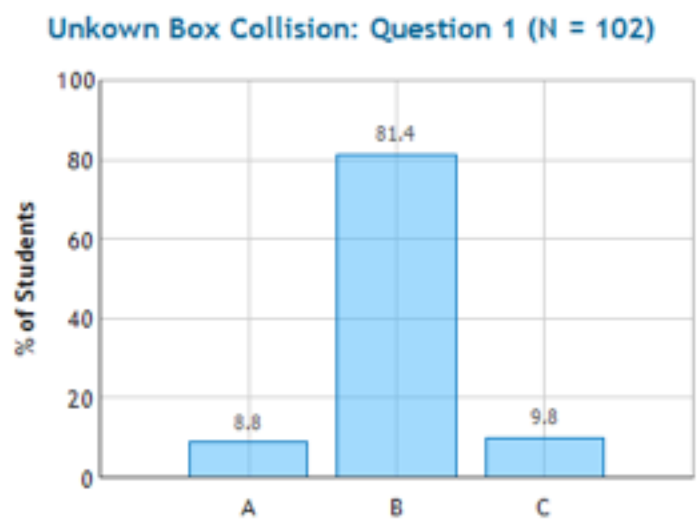


CheckPoint



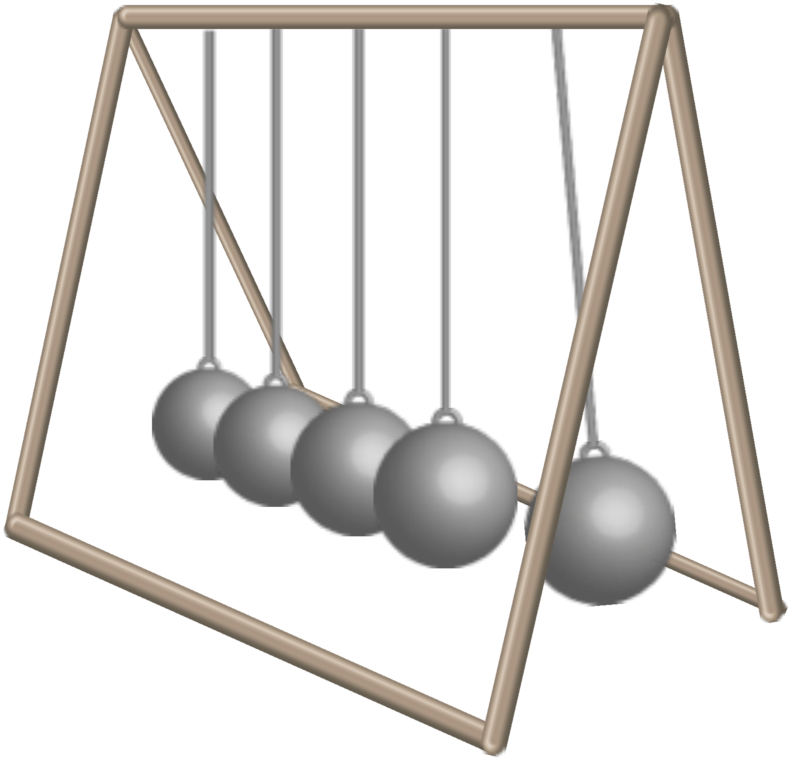
A) $m > M$ $\boxed{\text{B) } M = m}$ C) $M > m$ D) Need more information

B) In order to stop m in the collision, M must be the same as m. If m were smaller, it would bounce back and if m were bigger, it would continue to move forward.



Newton's Cradle

http://youtu.be/3CZShNRtT64
http://youtu.be/0MEVu_Elvwc



Centre-of-Mass Frame

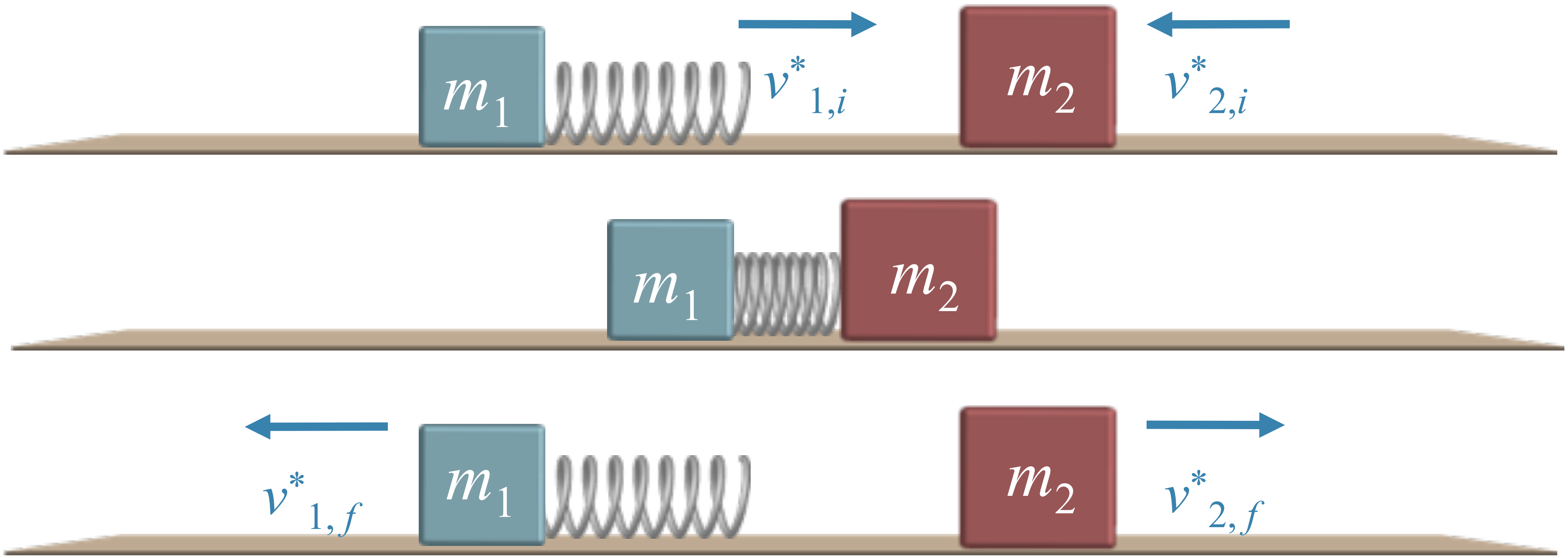
$$\vec{V}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\vec{P}_{tot}}{M_{tot}}$$

In the CM reference frame, $V_{CM} = 0$

$\boxed{\text{In the } CM \text{ reference frame, } P_{TOT} = 0}$

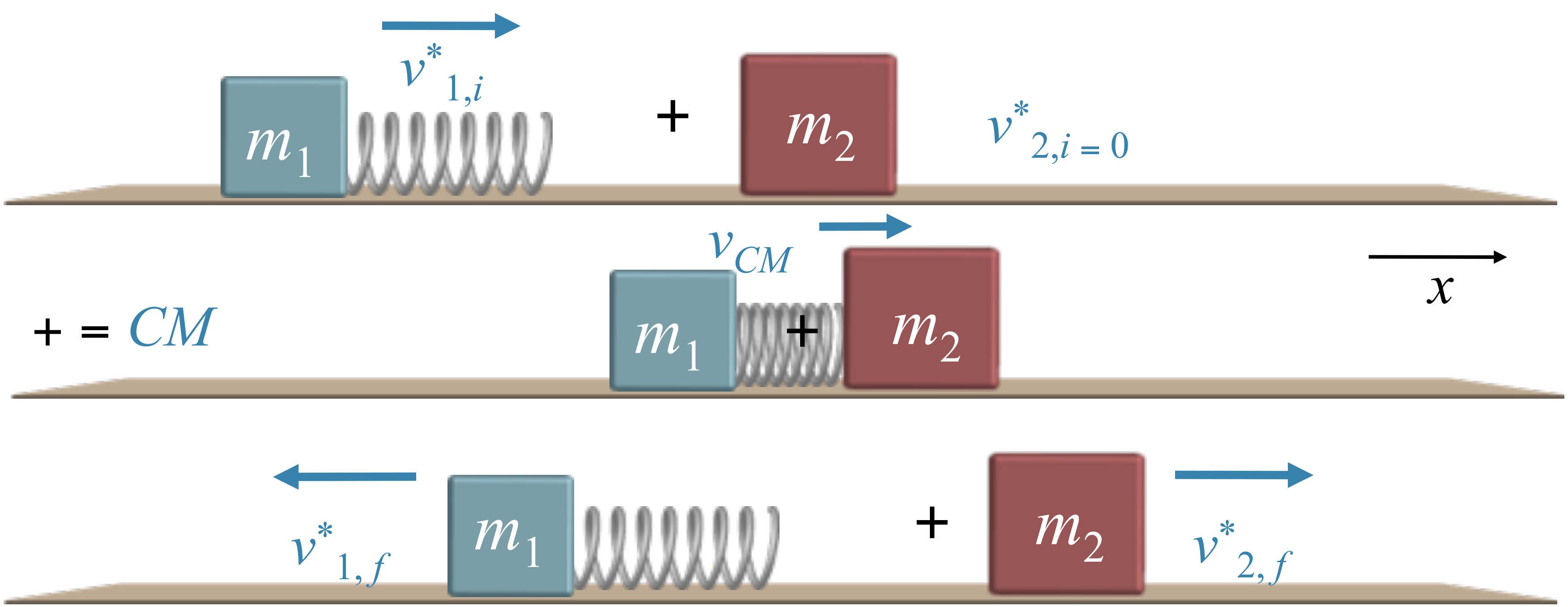
Centre of Mass Frame & Elastic Collisions

The speed of an object is the same before and after an elastic collision – if it is viewed in the CM frame:



Example: Using CM Reference Frame

A glider of mass $m_1 = 0.2$ kg slides on a frictionless track with initial velocity $v_{1,i} = 1.5$ m/s. It hits a stationary glider of mass $m_2 = 0.8$ kg. A spring attached to the first glider compresses and relaxes during the collision, but there is no friction (i.e., energy is conserved). What are the final velocities?



Example

Four step procedure:

Step 1: First figure out the velocity of the CM, V_{CM}.

$$V_{CM} = \left(\frac{1}{m_1 + m_2}\right)(m_1 v_{1,i} + m_2 v_{2,i})$$ but $v_{2,i} = 0$ so

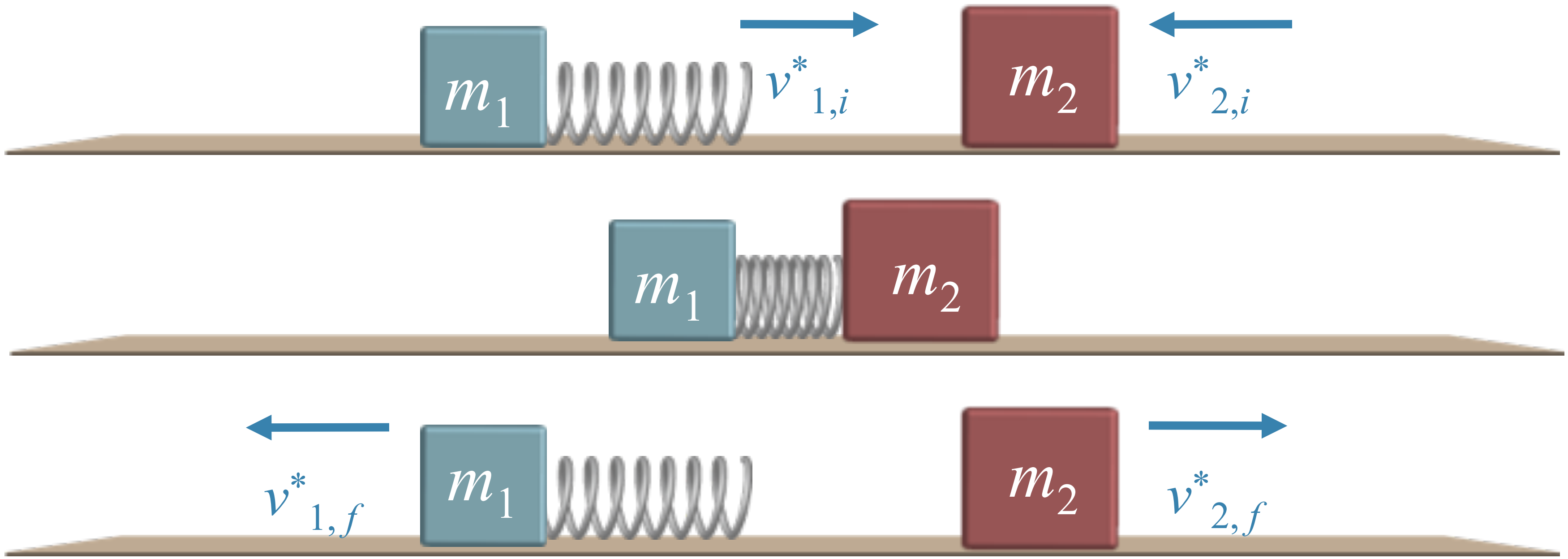
$$\boxed{V_{CM} = \left(\frac{m_1}{m_1 + m_2}\right) v_{1,i}}$$

(for $v_{2,i} = 0$ only)

So $V_{CM} = {}^1/_5\,(1.5 \text{ m/s}) = 0.3 \text{ m/s}$

Example

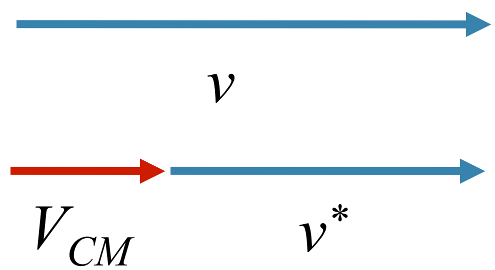
Now consider the collision viewed from a frame moving with the CM velocity V_{CM}.



Example

Step 2: Calculate the initial velocities in the *CM* reference frame (all velocities are in the *x* direction):

$$v = v^* + V_{CM}$$



$$v^* = v - V_{CM}$$

$$v^*_{1,i} = v_{1,i} - V_{CM} = 1.5 \text{ m/s} - 0.3 \text{ m/s} = 1.2 \text{ m/s}$$

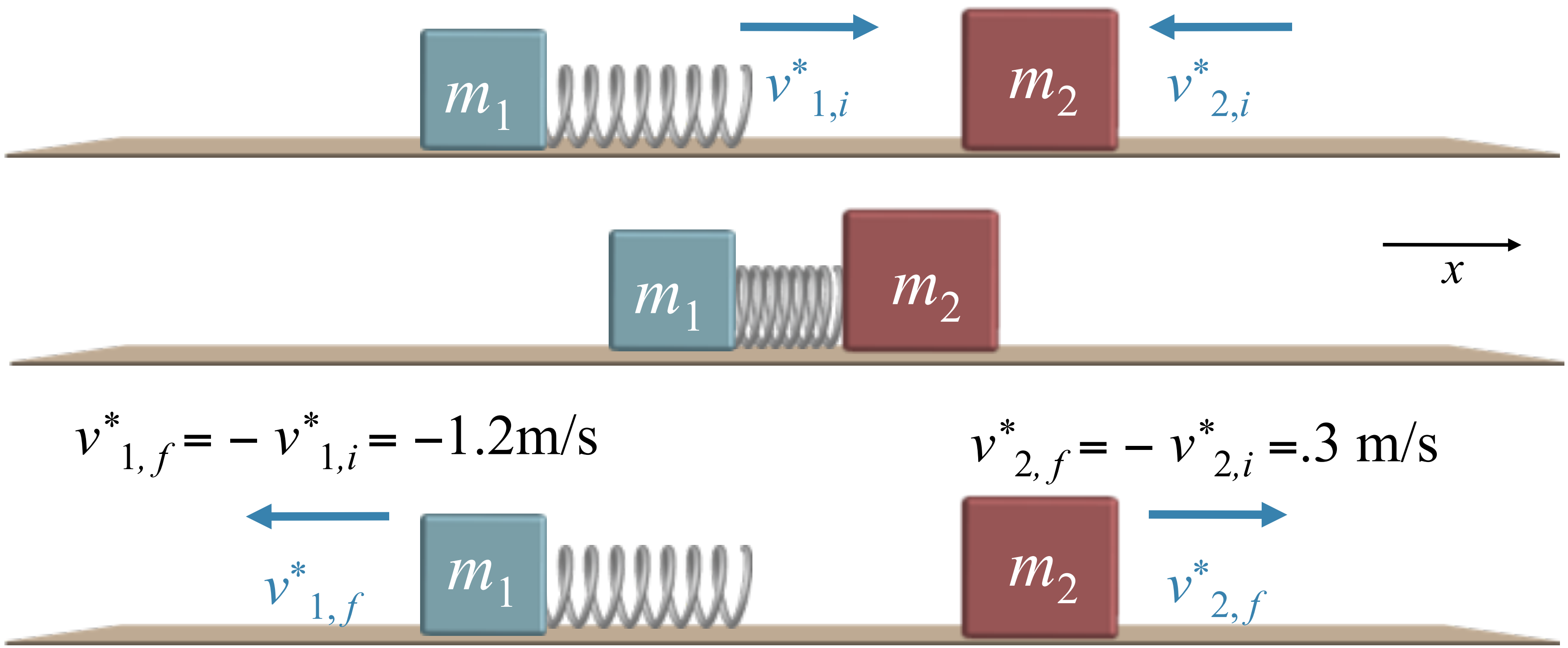
$$v^*_{2,i} = v_{2,i} - V_{CM} = 0 \text{ m/s} - 0.3 \text{ m/s} = -0.3 \text{ m/s}$$

$$\boxed{\begin{aligned} v^*_{1,i} &= 1.2 \text{ m/s} \\ v^*_{2,i} &= -0.3 \text{ m/s} \end{aligned}}$$

Example

Step 3: Use the fact that the speed of each block is the same before and after the collision in the *CM* frame.

$$v^*_{1,f} = -v^*_{1,i} \qquad v^*_{2,f} = -v^*_{2,i}$$



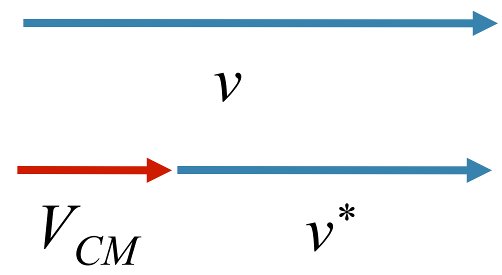
$$v^*_{1,f} = -\,v^*_{1,i} = -1.2\text{m/s} \qquad v^*_{2,f} = -\,v^*_{2,i} = .3\text{ m/s}$$

Example

Step 4:

Calculate the final velocities back in the lab reference frame:

$$v = v^* + V_{CM}$$



$$v_{1,f} = v^*_{1,f} + V_{CM} = -1.2 \text{ m/s} + 0.3 \text{ m/s} = -0.9 \text{ m/s}$$

$$v_{2,f} = v^*_{2,f} + V_{CM} = 0.3 \text{ m/s} + 0.3 \text{ m/s} = 0.6 \text{ m/s}$$

$$\boxed{\begin{aligned} v_{1,f} &= -0.9 \text{ m/s} \\ v_{2,f} &= 0.6 \text{ m/s} \end{aligned}}$$

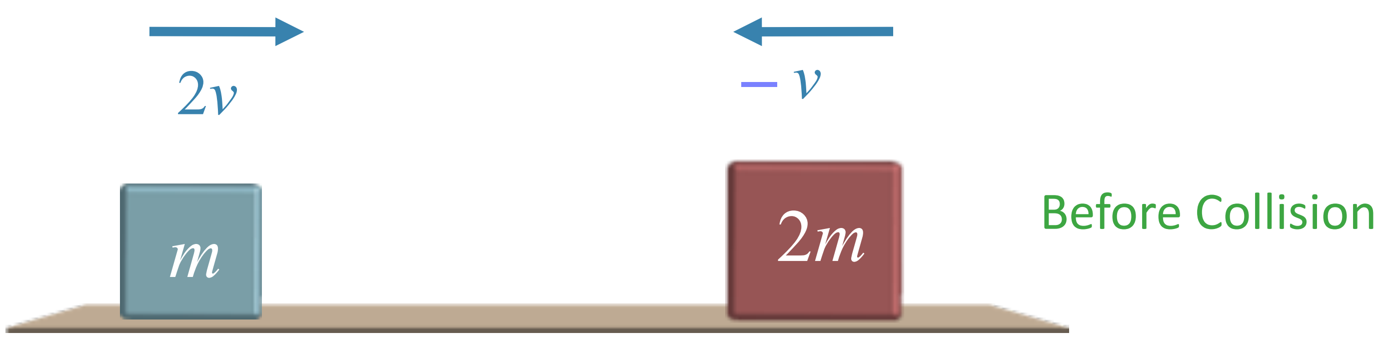
Four easy steps! No need to solve a quadratic equation!

CheckPoint

Two blocks on a horizontal frictionless track head toward each other as shown. One block has twice the mass and half the velocity of the other.

The velocity of the centre of mass of this system before the collision is.

A) Toward the left B) Toward the right C) zero



CheckPoint Response

The velocity of the centre of mass of this system before the collision is

A) Toward the left B) Toward the right $\boxed{\text{C) zero}}$

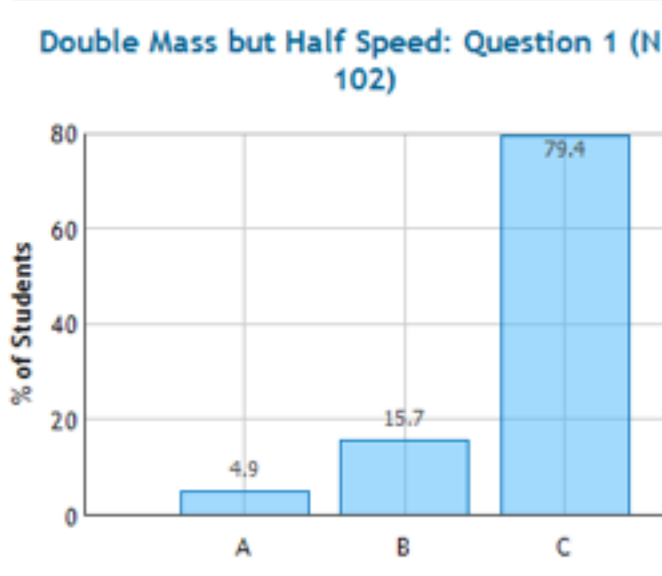
This is the *CM* frame

Before Collision

A) because it has twice the mass

B) the net velocity is towards the right.

C) The total momentum of the system is zero. This means that the velocity of the centre of mass must be zero.



CheckPoint

Two blocks on a horizontal frictionless track head toward each other as shown. One block has twice the mass and half the velocity of the other.

Suppose the blocks collide elastically. Picking the positive direction to the right, what is the velocity of the bigger block after the collision takes place?

A) v B) $-v$ C) $2v$ D) $-2v$ E) zero

$\longrightarrow$ +

Before Collision

This is the CM frame

CheckPoint Response

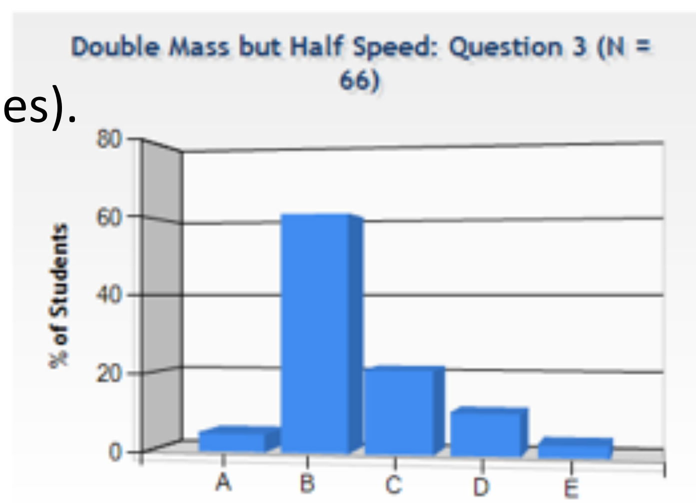
Suppose the blocks collide elastically. Picking the positive direction to the right, what is the velocity of the bigger block after the collision takes place?

A) v B) $-v$ C) $2v$ D) $-2v$ E) zero

Before Collision

This is the CM frame

A) Since the collision is elastic, and the velocity of the centre of mass is zero then the blocks simply travel backwards at their same speeds, (or opposite velocities).



B) the magnitude of the velocity stays the same but the direction changes

E) in order to conserve momentum, the blocks will both stop.

Bumper Cars

A bumper car with mass m_1 = 107 kg is moving to the right with a velocity of v_1 = 5 m/s. A second bumper car with mass m_2 = 86 kg is moving to the left with a velocity of v_2 = -3.6 m/s. The two cars have an elastic collision. Assume the surface is frictionless.

$$v_{CM} = \frac{m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i}}{m_1 + m_2}$$

1) What is the velocity of the center of mass of the system?

1.17 m/s Submit OK ✓ Correct

2) What is the initial velocity of car 1 in the center-of-mass reference frame?

$$v^*_{1,i} = v_{1,i} - v_{cm}$$

3.83 m/s Submit OK ✓ Correct

3) What is the final velocity of car 1 in the center-of-mass reference frame?

$$v^*_{1,f} = -v^*_{1,i}$$

-3.83 m/s Submit OK ✓ Correct

4) What is the final velocity of car 1 in the ground (original) reference frame?

$$v_{1,f} = v^*_{1,f} + v_{cm}$$

-2.66 m/s Submit OK ✓ Correct

Bumper Cars

A bumper car with mass m_1 = 107 kg is moving to the right with a velocity of v_1 = 5 m/s. A second bumper car with mass m_2 = 86 kg is moving to the left with a velocity of v_2 = -3.6 m/s. The two cars have an elastic collision. Assume the surface is frictionless.

5) What is the final velocity of car 2 in the ground (original) reference frame?

[ ] m/s Submit

Do same steps for car 2

6) In a new (inelastic) collision, the same two bumper cars with the same initial velocities now latch together as they collide.

What is the final speed of the two bumper cars after the collision?

[ ] m/s Submit

v_{cm} is the same = final speed

7) Compare the loss in energy in the two collisions:

- ○ $|\Delta KE_{elastic}| = |\Delta KE_{inelastic}|$
- ○ $|\Delta KE_{elastic}| > |\Delta KE_{inelastic}|$
- ○ $|\Delta KE_{elastic}| < |\Delta KE_{inelastic}|$

Compare $^1/_2\, mv^2$ before and after for both cases