Surname: Solutions

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Question #	Mark	Maximum Mark
Multiple Choice		20
11		10
12		10

Total = /40

Your SPECIAL CODE is 1111. Bubble in your Name, Student Number and Special Code on the Bubble Sheet and check it. Enter your answers to the multiple choice questions on the bubble sheet by blackening in the circle corresponding to the best answer. There is only one correct answer per question.

There are 10 multiple choice questions. Select the correct answer for each one and mark it on the bubble form on the cover sheet. Each question has only one correct answer. (2 marks each)

1. An archer pulls the bowstring back a distance of 0.2 m before releasing the arrow. Consider that the bow and string act like a spring with a spring constant of 230 N/m. The arrow has a mass of 0.03 kg.

   What is the magnitude of the spring potential energy of the drawn bow?

   (a) 2.3 J
   (b) 4.6 J Correct
   (c) 6.2 J
   (d) 7.4 J
   (e) 9.2 J

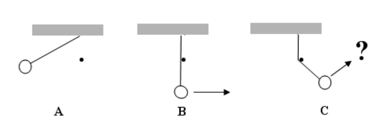
   Model the bow as an ideal spring. Energy stored is $U_{\text{spring}} = \frac{1}{2}kx^2$. x is the extension of the spring from equilibrium. Energy stored for an extension of 0.2 m is $U(0.2\text{ m})_{\text{spring}} = 0.5 \times 230 \times 0.2^2 = 4.6$ J.

2. What is speed of the arrow when it leaves the bow?

   (a) 8.5 m/s
   (b) 10.2 m/s
   (c) 17.5 m/s Correct
   (d) 19.3 m/s
   (e) 23.4 m/s

   The energy stored in the previous problem is turned into kinetic energy: $\frac{1}{2}mv^2 = 4.6$ J. $v = \sqrt{\frac{2\times4.6}{0.03}}$.

3. A pendulum is released from rest, as shown in Figure A. When the bob is directly below the pivot point, the ideal string encounters an ideal peg, about which it begins to wrap, as shown in Figures B and C. The peg is higher than the initial height of the pendulum.

   In Figure C, the bob rises up to a height

(a) equal to the original height of the bob in Figure A. Correct

(b) greater than the original height of the bob in Figure A.

(c) less than the original height of the bob in Figure A.

In order for to use up all the original potential energy that the ball had when it started swinging is must return to the same height, no matter the path it takes.

4. Two carts collide. During the collision the centre of mass of the carts travels at a constant velocity and is unchanged throughout. From this we can conclude that before and after the collision

   (a) total momentum of the carts is conserved

   (b) total kinetic energy is conserved

   (c) the relative velocities of the carts is conserved

   (d) all of the above

   (e) none of the above

Total momentum is conserved as long as no external forces are acting on the system. Kinetic energy is only conserved in elastic collisions. Relative velocities of the carts has the same magnitude, but opposite sign, so it is not conserved.

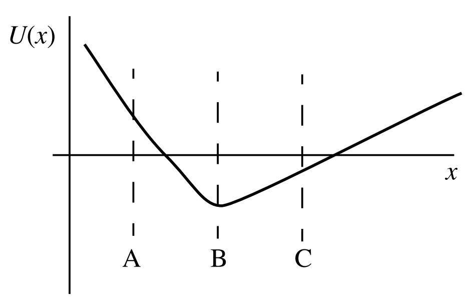
5. The Work Kinetic-Energy Theorem is invalid if work is done by a non-conservative force.

   (a) True

   (b) False

Non-conservative forces are those which dissipate energy to heat or forms other than kinetic.

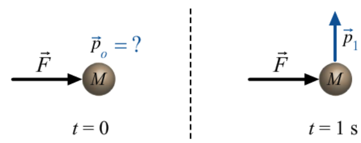
6. The potential energy $U(x)$ of an object as a function of x looks like the plot shown below



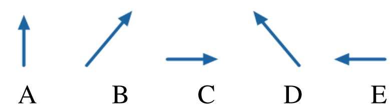
At which of the x values shown by a dashed line is the magnitude of the force the greatest?

$F(x) = -dU(x)/dx$. The slope is greatest at point A. It's magnitude is least where the slope is zero, point B.

7. A force $\vec{F}$ acts on mass M during the time period $t = 0$ s to $t = 1$ s. at $t = 1s$ the mass moves with momentum $\vec{p}_1$ as shown.



Which of the following vectors best represents $\vec{p}_0$, the momentum of M at $t = 0$.



The force $\vec{F}$ is to the right can only add to the x component of the initial velocity. Therefore the initial velocity must have a component in the $-x$ direction in order for it to end up with no x component.

8. In case A a block is pushed by force $\vec{F}$ for 1 second. In case B a block of one-half the mass is pushed by the same force $\vec{F}$ for 1 second. In both cases the track is frictionless and the blocks are initially at rest.

Compare the magnitudes of the final momenta of the two blocks:

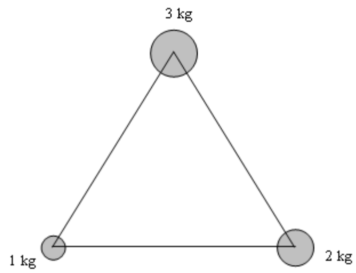
   (a) $p_A < p_B$

   (b) $p_A > p_B$

   (c) $p_A = p_B$

The force times time is the impulse and equals the change in momentum. Thus the momentum change is the same for both block. Since both blocks are initially at rest, their final momenta are the same.

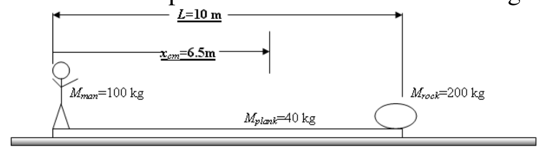
9. The centres of three spheres having masses 1 kg, 2 kg, and 3 kg are placed at the corners of an equilateral triangle whose sides are each 1 meter long, as shown below. How far from the centre of the 1 kg sphere is the centre of mass of the system?



   (a) 0.33 m
   (b) 0.50 m
   (c) 0.67 m
   (d) 0.73 m Correct
   (e) 1.00 m

   Find the y-coordinate of the c.m. which is half=way between the top and bottom, $y_{\rm cm} = \frac{1}{2}\cos 60^\circ = 0.43$ m. Then find the x component: $x_{\rm cm} = (0.5 \times 3 + 2 \times 2)/(1 + 2 + 3) = 0.58$ m. Distance = $\sqrt{0.43^2 + 0.58^2} = 0.722$ m.

10. A man is standing at one end of a plank of length $L = 10$ m. The man has mass $M_{\rm man} = 100$ kg and the plank has mass $M_{\rm plank} = 40$ kg and the plank is atop a frictionless sheet of ice. At the other end of the plank sits a large rock of mass $M_{\rm rock} = 200$ kg. The centre of mass of the man+plank+rock is 6.5 m from the end of the plank where the man is standing.



   The man walks to the other end of the plank and sits down on the rock. How far did the plank move along the ice?

   (a) 0 m
   (b) 1.3 m
   (c) 1.7 m
   (d) 2.9 m Correct
   (e) 3.3 m

   Find the c.m. of the boat-man-rock system before the man moves:

$$x_{\rm cm} = (m_{boat}\frac{L}{2} + m_{rock}L)/(m_{man} + m_{boat} + m_{rock})$$

   Now find where it is, relative to the boat, after the guy moves:

$$x'_{\rm cm} = (m_{boat}\frac{L}{2} + +m_{man}L + m_{rock}L)/(m_{man} + m_{boat} + m_{rock})$$

   . Subtract to get the change in the c.m. position of the system:

$$\Delta x_{\rm cm} = m_{man}L/((m_{man} + m_{boat} + m_{rock}) = 100 \times 10/340 = 2.9 \text{ m}$$

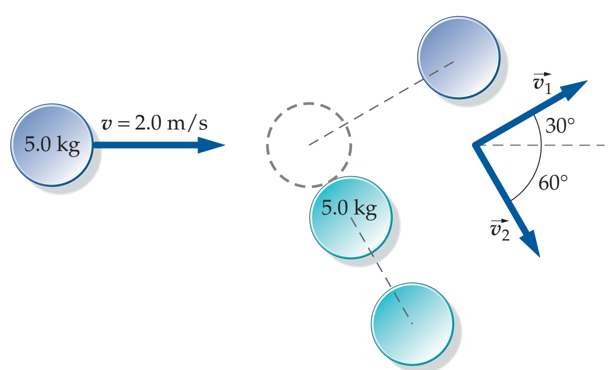
   .

   The boat is on ice so the c.m. relative to the ice must remain the same. Therefore, the boat moves by the distance that the c.m. shifts.

There are two written problems. Show all your work and explain your reasoning to get full credit.

11. A puck of mass 5.0 kg moving at 2.0 m/s approaches an identical puck that is stationary on frictionless ice. After the collision, the first puck leaves with a speed v_1 at 30° to the original line of motion; the second puck leaves with speed v_2 at 60°, as in the figure.

   (a) Calculate the speeds v_1 and v_2 . [5 pts]



   (b) How much kinetic energy was lost or gained in the collision, if any? [2 pts]

Picture the Problem Let the direction of motion of the puck that is moving before the collision be the $+x$ direction. Applying conservation of momentum to the collision in both the x and y directions will lead us to two equations in the unknowns v_1 and v_2 that we can solve simultaneously. We can decide whether the collision was elastic by either calculating the system's kinetic energy before and after the collision or by determining whether the angle between the final velocities is 90°.

(*a*) Use conservation of linear momentum in the x direction to obtain:

$$\boldsymbol{p}_{\mathrm{xi}} = \boldsymbol{p}_{\mathrm{xf}}$$
or
$$\boldsymbol{mv} = \boldsymbol{mv}_1 \cos 30° + \boldsymbol{mv}_2 \cos 60°$$

Simplify further to obtain:

$$\boldsymbol{v} = \boldsymbol{v}_1 \cos 30° + \boldsymbol{v}_2 \cos 60° \qquad (1)$$

Use conservation of momentum in the y direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$\boldsymbol{p}_{\mathrm{yi}} = \boldsymbol{p}_{\mathrm{yf}}$$
or
$$0 = \boldsymbol{mv}_1 \sin 30° - \boldsymbol{mv}_2 \sin 60°$$

Simplifying further yields:

$$0 = \boldsymbol{v}_1 \sin 30° - \boldsymbol{v}_2 \sin 60° \qquad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$\boldsymbol{v}_1 = \boxed{1.7\,\mathrm{m/s}} \text{ and } \boldsymbol{v}_2 = \boxed{1.0\,\mathrm{m/s}}$$

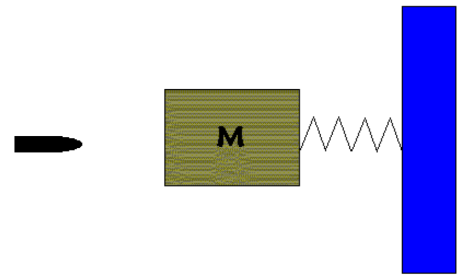
(*b*) Because the angle between $\vec{\boldsymbol{v}}_1$ and $\vec{\boldsymbol{v}}_2$ is 90°, the collision was elastic.

The fact that the collision is elastic can be determined by totalling the kinetic energies before and after and finding that they are about the same. (Neglecting round-off errors.) or that the angles between the two final velocities are 90 degrees and that the masses are the same. Which occurs if and only if the collision is elastic for equal masses.

(c) If the two pucks are in contact for 10 milliseconds, what would be the magnitude of the average force by the incoming puck on the puck that was hit, and the direction of that force? Explain your reasoning.[3 pts]

The change of momentum of puck 2 is $\Delta p = m_2 \Delta v_2$ = 5 kg-m/s.
$F_{avg} = \Delta p/\Delta t = 5/0.010$ = 500N, or 5/0.015 = 333 N.

12. A bullet with mass m = 20 grams and velocity v = 100 m/s collides with a wooden block of mass M = 2 kg and stays embedded in it. The wooden block is initially at rest, and is connected to a spring with k = 800 N/m. The other end of the spring is attached to an immovable wall. Note: You may assume that the spring is massless and that the collision between the bullet and the wooden block is completely inelastic.
    (a) Calculate the momentum of the bullet before the collision. [2]
    $p = mv = 0.02 \times 100 = 2$ kg-m/s.
    For version b: $p = mv = 0.025 \times 90 = 2.25$ kg-m/s.



    (b) What is the kinetic energy of the bullet before collision? [2]
    $\frac{1}{2}mv^2 = .5 \times 0.020 \times 100^2 = 100.$ J
    Version b: $\frac{1}{2}mv^2 = .5 \times 0.025 \times 90^2 = 101.25$ J.

    (c) What is the kinetic energy of the block after the collision? [3]
    The momentum is conserved during the impact of the bullet.

$$mv = (m + M)V$$

$$V = \frac{m}{m + M}v \approx 1 \text{ m/s}$$

(We neglected the bullet in the approximation. Including the bullet gives $V = 0.99$ m/s. The difference is insignificant.)

$$\frac{1}{2}MV^2 = 0.5 \times 2 \times 1^2 = 1.0 \text{ J}$$

Version b:

$$V = \frac{m}{m + M}v = 1.111 \text{ m/s}$$

$$\frac{1}{2}MV^2 = 0.5 \times 2 \times 1^2 = 1.23 \text{ J}$$

    (d) What is the maximum compression of the spring? [3]
    Let x be the maximum compression.

$$\frac{1}{2}kx^2 = 1 \text{ J}$$

$$x = \sqrt{2/k} = \sqrt{1/400} = 1/20 \text{ m}$$

Version b:

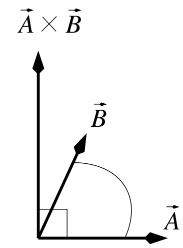
$$x = \sqrt{2 \times 1.23/k} = \sqrt{1.23/350} = 0.0598 \text{ m}$$

Formula Sheet

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = AB\cos\theta$

$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$ $\quad \left|\vec{A} \times \vec{B}\right| = AB\sin\theta$

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



$\vec{v} = \dfrac{d\vec{x}}{dt}$ $\quad \vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{x}}{dt^2}$ $\quad \Delta v = a_{\text{avg}}\,\Delta t$

For constant a: $\quad \Delta x = \dfrac{1}{2}[v_{\text{i}} + v_{\text{f}}]\,\Delta t = v_{\text{i}}\,\Delta t + \dfrac{1}{2}a\,\Delta t^2$

For constant a: $\quad v_{\text{f}}^2 = v_{\text{i}}^2 + 2a\,\Delta x$

Newton's 2nd: $\quad \sum_n \vec{F}_n = m\vec{a} = \dfrac{d\vec{p}}{dt}$ $\quad \vec{p} = m\vec{v}$

Friction: $\quad f_{\text{k}} = \mu_{\text{k}} F_{\text{N}}$ $\quad f_{\text{s,max}} = \mu_{\text{s}} F_{\text{N}}$

$K = \dfrac{1}{2}mv^2$ $\quad U_{\text{g}} = mgy$ $\quad U_{\text{g}} = -G\dfrac{Mm}{r}$

$\vec{J} = \displaystyle\int_{t_1}^{t_2} \vec{F}\,dt$ $\quad W = \displaystyle\int_{x_{\text{i}}}^{x_{\text{f}}} \vec{F} \cdot d\vec{x}$ $\quad P = \dfrac{dW}{dt}$

$\Delta U = -\displaystyle\int_{x_{\text{i}}}^{x_{\text{f}}} \vec{F} \cdot d\vec{x}$ $\quad \vec{r}_{\text{cm}} = \dfrac{\sum_n m_n \vec{r}_n}{\sum_n m_n}$

$\vec{\alpha} = \dfrac{d\vec{\omega}}{dt} = \dfrac{d^2\vec{\theta}}{dt^2}$ $\quad \Delta\omega = \alpha_{\text{avg}}\,\Delta t$ $\quad \omega = \dfrac{d\theta}{dt}$

For constant α: $\quad \Delta\theta = \omega_{\text{i}}\,\Delta t + \dfrac{1}{2}\alpha\,\Delta t^2$

$I = \sum_n m_n r_n^2$ $\quad \sum_n \tau_n = I\alpha$ $\quad \vec{L} = \vec{r} \times \vec{p}$

$\vec{\tau} = \dfrac{d\vec{L}}{dt}$ $\quad L = mv_{\text{t}}r \quad L = I\omega \quad K = \dfrac{1}{2}I\omega^2$

(In general $\theta, \omega, \alpha, \tau$ are vectors, but most physics 120 problems only need their magnitudes.)

Uniform Circular Motion: $\quad F_{\text{c}} = \dfrac{mv^2}{r}$

SHM: $F_{\text{s}} = -k\,\Delta s$ $\quad \omega = \sqrt{\dfrac{k}{m}}$ $\quad U_{\text{s}} = \dfrac{1}{2}k\,\Delta s^2$

$\omega = 2\pi f$ $\quad f = 1/T$

Pendulum, small amplitudes: $\quad \omega = \sqrt{\dfrac{g}{l}}$

Universal Gravitation: $\quad F_{12} = -\dfrac{Gm_1 m_2}{r_{12}^2}$ $\quad G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$

$M_{\text{earth}} = 5.09 \times 10^{24}$ kg $\quad r_{\text{earth}} = 6.37 \times 10^6$ m (mean) $\quad g_{\text{earth}} = 9.81$ N/kg (mean)

$M_{\text{moon}} = 7.34 \times 10^{22}$ kg $\quad r_{\text{moon}} = 1.74 \times 10^6$ m (mean) $\quad c = 3.0 \times 10^8$ m/s

Moments of Inertia (Rotational Inertias) about the centre of mass.
Uniform disk: $I = \frac{1}{2}MR^2$, Hoop: $I = MR^2$, Rod: $\frac{1}{12}M\ell^2$, Solid Sphere: $I = \frac{2}{5}MR^2$, Hollow Sphere: $I = \frac{2}{3}MR^2$