33.10. Solve: From Example 33.5, the on-axis magnetic field of a current loop is

$$B_{\text{loop}}(z) = \frac{\mu_0}{2}\frac{IR^2}{\left(z^2 + R^2\right)^{\frac{3}{2}}}.$$

We want to find the value of z such that $B(z) = 2B(0)$.

$$\frac{\mu_0}{2}\frac{IR^2}{\left(z^2 + R^2\right)^{\frac{3}{2}}} = 2\frac{\mu_0}{2}\frac{IR^2}{\left(R^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \left(z^2 + R^2\right)^{\frac{3}{2}} = \frac{R^3}{z} \Rightarrow z^2 + R^2 = \frac{R^2}{2^{\frac{2}{3}}} \Rightarrow z = R\left(2^{-\frac{2}{3}} - 1\right)^{\frac{1}{2}} = 0.77\text{ R}$$

33.12. Model: The magnetic field is the superposition of the magnetic fields of three wire segments.
Visualize: Please refer to Figure EX33.12.
Solve: The magnetic field of the horizontal wire, with current *I*, encircles the wire. Because the dot is on the axis of the wire, the input current creates no magnetic field at this point. The current divides at the junction, with *I*/2 traveling upward and *I*/2 traveling downward. The right-hand rule tells us that the upward current creates a field at the dot that is into the page; the downward current creates a field that is out of the page. Although we could calculate the strength of each field, the symmetry of the situation (the dot is the same distance and direction from the base of each wire) tells us that the fields of the upward and downward current must have the same strength. Since they are in opposite directions, their sum is $\vec{0}$. Altogether, then, the field at the dot is $\vec{B} = \vec{0}\text{ T}$.

33.22. Model: Only the two currents enclosed by the closed path contribute to the line integral.
Visualize: Please refer to Figure EX33.22.
Solve: Ampere's law gives the line integral of the magnetic field around the closed path:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} = 1.38 \times 10^{-5} \text{ T m } = \mu_0 (I_2 + I_3) = (4\pi \times 10^{-7} \text{ T m/A})(8.0 \text{ A} + I_3)$$

$$\Rightarrow (I_3 + 8.0 \text{ A}) = \frac{1.38 \times 10^{-5} \text{ T m}}{4\pi \times 10^{-7} \text{ T m/A}} \Rightarrow I_3 = 3.0 \text{ A, out of the page.}$$

Assess: The right-hand rule was used above to assign a positive sign to I_2. Since I_3 is also positive, it is in the same direction as I_2.

33.26. Model: A magnetic field exerts a magnetic force on a moving charge.
Visualize: Please refer to Figure EX33.26.
Solve: **(a)** The force on the charge is

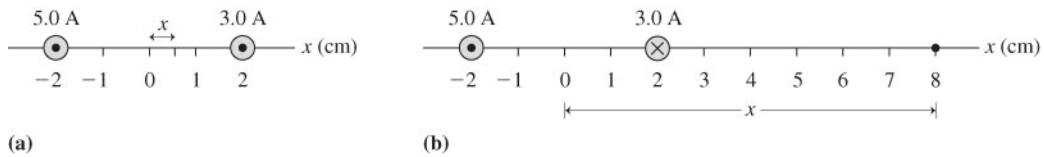
$$\vec{F}_{\text{on q}} = q\vec{v}\times\vec{B} = \left(1.60\times10^{-19}\text{ C}\right)\left(1.0\times10^{7}\text{ m/s}\right)\left(\cos45°\hat{\imath}+\sin45°\hat{k}\right)\times\left(0.50\hat{\imath}\text{ T}\right)$$

$$= \left(1.60\times10^{-19}\text{ C}\right)\left(0.50\text{ T}\right)\frac{\left(1.0\times10^{7}\text{ m/s}\right)}{\sqrt{2}}\left(\hat{\imath}\times\hat{\imath}+\hat{k}\times\hat{\imath}\right) = +5.7\times10^{-13}\,\hat{\jmath}\text{ N}$$

(b) Because the cross product $\hat{\imath}\times\hat{\imath}$ in the equation for the force is zero, $\vec{F}_{\text{on q}} = \vec{0}$ N.

33.42. Model: The magnetic field is that of two long wires that carry current.

Visualize:



Solve: **(a)** For $x > +2$ cm and for $x < -2$ cm, the magnetic fields due to the currents in the two wires add. The point where the two magnetic fields cancel lies on the *x*-axis in between the two wires. Let that point be a distance *x* away from the origin. Because the magnetic field of a long wire is $B = \mu_0 I/2\pi r$, we have

$$\frac{\mu_0}{2\pi}\frac{(5.0\text{ A})}{(0.020\text{ m}+x)} = \frac{\mu_0}{2\pi}\frac{(3.0\text{ A})}{(0.020\text{ m}-x)} \Rightarrow 5(0.020\text{ m}-x) = 3(0.020\text{ m}+x) \Rightarrow x = 0.0050\text{ m} = 0.50\text{ cm}$$

(b) The magnetic fields due to the currents in the two wires add in the region -2.0 cm $< x < 2.0$ cm. For $x < -2.0$ cm, the magnetic fields subtract, but the field due to the 5.0 A current is always larger than the field due to the 3.0 A current. However, for $x > 2.0$ m, the two fields will cancel at a point on the *x*-axis. Let that point be a distance *x* away from the origin, so

$$\frac{\mu_0}{2\pi}\frac{5.0\text{ A}}{x+0.020\text{ m}} = \frac{\mu_0}{2\pi}\frac{3.0\text{ A}}{x-0.020\text{ m}} \Rightarrow 5(x-0.020\text{ m}) = 3(x+0.020\text{ m}) \Rightarrow x = 8.0\text{ cm}$$

33.46. Model: Use the Biot-Savart law for a current carrying segment.
Visualize: Please refer to Figure P33.46.
Solve: **(a)** The Biot-Savart law (Equation 33.6) for the magnetic field of a current segment $\Delta\vec{s}$ is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

where the unit vector $\hat{r}$ points from current segment Δs to the point, a distance r away, at which we want to evaluate the field. For the two linear segments of the wire, $\Delta\vec{s}$ is in the same direction as $\hat{r}$, so $\Delta\vec{s}\times\hat{r} = 0$. For the curved segment, $\Delta\vec{s}$ and $\hat{r}$ are always perpendicular, so $\Delta\vec{s}\times\hat{r} = \Delta s$. Thus

$$B = \frac{\mu_0}{4\pi}\frac{I\Delta s}{r^2}$$

Now we are ready to sum the magnetic field of all the segments at point P. For all segments on the arc, the distance to point P is $r = R$. The superposition of the fields is

$$B = \frac{\mu_0}{4\pi}\frac{I}{R^2}\int_{\text{arc}} ds = \frac{\mu_0}{4\pi}\frac{IL}{R^2} = \frac{\mu_0 I\theta}{4\pi R}$$

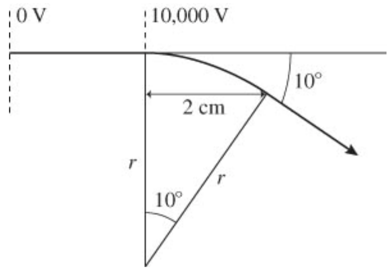
where $L = R\theta$ is the length of the arc.
(b) Substituting $\theta = 2\pi$ in the above expression,

$$B_{\text{loop center}} = \frac{\mu_0 I 2\pi}{4\pi R} = \frac{\mu_0}{2}\frac{I}{R}$$

This is Equation 33.7, which is the magnetic field at the center of a 1-turn coil.

33.59. **Model:** Energy is conserved as the electron moves between the two electrodes. Assume the electron starts from rest. Once in the magnetic field, the electron moves along a circular arc.

Visualize:



The electron is deflected by 10° after moving along a circular arc of angular width 10°.

Solve: Energy is conserved as the electron moves from the 0 V electrode to the 10,000 V electrode. The potential energy is $U = qV$ with $q = -e$, so

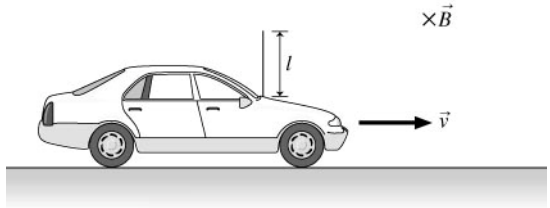
$$K_f + U_f = K_i + U_i \Rightarrow \tfrac{1}{2}mv^2 - eV = 0 + 0$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\ \text{C})(10{,}000\ \text{V})}{9.11\times10^{-31}\ \text{kg}}} = 5.93\times10^{7}\ \text{m/s}$$

The radius of cyclotron motion in a magnetic field is $r = mv/eB$. From the figure we see that the radius of the circular arc is $r = (2.0\ \text{cm})/\sin10°$. Thus

$$B = \frac{mv}{er} = \frac{(9.11\times10^{-31}\ \text{kg})(5.93\times10^{7}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.020\ \text{m})/\sin10°} = 2.9\times10^{-3}\ \text{T}$$

34.1. Visualize:



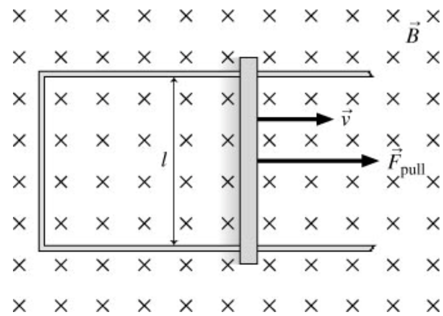
To develop a motional emf the magnetic field needs to be perpendicular to both, so let's say its direction is into the page.

Solve: This is a straightforward use of Equation 34.3. We have

$$v = \frac{\mathcal{E}}{lB} = \frac{1.0\ \text{V}}{(1.0\ \text{m})(5.0\times10^{-5}\ \text{T})} = 2.0\times10^{4}\ \text{m/s}$$

Assess: This is an unreasonable speed for a car. It's unlikely you'll ever develop a volt.

34.3. Visualize:



The wire is pulled with a constant force in a magnetic field. This results in a motional emf and produces a current in the circuit. From energy conservation, the mechanical power provided by the puller must appear as electrical power in the circuit.

Solve: (a) Using Equation 34.6,

$$P = F_{\text{pull}}v \Rightarrow v = \frac{P}{F_{\text{pull}}} = \frac{4.0 \text{ W}}{1.0 \text{ N}} = 4.0 \text{ m/s}$$

(b) Using Equation 34.6 again,

$$P = \frac{v^2 l^2 B^2}{R} \Rightarrow B = \sqrt{\frac{RF_{\text{pull}}}{vl^2}} = \sqrt{\frac{(0.20\ \Omega)(1.0 \text{ N})}{(4.0 \text{ m/s})(0.10 \text{ m})^2}} = 2.2 \text{ T}$$

Assess: This is reasonable field for the circumstances given.

34.5. Model: Consider the solenoid to be long so the field is constant inside and zero outside.

Visualize: Please refer to Figure Ex34.5. The field of a solenoid is along the axis. The flux through the loop is only nonzero inside the solenoid. Since the loop completely surrounds the solenoid, the total flux through the loop will be the same in both the perpendicular and tilted cases.

Solve: The field is constant inside the solenoid so we will use Equation 34.10. Take $\vec{A}$ to be in the same direction as the field. The magnetic flux is

$$\Phi = \vec{A}_{\text{loop}} \cdot \vec{B}_{\text{loop}} = \vec{A}_{\text{sol}} \cdot \vec{B}_{\text{sol}} = \pi r_{\text{sol}}^2 B_{\text{sol}} \cos\theta = \pi(0.010 \text{ m})^2 (0.20 \text{ T}) = 6.3 \times 10^{-5} \text{ Wb}$$

When the loop is tilted the component of $\vec{B}$ in the direction of $\vec{A}$ is less, but the effective area of the loop surface through which the magnetic field lines cross is increased by the same factor.

34.12. Model: Assume the field is uniform.

Visualize: Please refer to Figure Ex34.12. The motion of the loop changes the flux through it. This results in an induced emf and current.

Solve: The induced emf is $\mathcal{E} = |d\Phi/dt|$ and the induced current is $I = \mathcal{E}/R$. The area A is changing, but the field B is not. Take $\vec{A}$ as being out of the page and parallel to $\vec{B}$, so $\Phi = AB$ and $d\Phi/dt = B(dA/dt)$. The flux is through that portion of the loop where there is a field, that is, $A = lx$. The emf and current are

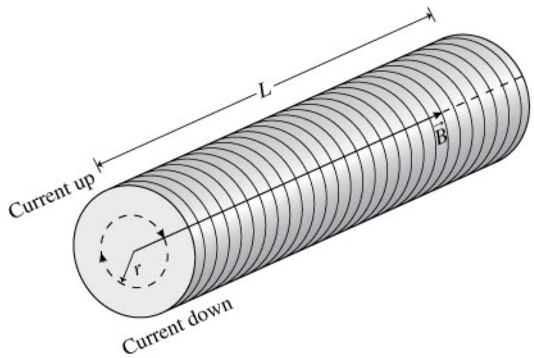
$$\mathcal{E} = B\left|\frac{dA}{dt}\right| = B\left|\frac{d(lx)}{dt}\right| = Blv = \left|(0.20\text{ T})(0.050\text{ m})(50\text{ m/s})\right| = 0.50\text{ V}$$

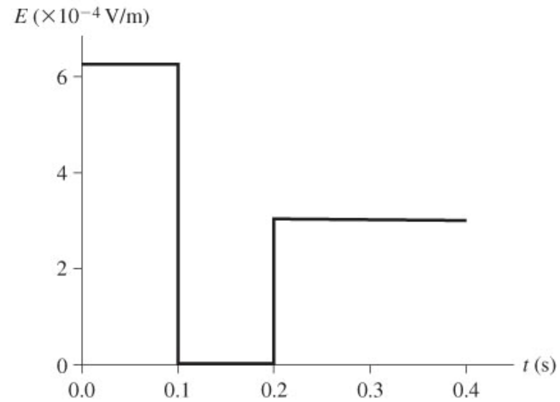
$$I = \frac{\mathcal{E}}{R} = \frac{0.50\text{ V}}{0.10\Omega} = 5.0\text{ A}$$

The field is out of the page. As the loop moves the flux increases because more of the loop area has field through it. To *prevent the increase*, the induced field needs to point into the page. Thus, the induced current flows *clockwise*.

Assess: This seems reasonable since there is rapid motion of the loop.

34.15. Model: A changing magnetic field creates an electric field.
Visualize: Please refer to Figure Ex34.15 in your textbook.





Solve: The magnetic field produced by a current I in a solenoid is

$$B_{\text{solenoid}} = \frac{\mu_0 NI}{L}$$

The magnitude of the induced electric field inside a solenoid is given by Equation 34.26. The direction can be found using Lenz's law, but the problem only asks for field strength, which is positive.
Only the current is changing, thus

$$E = \frac{(4\pi \times 10^{-7}\ \text{T m/A})(0.010\ \text{m})(400)}{2(0.20\ \text{m})}\left|\frac{dI}{dt}\right| = \left(1.26\times 10^{-5}\ \frac{\text{V s}}{\text{A m}}\right)\left|\frac{dI}{dt}\right|$$

In the interval $t = 0$ s to $t = 0.1$ s,

$$\left|\frac{dI}{dt}\right| = \frac{5\ \text{A}}{0.1\ \text{s}} = 50\ \text{A/s}$$

In the interval t = 0.1 s to 0.2 s, the change in the current is zero. In the interval t = 0.2 s to 0.4 s, $|dI/dt| = 25$ A/s.

Thus in the interval $t = 0$ s to $t = 0.1$ s, $E = 6.3 \times 10^{-4}$ V/m. In the interval $t = 0.1$ s to 0.2 s, E is zero. In the interval t = 0.2 s to 0.4 s, $E = 3.1 \times 10^{-4}$ V/m. The E-versus-t graph is shown earlier, in the Visualize step.

34.47. Model: Assume the field changes abruptly at the boundary and is uniform.

Visualize: Please refer to Figure P34.47. As the loop enters the field region the amount of flux will change as more area has field penetrating it. This change in flux will create an induced emf and corresponding current. While the loop is moving at constant speed, the rate of change of the area is not constant because of the orientation of the loop. The loop is moving along the *x*-axis.

Solve: **(a)** If the edge of the loop enters the field region at $t = 0$ s, then the leading corner has moved a distance $x = v_0 t$ at time *t*. The area of the loop with flux through it is

$$A = 2\left(\tfrac{1}{2}\right)yx = x^2 = \left(v_0 t\right)^2$$

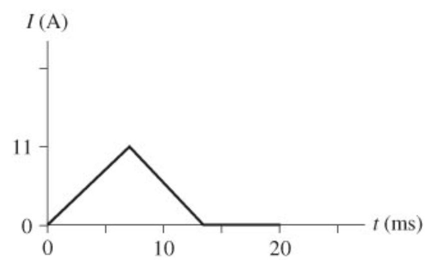
where we have used the fact that $y = x$ since the sides of the loop are oriented at $45°$ to the horizontal. Take the surface normal of the loop to be into the page so that $\Phi = \vec{A}\cdot\vec{B} = BA$. The current in the loop is

$$I = \frac{\mathcal{E}}{R} = \frac{1}{R}\left|\frac{d\Phi}{dt}\right| = \frac{1}{R}B\left|\frac{dA}{dt}\right| = \frac{1}{R}B\left|\frac{d}{dt}\left(v_0 t\right)^2\right| = \frac{1}{R}B(2)v_0^2 t = \left(\frac{2(0.80\text{ T})(10\text{ m/s})^2}{0.10\Omega}\right)t = \left(1.6\times10^3\text{ A}\right)t$$

The current is increasing at a constant rate. This expression is good until the loop is halfway into the field region. The time for the loop to be halfway is found as follows:

$$\frac{10\text{ cm}}{\sqrt{2}} = v_0 t = (10\text{ m/s})t \quad\Rightarrow\quad t = 7.1\times10^{-3}\text{ s} = 7.1\text{ ms}$$

At this time the current is 11 A. While the second half of the loop is moving into the field, the flux continues to increase, but at a slower rate. Therefore, the current will decrease at the same rate as it increased before, until the loop is completely in the field at $t = 14$ ms. After that the flux will not change and the current will be zero.



(b) The maximum current of 11 A occurs when the flux is changing the fastest and this occurs when the loop is halfway into the region of the field.

34.51. Model: Assume that the magnetic field is uniform in the region of the loop.
Visualize: Please refer to Figure P34.51. The rotating semicircle will change the area of the loop and therefore the flux through the loop. This changing flux will produce an induced emf and corresponding current in the bulb.
Solve: (a) The spinning semicircle has a normal to the surface that changes in time, so while the magnetic field is constant, the area is changing. The flux through in the lower portion of the circuit does not change and will not contribute to the emf. Only the flux in the part of the loop containing the rotating semicircle will change. The flux associated with the semicircle is

$$\Phi = \vec{A} \cdot \vec{B} = BA = BA\cos\theta = BA\cos(2\pi ft)$$

where $\theta = 2\pi ft$ is the angle between the normal of the rotating semicircle and the magnetic field and A is the area of the semicircle. The induced current from the induced emf is given by Faraday's law. We have

$$I = \frac{\mathcal{E}}{R} = \frac{1}{R}\left|\frac{d\Phi}{dt}\right| = \frac{1}{R}\left|\frac{d}{dt}BA\cos(2\pi ft)\right| = \frac{B}{R}\frac{\pi r^2}{2}2\pi f\sin(2\pi ft)$$
$$= \frac{2(0.20\text{ T})\pi^2(0.050\text{ m})^2}{2(1.0\Omega)} f\sin(2\pi ft) = 4.9\times10^{-3} f\sin(2\pi ft)\text{ A}$$

where the frequency f is in Hz.
(b) We can now solve for the frequency necessary to achieve a certain current. From our study of DC circuits we know how power relates to resistance:

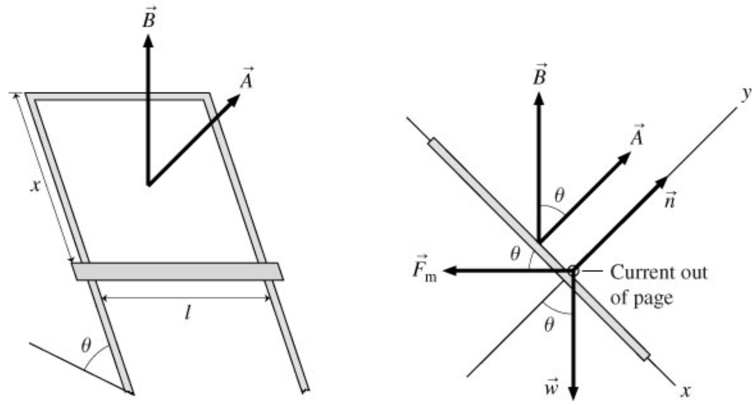
$$P = I^2R \Rightarrow I = \sqrt{P/R} = \sqrt{4.0\text{ W}/1.0\Omega} = 2.0\text{ A}$$

The maximum of the sine function is +1, so the maximum current is

$$I_{\text{max}} = 4.9\times10^{-3} f\ \text{ A s} = 2.0\text{ A} \Rightarrow f = \frac{2.0\text{ A}}{4.9\times10^{-3}\text{ A s}} = 4.1\times10^{2}\text{ Hz}$$

Assess: This is not a reasonable frequency to obtain by hand.

34.53. Model: Assume the magnetic field is uniform in the region of the loop.
Visualize:



The moving wire creates a changing area and corresponding change in flux. This produces an induced emf and induced current. The flux through the loop depends on the size and orientation of the loop.

Solve: **(a)** The normal to the surface is perpendicular to the loop and the flux is $\Phi_{\text{inner}} = \vec{A}\cdot\vec{B} = AB\cos\theta$. We can get the current from Faraday's law. Since the loop area is $A = lx$, We have

$$I = \frac{\mathcal{E}}{R} = \frac{1}{R}\left|\frac{d\Phi}{dt}\right| = \frac{1}{R}\left|\frac{d}{dt}lxB\cos\theta\right| = \frac{Bl\cos\theta}{R}\left|\frac{dx}{dt}\right| = \frac{Blv\cos\theta}{R}$$

(b) Using the free-body diagram shown in the figure, we can apply Newton's second law. The magnetic force on a straight, current-carrying wire is $F_{\text{m}} = IlB$ and is horizontal. Using the current I from part (a) gives

$$\sum F_x = -F_{\text{m}}\cos\theta + mg\sin\theta = -\frac{B^2l^2v\cos^2\theta}{R} + mg\sin\theta = ma_x$$

Terminal speed is reached when a_x drops to zero. In this case, the two terms are equal and we have

$$v_{\text{term}} = \frac{mgR\tan\theta}{l^2B^2\cos\theta}$$