

Physics 121 - Assignment #3

Include your name, Student ID number, T.A. name, and tutorial section number (e.g. D108) on the first page of your assignment.

Show all your work for full credit.

Do the following problems from *Physics for Scientists and Engineers Second Edition - A Strategic Approach* by Randall Knight.

1. 23.2	5. 23.48
2. 23.14	6. 23.50
3. 23.16	
4. 23.20	7. 23.56

23.2 Light rays travel in straight lines in each medium. So, let the times in each medium be given by $t_{glass}, t_{oil}, t_{plastic}$. Using glass as an example we have

$$t_{glass} = \frac{\Delta x}{v_{glass}}$$

However, we know the speed of light in glass given its index of refraction:

$$v_{glass} = \frac{c}{n_{glass}}$$

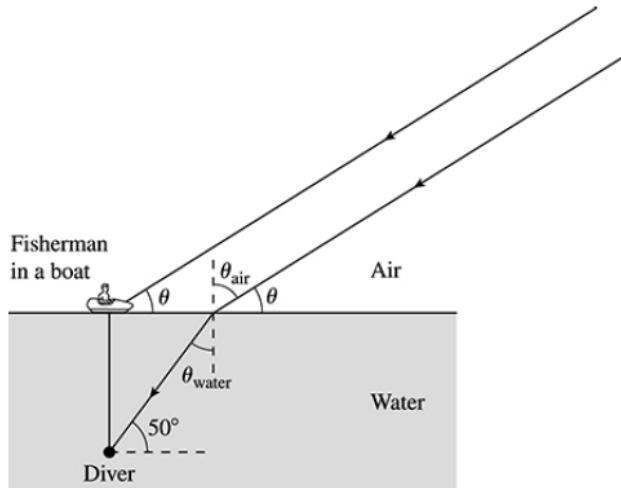
So, we can write

$$t_{glass} = \frac{\Delta x}{c/n_{glass}} = \frac{\Delta x n_{glass}}{c} = \frac{1.0 \times 10^{-2} (1.50)}{3.0 \times 10^8 \text{ m/s}} = 0.050 \text{ ns}$$

Similarly, $t_{oil} = 0.243 \text{ ns}$ and $t_{plastic} = 0.106 \text{ ns}$. Thus, $t_{total} = t_{glass} + t_{oil} + t_{plastic} = 0.40 \text{ ns}$.

23.14 Question 2

Visualize:



A ray that arrives at the diver 50° above horizontal refracted into the water at $\theta_{\text{water}} = 40^\circ$.

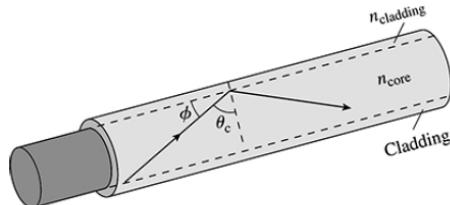
Solve: Using Snell's law at the water-air boundary

$$\begin{aligned} n_{\text{air}} \sin \theta_{\text{air}} &= n_{\text{water}} \sin \theta_{\text{water}} \Rightarrow \sin \theta_{\text{air}} = \frac{n_{\text{water}}}{n_{\text{air}}} \sin \theta_{\text{water}} = \left(\frac{1.33}{1.0} \right) \sin 40^\circ \\ &\Rightarrow \theta_{\text{air}} = 58.7^\circ \end{aligned}$$

Thus the height above the horizon is $\theta = 90^\circ - \theta_{\text{air}} = 31.3^\circ \approx 31^\circ$. Because the sun is far away from the fisherman (and the diver), the fisherman will see the sun at the same angle of 31° above the horizon.

23.16. Model: Use the ray model of light. For an angle of incidence greater than the critical angle, the ray of light undergoes total internal reflection.

Visualize:



Solve: The critical angle of incidence is given by Equation 23.9:

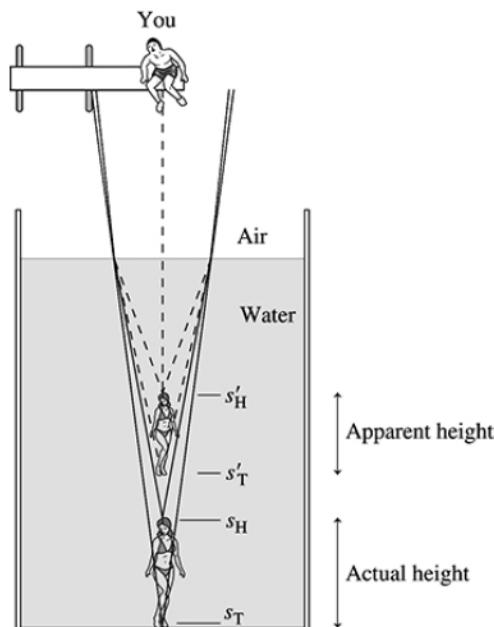
$$\theta_c = \sin^{-1} \left(\frac{n_{\text{cladding}}}{n_{\text{core}}} \right) = \sin^{-1} \left(\frac{1.48}{1.60} \right) = 67.7^\circ$$

Thus, the maximum angle a light ray can make with the wall of the core to remain inside the fiber is $90^\circ - 67.7^\circ = 23.3^\circ$.

Assess: We can have total internal reflection because $n_{\text{core}} > n_{\text{cladding}}$.

23.20. Model: Represent the diver's head and toes as point sources. Use the ray model of light.

Visualize:



Paraxial rays from the head and the toes of the diver refract into the air and then enter into your eyes. When these refracted rays are extended into the water, the head and the toes appear elevated toward you.

Solve: Using Equation 23.13,

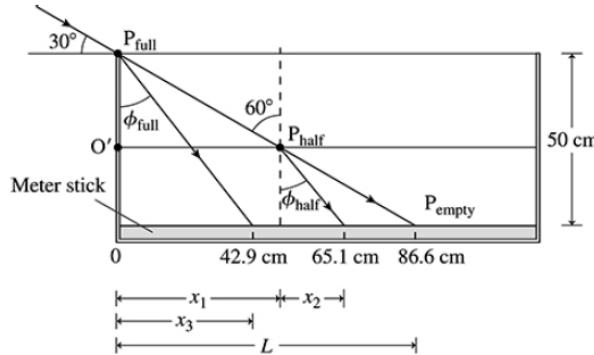
$$s'_T = \frac{n_2}{n_1} s_T = \frac{n_{\text{air}}}{n_{\text{water}}} s_T \quad s'_H = \frac{n_{\text{air}}}{n_{\text{water}}} s_H$$

Subtracting the two equations, her apparent height is

$$s'_H - s'_T = \frac{n_{\text{air}}}{n_{\text{water}}} (s_H - s_T) = \frac{1.0}{1.33} (150 \text{ cm}) = 113 \text{ cm}$$

23.48. Model: Use the ray model of light and the law of refraction.

Visualize:



Solve: (a) The ray of light strikes the meter stick at P_{empty} , which is a distance L from the zero mark of the meter stick. So,

$$\tan 60^\circ = \frac{L}{50 \text{ cm}} \Rightarrow L = (50 \text{ cm}) \tan 60^\circ = 86.6 \text{ cm}$$

(b) The ray of light refracts at P_{half} and strikes the meter stick a distance $x_1 + x_2$ from the zero of the meter stick. We can find x_1 from the triangle $P_{\text{full}}P_{\text{half}}O'$:

$$\tan 60^\circ = \frac{x_1}{25 \text{ cm}} \Rightarrow x_1 = (25 \text{ cm}) \tan 60^\circ = 43.30 \text{ cm}$$

We also have $x_2 = (25 \text{ cm}) \tan \phi_{\text{half}}$. Using Snell's law,

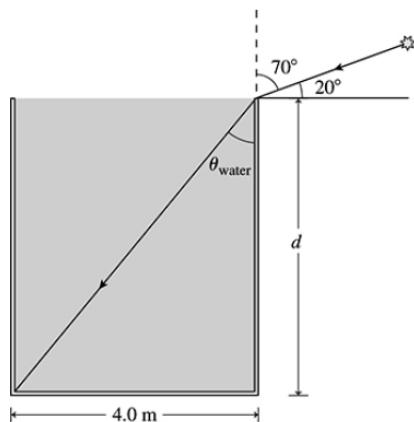
$$n_{\text{air}} \sin 60^\circ = n_{\text{water}} \sin \phi_{\text{half}} \Rightarrow \phi_{\text{half}} = \sin^{-1} \left(\frac{\sin 60^\circ}{1.33} \right) = 40.63^\circ$$

$$\Rightarrow x_2 = (25 \text{ cm}) \tan 40.63^\circ = 21.45 \text{ cm} \Rightarrow x_1 + x_2 = 43.30 \text{ cm} + 21.45 \text{ cm} = 64.8 \text{ cm}$$

(c) The ray of light experiences refraction at P_{full} and the angle of refraction is the same as in part (b). We get

$$\tan \phi_{\text{full}} = \frac{x_3}{50 \text{ cm}} \Rightarrow x_3 = (50 \text{ cm}) \tan 40.63^\circ = 42.9 \text{ cm}$$

23.50. Model: Use the ray model of light and the law of refraction. Assume the sun is a point source of light.
Visualize:



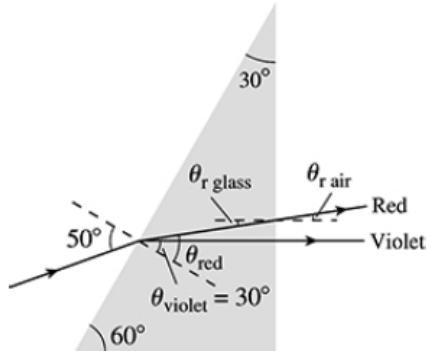
When the bottom of the pool becomes completely shaded, a ray of light that is incident at the top edge of the swimming pool does not reach the bottom of the pool after refraction.

Solve: The depth of the swimming pool is $d = 4.0 \text{ m} / \tan \theta_{\text{water}}$. We will find the angle by using Snell's law. We have

$$n_{\text{water}} \sin \theta_{\text{water}} = n_{\text{air}} \sin 70^\circ \Rightarrow \theta_{\text{water}} = \sin^{-1} \left(\frac{\sin 70^\circ}{1.33} \right) = 44.95^\circ \Rightarrow d = \frac{4.0 \text{ m}}{\tan 44.95^\circ} = 4.0 \text{ m}$$

23.56. Model: Use the ray model of light and the phenomena of refraction and dispersion.

Visualize:



Solve: Since violet light is perpendicular to the second surface, it must reflect at $\theta_{\text{violet}} = 30^\circ$ at the first surface. Using Snell's law at the air-glass boundary where the ray is incident,

$$n_{\text{air}} \sin \theta_{\text{air}} = n_{\text{violet}} \sin \theta_{\text{violet}} \Rightarrow n_{\text{violet}} = \frac{n_{\text{air}} \sin \theta_{\text{air}}}{\sin \theta_{\text{violet}}} = \frac{(1.0) \sin 50^\circ}{\sin 30^\circ} = 1.5321$$

Since $n_{\text{violet}} = 1.02 n_{\text{red}}$, $n_{\text{red}} = 1.5021$. Using Snell's law for the red light at the first surface

$$n_{\text{red}} \sin \theta_{\text{red}} = n_{\text{air}} \sin \theta_{\text{air}} \Rightarrow \theta_{\text{red}} = \sin^{-1} \left(\frac{1.0 \sin 50^\circ}{1.5021} \right) = 30.664^\circ$$

The angle of incidence on the rear face of the prism is thus $\theta_{\text{r glass}} = 30.664^\circ - 30^\circ = 0.664^\circ$. Using Snell's law once again for the rear face and for the red wavelength,

$$n_{\text{red}} \sin \theta_{\text{r glass}} = n_{\text{air}} \sin \theta_{\text{r air}} \Rightarrow \theta_{\text{r air}} = \sin^{-1} \left(\frac{n_{\text{red}} \sin \theta_{\text{r glass}}}{n_{\text{air}}} \right) = \sin^{-1} \left(\frac{1.5021 \sin 0.664^\circ}{1.0} \right) = 0.997^\circ$$

Because $\theta_{\text{v air}} = 0^\circ$ and $\theta_{\text{r air}} = 0.997^\circ$, $\phi = \theta_{\text{r air}} - \theta_{\text{v air}} = 0.997^\circ \cong 1.00^\circ$.