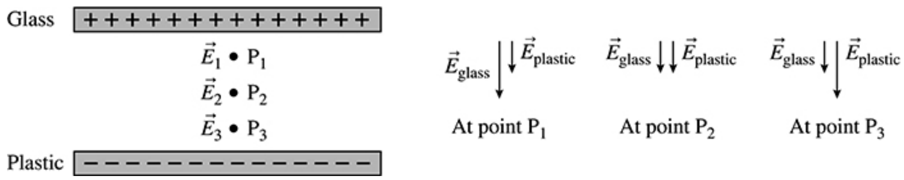
27.8. Model: The rods are thin. Assume that the charge lies along a *line*.

Visualize:



The electric field of the positively charged glass rod points away from the glass rod, whereas the electric field of the negatively charged plastic rod points toward the plastic rod. The electric field strength is the magnitude of the electric field and is always positive.

Solve: Example 27.3 shows that the electric field strength in the plane that bisects a charged rod is

$$E_{rod} = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r\sqrt{r^2+(L/2)^2}}$$

The electric field from the glass rod at $r = 1$ cm from the glass rod is

$$E_{glass} = (9.0\times10^9 \text{ N m}^2/\text{C}^2)\frac{10\times10^{-9}\text{ C}}{(0.01\text{ m})\sqrt{(0.01\text{ m})^2+(0.05\text{ m})^2}} = 1.765\times10^5 \text{ N/C}$$

The electric fields from the glass rod at $r = 2$ cm and $r = 3$ cm are 0.835×10^5 N/C and 0.514×10^5 N/C. The electric field from the plastic rod at distances 1 cm, 2 cm, and 3 cm from the plastic rod are the same as for the glass rod. Point P_1 is 1.0 cm from the glass rod and is 3.0 cm from the plastic rod, point P_2 is 2 cm from both rods, and point P_3 is 3 cm from the glass rod and 1 cm from the plastic rod. Because the direction of the electric fields at P_1 is the same, the net electric field strength 1 cm from the glass rod is the sum of the fields from the glass rod at 1 cm and the plastic rod at 3 cm. Thus

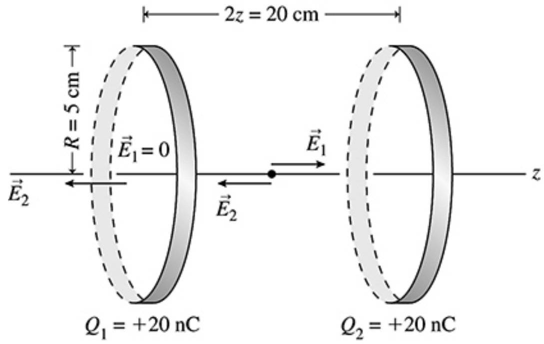
At 1.0 cm $\quad E = 1.765\times10^5 \text{ N/C} + 0.514\times10^5 \text{ N/C} = 2.3\times10^5 \text{ N/C}$

At 2.0 cm $\quad E = 0.835\times10^5 \text{ N/C} + 0.835\times10^5 \text{ N/C} = 1.67\times10^5 \text{ N/C}$

At 3.0 cm $\quad E = 0.514\times10^5 \text{ N/C} + 1.765\times10^5 \text{ N/C} = 2.3\times10^5 \text{ N/C}$

Assess: The electric field strength in the space between the two rods goes through a minimum. This point is exactly in the middle of the line connecting the two rods. Also, note that the arrows shown in the figure are not to scale.

27.12. Model: Assume that the rings are thin and that the charge lies along circle of radius R.
Visualize:



Solve: **(a)** Let the rings be centered on the z-axis. According to Example 27.5, the field of the left ring at z = 10 cm is

$$(E_1)_z = \frac{zQ_1}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}} = \frac{\left(9.0\times10^9 \text{ N m}^2/\text{C}^2\right)(0.10 \text{ m})\left(20\times10^{-9} \text{ C}\right)}{\left[(0.10 \text{ m})^2+(0.050 \text{ m})^2\right]^{3/2}} = 1.29\times10^4 \text{ N/C}$$

That is, $\vec{E}_1 = \left(1.29\times10^4 \text{ N/C, right}\right)$. Ring 2 has the same quantity of charge and is at the same distance, so it will produce a field of the same strength. Because Q_2 is positive, $\vec{E}_2$ will point to the left. The net field at the midpoint between the two rings is $\vec{E} = \vec{E}_1 + \vec{E}_2 = 0$ N/C.

(b) The field of the left ring at z = 0 cm is $(E_1)_z = 0$ N/C. The field of the right ring at z = 20 cm to its left is

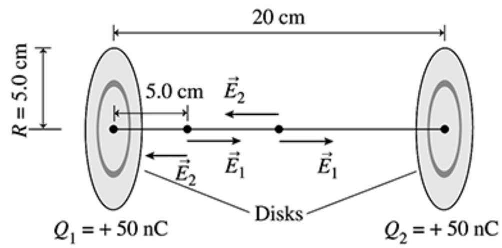
$$(E_2)_z = \frac{\left(9.0\times10^9 \text{ N m}^2/\text{C}^2\right)(0.20 \text{ m})\left(20\times10^{-9} \text{ C}\right)}{\left[(0.20 \text{ m})^2+(0.050)^2\right]^{3/2}} = 4.1\times10^3 \text{ N/C}$$

$$\Rightarrow \vec{E} = \vec{E}_1 + \vec{E}_2 = 0 \text{ N/C} + (4.1\times10^3 \text{ N/C, left})$$

So the electric field strength is 4.1×10^3 N/C.

27.14. Model: Model each disk as a uniformly charged disk. When the disk is positively charged, the on-axis electric field of the disk points away from the disk.

Visualize:



Solve: (a) The surface charge density on the disk is

$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{50\times10^{-9}\ \text{C}}{\pi(0.050\ \text{m})^2} = 6.366\times10^{-6}\ \text{C/m}^2$$

From Equation 27.23, the electric field of the left disk at z = 0.10 m is

$$(E_1)_z = \frac{\eta}{2\varepsilon_0}\left[1-\frac{1}{\sqrt{1+R^2/z^2}}\right] = \frac{6.366\times10^{-6}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)}\left[1-\frac{1}{\sqrt{1+(0.050\ \text{m}/0.10\ \text{m})^2}}\right] = 38{,}000\ \text{N/C}$$

Hence, $\vec{E}_1 = (38{,}000\ \text{N/C, right})$. Similarly, the electric field of the right disk at $z = 0.10$ m (to its left) is $\vec{E}_2 = (38{,}000\ \text{N/C, left})$. The net field at the midpoint between the two disks is $\vec{E} = \vec{E}_1 + \vec{E}_2 = 0$ N/C.

(b) The electric field of the left disk at z = 0.050 m is

$$(E_1)_z = \frac{6.366\times10^{-6}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N m}^2)}\left[1-\frac{1}{\sqrt{1+(0.050\ \text{m}/0.10\ \text{m})^2}}\right] = 1.05\times10^5\ \text{N/C} \Rightarrow \vec{E}_1 = (1.05\times10^5\ \text{N/C, right})$$

Similarly, the electric field of the right disk at z = 0.15 m (to its left) is $\vec{E}_2 = (1.85\times10^4\ \text{N/C, left})$. The net field is thus

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = (8.7\times10^4\ \text{N/C, right})$$

The field strength is 8.7×10^4 N/C.

27.22. Model: A uniform electric field causes a charge to undergo constant acceleration.
Solve: Kinematics yields the acceleration of the electron.

$$v_1^2 = v_0^2 + 2a\Delta x \Rightarrow a = \frac{v_1^2 - v_0^2}{2\Delta x} = \frac{(4.0\times10^7 \text{ m/s})^2 - (2.0\times10^7 \text{ m/s})^2}{2(0.012 \text{ m})} = 5.0\times10^{16} \text{ m/s}^2$$

The magnitude of the electric field required to obtain this acceleration is

$$E = \frac{F_{\text{net}}}{e} = \frac{m_e a}{e} = \frac{(9.11\times10^{-31} \text{ kg})(5.0\times10^{16} \text{ m/s}^2)}{1.6\times10^{-19} \text{ C}} = 2.8\times10^5 \text{ N/C}.$$

27.28. Model: The electric field is that of three point charges q_1, q_2, and q_3.

Visualize: Please refer to Figure P27.28. Assume the charges are in the x-y plane. The 5.0 nC charge is q_1, the 10 nC charge is q_3, and the −5.0 nC charge is q_2. The net electric field at the dot is $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. The procedure will be to find the magnitudes of the electric fields, to write them in component form, and to add the components.

Solve: **(a)** The electric field produced by q_1 is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{r_1^2} = \frac{\left(9.0\times10^9 \text{ N m}^2/\text{C}^2\right)\left(5.0\times10^{-9} \text{ C}\right)}{\left(0.020 \text{ m}\right)^2} = 112{,}500 \text{ N/C}$$

$\vec{E}_1$ points away from q_1, so in component form $\vec{E}_1 = 112{,}500\hat{i}$ N/C. The electric field produced by q_2 is $E_2 =$ 28,120 N/C. $\vec{E}_2$ points toward q_2, so $\vec{E}_2 = 28{,}120\hat{j}$ N/C . Finally, the electric field produced by q_3 is

$$E_3 = \frac{1}{4\pi\varepsilon_0}\frac{|q_3|}{r_3^2} = \frac{\left(9.0\times10^9 \text{ N m}^2/\text{C}^2\right)\left(10\times10^{-9} \text{ C}\right)}{\left(0.020 \text{ m}\right)^2 + \left(0.040 \text{ m}\right)^2} = 45{,}000 \text{ N/C}$$

$\vec{E}_3$ points away from q_3 and makes an angle $\phi = \tan^{-1}\left(4/2\right) = 63.43°$ with the x-axis. So,

$$\vec{E}_3 = E_3\cos\phi\hat{i} - E_3\sin\phi\hat{j} = \left(20{,}130\hat{i} - 40{,}250\hat{j}\right) \text{ N/C}$$

Adding these three vectors gives

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \left(132{,}600\hat{i} - 12{,}130\hat{j}\right) \text{ N/C} = \left(1.33\times10^5\hat{i} - 1.21\times10^4\hat{j}\right) \text{ N/C}$$

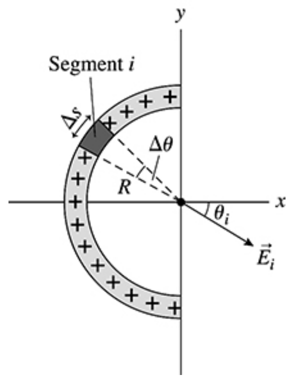
This is in component form.

(b) The magnitude of the field is

$$E_{\text{net}} = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(132{,}600 \text{ N/C}\right)^2 + \left(-12{,}130 \text{ N/C}\right)^2} = 133{,}200 \text{ N/C} = 1.33\times10^5 \text{ N/C}$$

and its angle from the x-axis is $\theta = \tan^{-1}\left(\left|E_x/E_y\right|\right) = 5.2°$. We can also write $\vec{E}_{\text{net}} = (1.33\times10^5$ N/C, 5.2° CW from the $+x$-axis).

27.46. Model: Assume that the semicircular rod is thin and that the charge lies along the semicircle of radius R.
Visualize:



The origin of the coordinate system is at the center of the circle. Divide the rod into many small segments of charge Δq and arc length Δs. Segment i creates a small electric field $\vec{E}_i$ at the origin. The line from the origin to segment i makes an angle θ with the x-axis.

Solve: Because every segment i at an angle θ above the axis is matched by segment j at angle θ below the axis, the
y-components of the electric fields will cancel when the field is summed over all segments. This leads to a net field pointing to the right with

$$E_x = \sum_i (E_i)_x = \sum_i E_i \cos\theta_i \qquad E_y = 0 \text{ N/C}$$

Note that angle θ_i depends on the location of segment i. Now all segments are at the same distance $r_i = R$ from the origin, so

$$E_i = \frac{\Delta q}{4\pi\varepsilon_0 r_i^2} = \frac{\Delta q}{4\pi\varepsilon_0 R^2}$$

The linear charge density on the rod is $\lambda = Q/L$, where L is the rod's length. This allows us to relate charge Δq to the arc length Δs through

$$\Delta q = \lambda\,\Delta s = (Q/L)\Delta s$$

Thus, the net field at the origin is

$$E_x = \sum_i \frac{(Q/L)\Delta s}{4\pi\varepsilon_0 R^2}\cos\theta_i = \frac{Q}{4\pi\varepsilon_0 LR^2}\sum_i \cos\theta_i \Delta s$$

The sum is over all the segments on the rim of a semicircle, so it will be easier to use polar coordinates and integrate over θ rather than do a two-dimensional integral in x and y. We note that the arc length Δs is related to the small angle $\Delta\theta$ by $\Delta s = R\Delta\theta$, so

$$E_x = \frac{Q}{4\pi\varepsilon_0 LR}\sum_i \cos\theta_i \Delta\theta$$

With $\Delta\theta \to d\theta$, the sum becomes an integral over all angles forming the rod. θ varies from $\Delta\theta = -\pi/2$ to $\theta = +\pi/2$. So we finally arrive at

$$E_x = \frac{Q}{4\pi\varepsilon_0 LR}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \frac{Q}{4\pi\varepsilon_0 LR}\sin\theta\Big|_{-\pi/2}^{\pi/2} = \frac{2Q}{4\pi\varepsilon_0 LR}$$

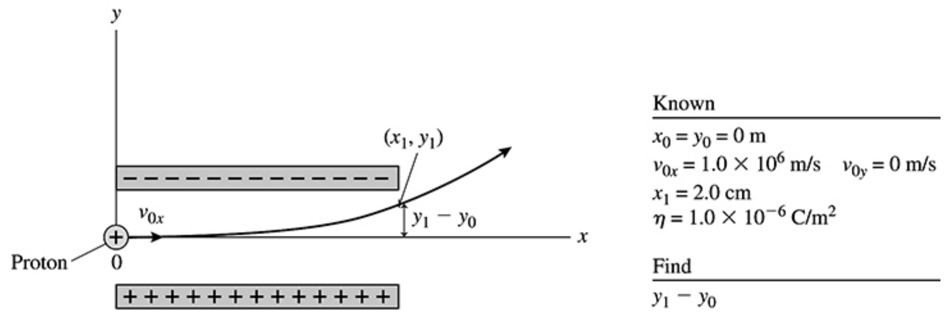
Since we're given the rod's length L and not its radius R, it will be convenient to let $R = L/\pi$. So our final expression for $\vec{E}$, now including the vector information, is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2\pi Q}{L^2}\right)\hat{\imath}$$

(b) Substituting into the above expression,

$$E = \frac{\left(9.0\times10^{9}\ \text{N m}^2/\text{C}^2\right)2\pi\left(30\times10^{-9}\ \text{C}\right)}{\left(0.10\ \text{m}\right)^2} = 1.70\times10^{5}\ \text{N/C}$$

27.52. Model: The electric field is uniform inside the capacitor, so constant-acceleration kinematic equations apply to the motion of the proton.
Visualize:



Solve: From Equation 27.29, the electric field between the parallel plates $\vec{E} = (\eta/\varepsilon_0)\hat{j}$. The force on the proton is

$$\vec{F} = m\vec{a} = q\vec{E} \Rightarrow \vec{a} = \frac{q\vec{E}}{m} = \frac{q\eta}{m\varepsilon_0}\hat{j} \Rightarrow a_y = \frac{q\eta}{m\varepsilon_0}$$

Using the kinematic equation $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$\Delta y = y_1 - y_0 = (0 \text{ m/s})(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0) = \frac{1}{2}\left(\frac{q\eta}{m\varepsilon_0}\right)(t_1 - t_0)^2$$

To determine $t_1 - t_0$, we consider the horizontal motion of the proton. The proton travels a distance of 2.0 cm at a constant speed of 1.0×10^6 m/s. The velocity is constant because the only force acting on the proton is due to the field between the plate along the *y*-direction. Using the same kinematic equation,

$$\Delta x = 2.0\times10^{-2} \text{ m} = v_{0x}(t_1 - t_0) + 0 \text{ m} \Rightarrow (t_1 - t_0) = \frac{2.0\times10^{-2} \text{ m}}{1.0\times10^{6} \text{ m/s}} = 2.0\times10^{-8} \text{ s}$$

$$\Rightarrow \Delta y = \frac{1}{2}\frac{(1.60\times10^{-19} \text{ C})(1.0\times10^{-6} \text{ C/m}^2)(2.0\times10^{-8} \text{ s})^2}{(1.67\times10^{-27} \text{ kg})(8.85\times10^{-12} \text{ C}^2/\text{Nm}^2)} = 2.2 \text{ mm}$$