29.28. Model: The electric potential at the dot is the sum of the potentials due to each charge.
Visualize: Please refer to Figure EX29.28.
Solve: The electric potential at the dot is

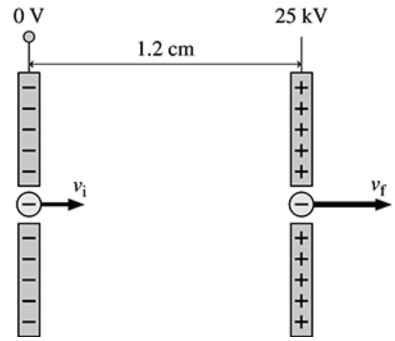
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

$$= \left(9.0\times10^9 \text{ Nm}^2/\text{C}^2\right)\left[\frac{q}{\sqrt{(0.020 \text{ m})^2 + (0.040 \text{ m})^2}} + \frac{-5.0\times10^{-9} \text{ C}}{0.040 \text{ m}} + \frac{5.0\times10^{-9} \text{ C}}{0.020 \text{ m}}\right] = 3140 \text{ V}$$

Solving yields $q = 1.00\times10^{-8}$ C = 10.0 nC.
Assess: Potential is a scalar quantity, so we found the net potential by adding three scalar quantities.

29.46. Model: Energy is conserved.

Visualize:



Solve: **(a)** The electric field inside a parallel-plate capacitor is constant with strength

$$E = \frac{\Delta V}{d} = \frac{(25 \times 10^3 \text{ V})}{0.012 \text{ m}} = 2.1 \times 10^6 \text{ V/m}.$$

(b) Assuming the initial velocity is zero, energy conservation yields

$$U_i = K_f + U_f$$

$$0 = \frac{1}{2} m_e v_f^2 + (-e)(Ed)$$

$$\Rightarrow v_f = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(2.1 \times 10^6 \text{ V/m})(0.012 \text{ m})}{9.11 \times 10^{-31} \text{ kg}}} = 9.4 \times 10^7 \text{ m/s}$$

Assess: This speed is about 31% the speed of light. At that speed, relativity must be taken into account.

30.14. Model: The electric field is the negative of the derivative of the potential function.

Solve: **(a)** From Equation 30.11, the component of the electric field in the *s*-direction is $E_s = -dV/ds$. For the given potential,

$$\frac{dV}{dx} = \frac{d}{dx}\left(100x^2 \text{ V}\right) = 200x \ \frac{\text{V}}{\text{m}} \Rightarrow E_x = -200x \ \text{V/m}$$

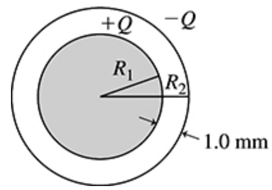
At $x = 0$ m, $E_x = 0$ V/m.

(b) At $x = 1$ m, $E_x = -200$ (1) V/m $= -200$ V/m.

Assess: The potential increases with x, so the electric field must point in the $-x$-direction.

30.56. Model: Capacitance is a geometric property of two electrodes.

Visualize:



Solve: The ratio of the charge to the potential difference is called the capacitance: $C = Q/\Delta V_C$. The potential difference across the capacitor is

$$\Delta V_C = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_1} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow C = 4\pi\varepsilon_0\left[\frac{1}{R_1} - \frac{1}{R_2}\right]^{-1} = 4\pi\varepsilon_0\frac{R_1R_2}{R_2 - R_1} = 4\pi\varepsilon_0\frac{R_1R_2}{1.0\times10^{-3}\text{ m}} = 100\times10^{-12}\text{ F}$$

$$\Rightarrow R_1R_2 = \left(100\times10^{-12}\text{ F}\right)\left(1.0\times10^{-3}\text{ m}\right)\left(9.0\times10^9\text{ N m}^2/\text{C}^2\right) = 900\times10^{-6}\text{ m}^2$$

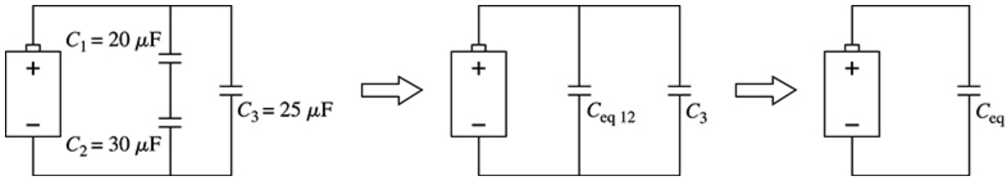
Using $R_2 = R_1 + 1.0$ mm,

$$R_1\left(R_1 + 1.0\times10^{-3}\text{ m}\right) = 900\times10^{-6}\text{ m} \Rightarrow R_1^2 + \left(1.0\times10^{-3}\text{ m}\right)R_1 - 900\times10^{-6}\text{ m} = 0$$

$$\Rightarrow R_1 = \frac{-1.0\times10^{-3}\text{ m} \pm \sqrt{\left(1.0\times10^{-3}\text{ m}\right)^2 + 3600\times10^{-6}\text{ m}^2}}{2} = 0.0295\text{ m} = 2.95\text{ cm}$$

The outer radius is $R_2 = R_1 + 0.001$ m = 0.0305 m = 3.05 cm. So, the diameters are 5.9 cm and 6.1 cm.

30.58. Visualize:



The pictorial representation shows how to find the equivalent capacitance of the three capacitors shown in the figure.

Solve: Because C_1 and C_2 are in series, their equivalent capacitance $C_{\text{eq 12}}$ is

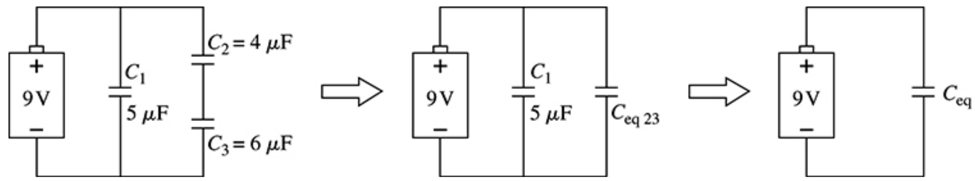
$$\frac{1}{C_{\text{eq 12}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{20\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}} = \frac{1}{12\ \mu\text{F}} \Rightarrow C_{\text{eq 12}} = 12\ \mu\text{F}$$

Then, $C_{\text{eq 12}}$ and C_3 are in parallel. So,

$$C_{\text{eq}} = C_{\text{eq 12}} + C_3 = 12\ \mu\text{F} + 25\ \mu\text{F} = 37\ \mu\text{F}$$

30.62. Model: Assume the battery is an ideal battery.

Visualize:



The pictorial representation shows how to find the equivalent capacitance of the three capacitors shown in the figure.

Solve: Because C_2 and C_3 are in series,

$$\frac{1}{C_{\text{eq }23}}=\frac{1}{C_2}+\frac{1}{C_3}=\frac{1}{4\ \mu\text{F}}+\frac{1}{6\ \mu\text{F}}=\frac{10}{24}(\mu\text{F})^{-1}\Rightarrow C_{\text{eq }23}=\tfrac{24}{10}\ \mu\text{F}=\ 2.4\ \mu\text{F}$$

$C_{\text{eq }23}$ and C_1 are in parallel, so

$$C_{\text{eq}}=C_{\text{eq }23}+C_1=2.4\ \mu\text{F}+5\ \mu\text{F}=7.4\ \mu\text{F}$$

A potential difference of $\Delta V_C = 9$ V across a capacitor of equivalent capacitance 7.4 μF produces a charge

$$Q=C_{\text{eq}}\Delta V_C=(7.4\ \mu\text{F})(9\ \text{V})=66.6\ \mu\text{C}$$

Because C_{eq} is a parallel combination of C_1 and $C_{\text{eq }23}$, these capacitors have $\Delta V_1=\Delta V_{\text{eq }23}=\Delta V_C=9$ V. Thus the charges on these two capacitors are

$$Q_1=(5\ \mu\text{F})(9\ \text{V})=45\ \mu\text{C}\qquad Q_{\text{eq }23}=(2.4\ \mu\text{F})(9\ \text{V})=21.6\ \mu\text{C}$$

Because $Q_{\text{eq }23}$ is due to a series combination of C_2 and C_3, $Q_2 = Q_3 = 21.6\ \mu$C. This means

$$\Delta V_2=\frac{Q_2}{C_2}=\frac{21.6\ \mu\text{C}}{4\ \mu\text{F}}=\ 5.4\ \text{V}\qquad \Delta V_3=\frac{Q_3}{C_3}=\frac{21.6\ \mu\text{C}}{6\ \mu\text{F}}=3.6\ \text{V}$$

In summary, $Q_1 = 45\ \mu$C, $V_1 = 9$ V; $Q_2 = 21.6\ \mu$C, $V_2 = 5.4$ V; and $Q_3 = 21.6\ \mu$C, $V_3 = 3.6$ V.

30.64. Model: Capacitance is a geometric property.

Visualize: Please refer to Figure P30.64. Shells R_1 and R_2 are a spherical capacitor C. Shells R_2 and R_3 are a spherical capacitor C'. These two capacitors are in series.

Solve: The ratio of the charge to the potential difference is called the capacitance: $C = Q/\Delta V_C$. The potential differences across the capacitors C and C' are

$$\Delta V_C = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_1} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Delta V_{C'} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_2} - \frac{1}{4\pi\varepsilon_0}\frac{Q}{R_3} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R_2} - \frac{1}{R_3}\right]$$

$$\Rightarrow C = (4\pi\varepsilon_0)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)^{-1} \qquad C' = (4\pi\varepsilon_0)\left(\frac{1}{R_2} - \frac{1}{R_3}\right)^{-1}$$

Because these two capacitors are in series,

$$\frac{1}{C_{\text{net}}} = \frac{1}{C} + \frac{1}{C'} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_2} - \frac{1}{R_3}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{R_3 - R_1}{R_1 R_3}\right)$$

$$C_{\text{net}} = 4\pi\varepsilon_0\left(\frac{R_1 R_3}{R_3 - R_1}\right) = \frac{1}{9.0\times 10^9 \text{ N m}^2/\text{C}^2}\left[\frac{(0.010 \text{ m})(0.030 \text{ m})}{0.030 \text{ m} - 0.010 \text{ m}}\right] = 1.67\times 10^{-12} \text{ F} = 1.67 \text{ pF}$$

Assess: C_{net} depends on only the inner and outer shells, not on R_2.