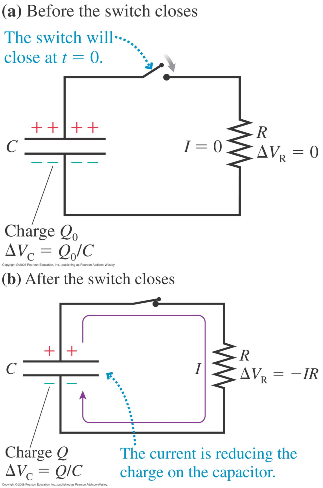
RC Circuits (32.9)



- We have only been discussing DC circuits so far. However, using a capacitor we can create an RC circuit.
- In this example, a capacitor is charged but the switch is open, meaning no current flows.
- The switch closes and current flows through the resistor, discharging the capacitor. The current stops once the capacitor is discharged.
- The voltage around the loop is

$$\Delta V_C + \Delta V_R = \frac{Q}{C} - IR = 0$$

RC Circuits

- The rate of charge leaving the capacitor is equal to the rate of charge passing through the circuit (the current)

$$I = -\frac{dQ}{dt}$$

- The loop then becomes (dividing by R and rearranging)

$$\begin{aligned}
\frac{Q}{C} - IR &= 0 \\
\frac{Q}{C} + \frac{dQ}{dt}R &= 0 \\
\frac{Q}{RC} + \frac{dQ}{dt} &= 0 \\
\frac{dQ}{Q} &= -\frac{1}{RC}dt
\end{aligned}$$

- We can solve this by integrating it.

- The integral is

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\frac{1}{RC}\int_0^t dt$$

- We can do that integral

$$\ln Q|_{Q_0}^{Q} = \ln Q - \ln Q_0 = \ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$$

- Solving for Q gives

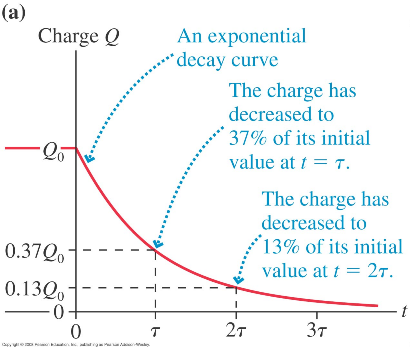
$$Q = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$$

  where $\tau = RC$ is the time constant of the circuit.

- The resistor current is then

$$I = -\frac{dQ}{dt} = \frac{Q_0}{\tau}e^{-t/\tau} = \frac{Q_0}{RC}e^{-t/\tau} = \frac{\Delta V_C}{R}e^{-t/\tau} = I_0 e^{-t/\tau}$$

RC Circuits


(a)
Charge Q
An exponential decay curve
The charge has decreased to 37% of its initial value at $t = \tau$.
The charge has decreased to 13% of its initial value at $t = 2\tau$.
Q_0
$0.37Q_0$
$0.13Q_0$
0
t
0
τ
2τ
3τ
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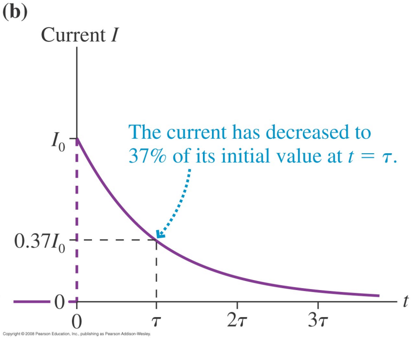
(b)
Current I
The current has decreased to 37% of its initial value at $t = \tau$.
I_0
$0.37I_0$
0
t
0
τ
2τ
3τ
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Charging a Capacitor