We have already seen an induced electric dipole. Natural dipoles also exist. What kind of electric field do they produce?
The Electric Field of a Dipole

A water molecule is a *permanent* dipole because the negative electrons spend more time with the oxygen atom.

This dipole is *induced*, or stretched, by the electric field acting on the + and − charges.

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Overall the dipole is neutral.
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Overall the dipole is neutral.

But, the test charge (left) is closer to the positive charge than it is to the negative. A force results.
Let’s calculate the electric field at the point on the $y$ axis with

$$r_+ = y - \frac{s}{2}$$

$$r_- = y + \frac{s}{2}$$
The Electric Field of a Dipole

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  $r_+ = y - \frac{s}{2}$

  $r_- = y + \frac{s}{2}$

- The sum of the fields is
  
  $$(E_{\text{dipole}})_y = \frac{1}{4\pi\epsilon_0} \frac{q}{(y - \frac{1}{2} s)^2} + \frac{1}{4\pi\epsilon_0} \frac{(-q)}{(y + \frac{1}{2} s)^2}$$
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$$= \frac{q}{4\pi\varepsilon_0} \left[ \frac{1}{(y - \frac{1}{2}s)^2} - \frac{1}{(y + \frac{1}{2}s)^2} \right]$$
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$$= \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{(y - \frac{1}{2}s)^2} - \frac{1}{(y + \frac{1}{2}s)^2} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[ \frac{2ys}{(y - \frac{1}{2}s)^2(y + \frac{1}{2}s)^2} \right]$$
The Electric Field of a Dipole

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(E_{\text{dipole}})_y = \frac{q}{4\pi\epsilon_0} \left[ \frac{2ys}{(y - \frac{1}{2}s)^2(y + \frac{1}{2}s)^2} \right]
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- For distances much larger than the charge separation \((y \gg s)\) the \(y - \frac{1}{2}s\) is just \(y\) and

\[
(E_{\text{dipole}})_y = \frac{1}{4\pi\epsilon_0} \frac{2qs}{y^3}
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- We define the dipole moment as \(\vec{p} = qs\) (direction negative to positive) \(\vec{p} = qs \hat{j}\) in this case, so that

\[
\vec{E}_{\text{dipole}} = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}, \text{(on axis of dipole)}
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The Electric Field of a Dipole

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\[\vec{E}_{\text{dipole}} = \frac{1}{4\pi\varepsilon_0} \frac{2\vec{p}}{r^3}, \text{(on axis of dipole)}\]

- In the perpendicular plane that bisects the dipole we can also show

\[\vec{E}_{\text{dipole}} = \frac{1}{4\pi\varepsilon_0} \frac{(-\vec{p})}{r^3}, \text{(perpendicular to dipole)}\]
Picturing the Electric Field

We have a couple of different ways to represent an electric field:

- Electric field lines are continuous curves tangent to electric field vectors.

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The electric field vectors are tangent to the electric field lines.
Picturing the Electric Field

We have a couple of different ways to represent an electric field:

- Electric field lines are continuous curves tangent to electric field vectors.
- Closely spaced field lines represent larger field strength.
- Electric field lines never cross.
- Electric field lines start on positive charges and end on negative charges.
For a continuous object we cannot look at every single charge individually. Instead define linear charge density and surface charge density

\[
\lambda = \frac{Q}{L}, \quad \eta = \frac{Q}{A}
\]

Where \(Q\) is the total charge on an object, not a single-particle charge.
(27.3)

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These definitions assume that the object is uniformly charged.
The Electric Field of a Continuous Charge Distribution (27.3)

For a continuous object we cannot look at every single charge individually. Instead define linear charge density and surface charge density

\[ \lambda = \frac{Q}{L}, \quad \eta = \frac{Q}{A} \]

Where \( Q \) is the total charge on an object, not a single-particle charge.

- These definitions assume that the object is uniformly charged.
- We have some tricks to break the distributions into pieces, then build it back up again.
Example 27.3 - The Electric Field of a Line of Charge

Find the electric field strength at a distance \( d \) in the plane that bisects a rod of length \( L \) and total charge \( q \).

- The rod is thin, so assume the charge lies along a line
Example 27.3

Find the electric field strength at a distance $d$ in the plane that bisects a rod of length $L$ and total charge $q$.

- The rod is thin, so assume the charge lies along a line
- The charge density of the line is $\lambda = \frac{Q}{L}$
Model each little segment of charge \((i)\) as a point charge

\[
(E_i)_x = E_i \cos \theta_i = \frac{1}{4\pi\varepsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i
\]
Model each little segment of charge \((i)\) as a point charge

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(E_i)_x = E_i \cos \theta_i = \frac{1}{4\pi\varepsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i
\]

We can express \(r_i^2\) and \(\cos \theta_i\) as

\[
r_i = (y_i^2 + d^2)^{1/2}, \quad \cos \theta_i = \frac{d}{r} = \frac{d}{(y_i^2 + d^2)^{1/2}}
\]
Example 27.3

Plugging these into the electric field formula gives

\[
(E_i)_x = \frac{1}{4\pi\varepsilon_0} \frac{\Delta Q}{y_i^2 + d^2} \frac{d}{\sqrt{y_i^2 + d^2}}
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= \frac{1}{4\pi \varepsilon_0} \frac{d\Delta Q}{(y_i^2 + d^2)^{3/2}}
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- Now we can sum over all of the little segments

\[
E_x = \frac{1}{4\pi \varepsilon_0} \sum_{i=1}^{N} \frac{d\Delta Q}{(y_i^2 + d^2)^{3/2}}
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- Of course, the rod is not really in little segments. We should make those infinitely small and integrate.
The problem with trying to integrate this:

\[ E_x = \frac{1}{4\pi \varepsilon_0} \sum_{i=1}^{N} \frac{d\Delta Q}{(y_i^2 + d^2)^{3/2}} \]

is that we don’t know how to integrate over \( Q \).
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- We need to change the variable using the charge density.

\[ \Delta Q = \lambda \Delta y = \frac{Q}{L} \Delta y \]
Example 27.3

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is that we don’t know how to integrate over \( Q \).

- We need to change the variable using the charge density.

\[ \Delta Q = \lambda \Delta y = \frac{Q}{L} \Delta y \]

- Giving:

\[ E_x = \frac{Q/L}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{d\Delta y}{(y_i^2 + d^2)^{3/2}} \]
Example 27.3

1. Choose a coordinate system with the origin at the center of the rod.

2. Identify the point at which we’re going to calculate the field.

3. Divide the rod into $N$ small segments of length $\Delta y$ and charge $\Delta Q = \lambda \Delta y$.

4. Draw the field vector of charge segment.

5. Note that the field from a symmetrically located charge segment will cancel $(E_i)_y$.

\[
E_x = \frac{Q/L}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} \frac{d}{(y_i^2 + d^2)^{3/2}} dy
\]
Example 27.3

- We can actually do that integral:

\[ E_x = \frac{Q/L}{4\pi\epsilon_0} \int_{-L/2}^{L/2} d\sqrt{y_i^2 + d^2} \]

We should check this at the far-away limit, \( d \gg L \).
Example 27.3

- We can actually do that integral:

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E_x = \frac{Q/L}{4\pi\varepsilon_0} \int_{-L/2}^{L/2} y \sqrt{y_i^2 + d^2} \, dy
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Example 27.3

We can actually do that integral:

\[
E_x = \frac{Q}{L} \frac{y}{4\pi \varepsilon_0} \frac{d}{\sqrt{y^2 + d^2}} \bigg|_{-L/2}^{L/2} = \frac{Q}{L} \left[ \frac{L/2}{d \sqrt{(L/2)^2 + d^2}} - \frac{-L/2}{d \sqrt{(-L/2)^2 + d^2}} \right] \]

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Example 27.3

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\[ E_x = \frac{Q/L}{4\pi\epsilon_0} \left[ \int_{-L/2}^{L/2} \frac{y}{d\sqrt{y_i^2 + d^2}} \right] \]

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\[ = \frac{1}{4\pi\epsilon_0} \frac{Q}{d\sqrt{d^2 + (L/2)^2}} \]

- We should check this at the far-away limit, \( d \gg L \)

\[ E_x = \frac{1}{4\pi\epsilon_0} \frac{Q}{d^2} \]

Back to a point charge!!
Let’s consider an infinitely long wire of the same charge density $\lambda$. We can use the formula for the wire in the extreme limit

$$E_{\text{line}} = \lim_{L \to \infty} \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r \sqrt{r^2 + (L/2)^2}} = \frac{1}{4\pi\epsilon_0} \frac{|Q|}{rL/2} = \frac{1}{4\pi\epsilon_0} \frac{2|\lambda|}{r}$$
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Notice that the field goes like $1/r$ instead of $1/r^2$. 
An Infinite Line of Charge

The field points straight away from the line at all points.

The field strength decreases with distance.

Of course, no line of charge is really infinite.
An Infinite Line of Charge

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The contributions from charges far down the wire are very small (\(1/r^2\)), so a long wire exerts roughly the same force as an infinite one.
An Infinite Line of Charge

1. Of course, no line of charge is really infinite.
2. The contributions from charges far down the wire are very small (like $1/r^2$), so a long wire exerts roughly the same force as an infinite one.
3. There are problems with this close to the ends of the finite wire.
Example 27.5

A thin ring of radius $R$ is uniformly charged with total charge $Q$. Find the electric field at a point on the axis of the ring.
Electric Field from A Thin Ring

- The linear charge density along the ring is

\[ \lambda = \frac{Q}{2\pi R} \]

1. Choose a coordinate system.
2. Identify the point at which to calculate the field.
3. Divide the ring into segments.
4. Draw the field vector of charge segment \( i \).
5. Note that the field from a symmetrically located charge segment will cancel \((E)_i\).
The linear charge density along the ring is

$$\lambda = \frac{Q}{2\pi R}$$

Divide the ring into $N$ small segments and the $z$ component of the $ith$ segment is

$$(E_i)_z = E_i \cos \theta_i = \frac{1}{4\pi \epsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i$$
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Every point on the ring is equidistant from the axis!

\[ r_i = \sqrt{z^2 + R^2} \]
Electric Field from A Thin Ring

- The linear charge density along the ring is

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- Divide the ring into \( N \) small segments and the \( z \) component of the \( i^{th} \) segment is

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  \[ r_i = \sqrt{z^2 + R^2} \]
  \[ \cos \theta_i = \frac{z}{r_i} = \frac{z}{\sqrt{z^2 + R^2}} \]
Electric Field from A Thin Ring

- Substituting we have:

\[
(E_i)_z = \frac{1}{4\pi\varepsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i
\]

This needs to be summed over all segments:

\[
E_z = \frac{1}{4\pi\varepsilon_0} z \frac{Q}{(z^2 + R^2)^{3/2}} \Delta Q
\]

Note that all points on the ring are the same distance from the axis. Who needs an integral??
Electric Field from A Thin Ring

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E_z = \frac{1}{4\pi\varepsilon_0} \frac{z}{(z^2 + R^2)^{3/2}} \sum_{i=1}^{N} \Delta Q
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