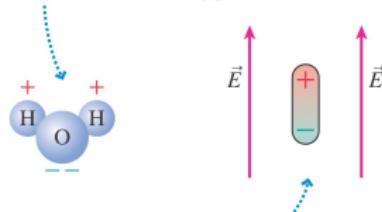


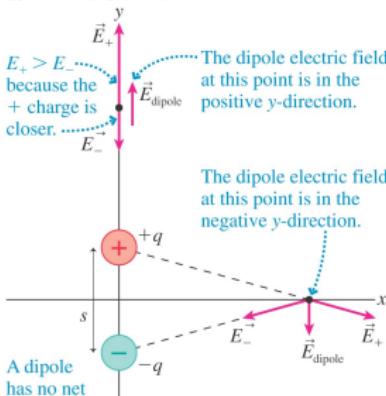
The Electric Field of a Dipole

A water molecule is a *permanent* dipole because the negative electrons spend more time with the oxygen atom.



This dipole is *induced*, or stretched, by the electric field acting on the + and - charges.

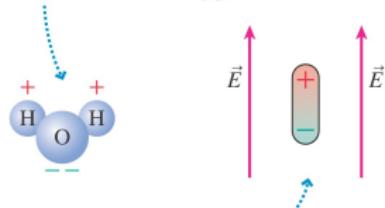
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- We have already seen an induced electric dipole. Natural dipoles also exist. What kind of electric field do they produce?

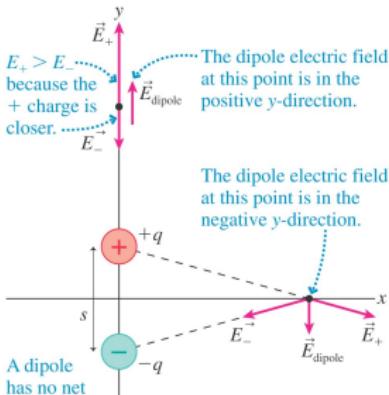
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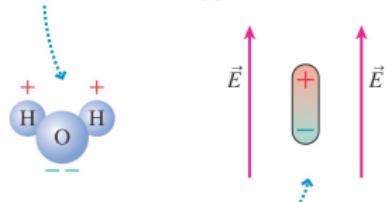


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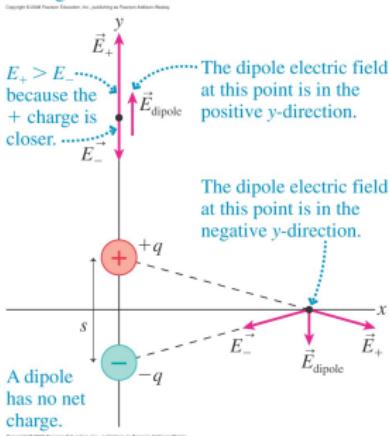
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- Overall the dipole is neutral.

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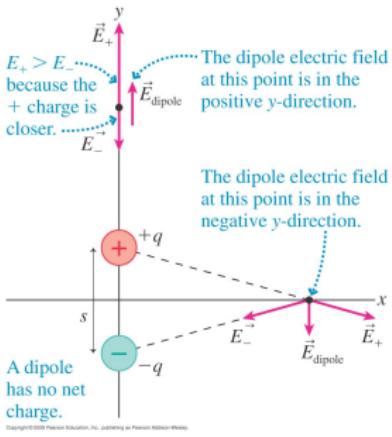


- We have already seen an induced electric dipole. Natural dipoles also exist. What kind of electric field do they produce?
- Overall the dipole is neutral.
- But, the test charge (left) is closer to the positive charge than it is to the negative. A force results.

The Electric Field of a Dipole

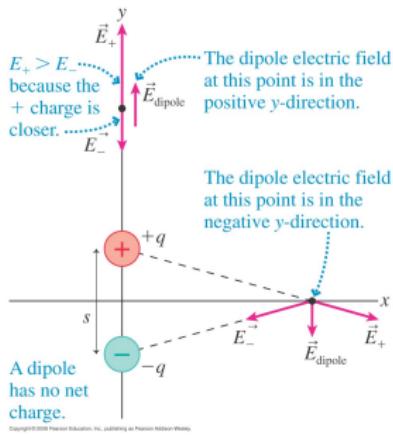
- Let's calculate the electric field at the point on the y axis with

$$r_+ = y - \frac{s}{2}$$
$$r_- = y + \frac{s}{2}$$



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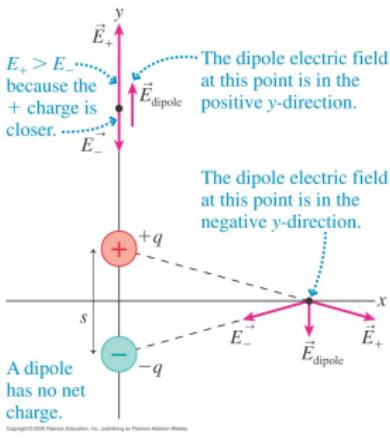
$$r_+ = y - \frac{s}{2}$$
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- The sum of the fields is

$$(E_{\text{dipole}})_y = \frac{1}{4\pi\epsilon_0} \frac{q}{(y - \frac{1}{2}s)^2} + \frac{1}{4\pi\epsilon_0} \frac{(-q)}{(y + \frac{1}{2}s)^2}$$

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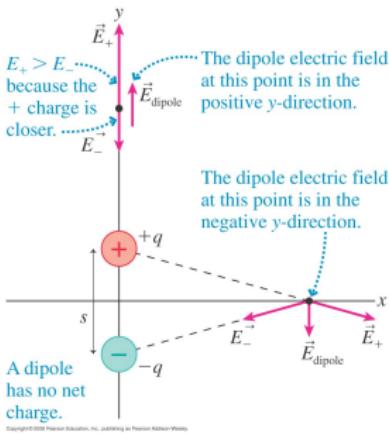
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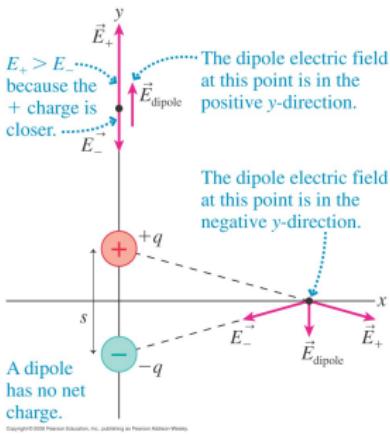
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$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(y - \frac{1}{2}s)^2} - \frac{1}{(y + \frac{1}{2}s)^2} \right]$$

The Electric Field of a Dipole

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The Electric Field of a Dipole

$$(E_{\text{dipole}})_y = \frac{q}{4\pi\epsilon_0} \left[\frac{2ys}{(y - \frac{1}{2}s)^2(y + \frac{1}{2}s)^2} \right]$$

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- We define the **dipole moment** as $\vec{p} = qs$ (direction negative to positive) $\vec{p} = qs\hat{j}$ in this case, so that

$$\vec{E}_{\text{dipole}} = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}, \text{ (on axis of dipole)}$$

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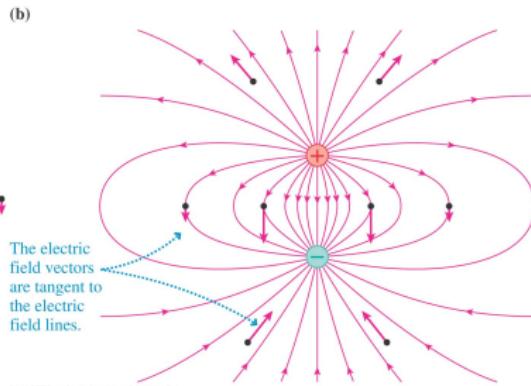
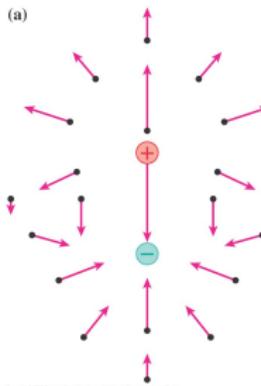
$$\vec{E}_{\text{dipole}} = \frac{1}{4\pi\epsilon_0} \frac{2\vec{p}}{r^3}, \text{ (on axis of dipole)}$$

- In the perpendicular plane that bisects the dipole we can also show

$$\vec{E}_{\text{dipole}} = \frac{1}{4\pi\epsilon_0} \frac{(-\vec{p})}{r^3}, \text{ (perpendicular to dipole)}$$

Picturing the Electric Field

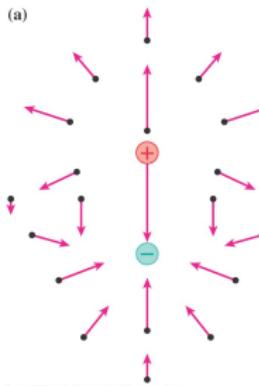
We have a couple of different ways to represent an electric field:



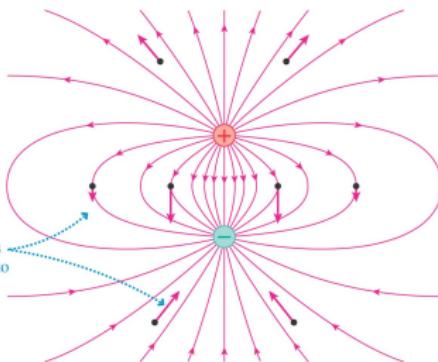
- Electric field lines are continuous curves tangent to electric field vectors

Picturing the Electric Field

We have a couple of different ways to represent an electric field:



(b)

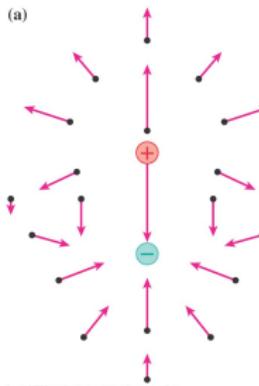


The electric
field vectors
are tangent to
the electric field
lines.

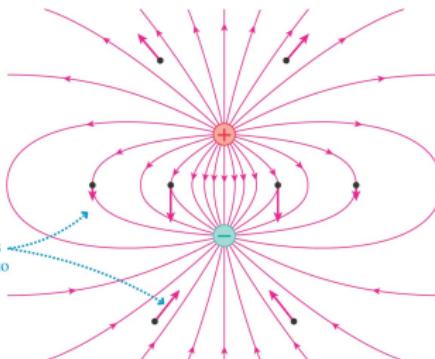
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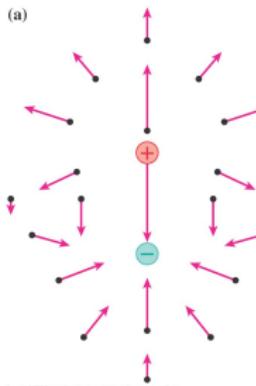
(b)



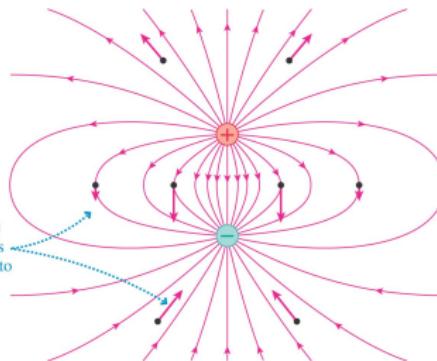
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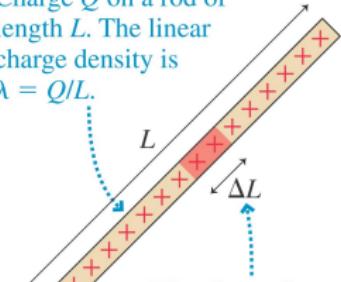
(b)



- Electric field lines are continuous curves tangent to electric field vectors
- Closely spaced field lines represent larger field strength
- Electric field lines never cross
- Electric field lines start on positive charges and end on negative charges.

The Electric Field of a Continuous Charge Distribution (27.3)

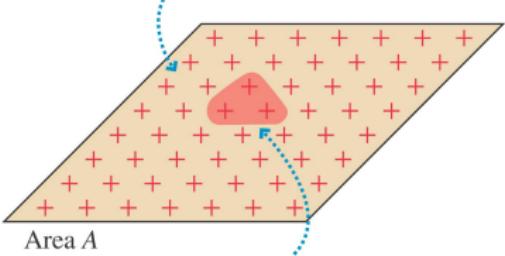
(a) Charge Q on a rod of length L . The linear charge density is $\lambda = Q/L$.



The diagram shows a long, thin, straight rod oriented diagonally. The rod is covered with red '+' signs representing charge. A small segment of the rod is highlighted in pink, with a length labeled ΔL and a charge label ΔQ . The total length of the rod is labeled L . A dotted line extends from the rod to the right, indicating its continuation.

The charge in a small length ΔL is $\Delta Q = \lambda \Delta L$.

(b) Charge Q on a surface of area A . The surface charge density is $\eta = Q/A$.



The diagram shows a flat, triangular surface representing a plane. The surface is covered with red '+' signs. A small triangular area on the surface is highlighted in pink, with a surface area labeled ΔA and a charge label ΔQ . The total area of the surface is labeled A . A dotted line extends from the surface to the right, indicating its continuation.

The charge in a small area ΔA is $\Delta Q = \eta \Delta A$.

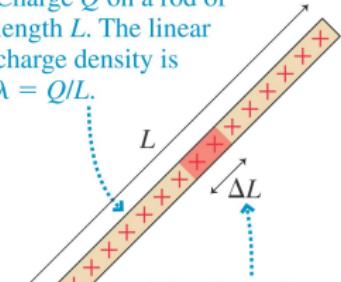
- For a continuous object we cannot look at every single charge individually. Instead define **linear charge density** and **surface charge density**

$$\lambda = \frac{Q}{L}, \quad \eta = \frac{Q}{A}$$

Where Q is the total charge on an object, not a single-particle charge.

The Electric Field of a Continuous Charge Distribution (27.3)

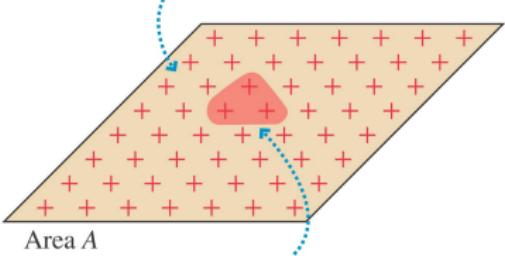
(a) Charge Q on a rod of length L . The linear charge density is $\lambda = Q/L$.



The diagram shows a long, thin, slightly curved rod with a uniform distribution of positive charges represented by red '+' symbols. The rod is oriented diagonally. A small segment of length ΔL is highlighted with a pink shaded area. The total length of the rod is labeled L . A dotted line with arrows indicates the direction of the rod's length.

The charge in a small length ΔL is $\Delta Q = \lambda \Delta L$.

(b) Charge Q on a surface of area A . The surface charge density is $\eta = Q/A$.



The diagram shows a flat, triangular-shaped surface representing a plane. The surface is covered with a uniform distribution of positive charges, shown as red '+' symbols. A small, circular region of area ΔA is highlighted with a pink shaded area. The total area of the surface is labeled A . A dotted line with arrows indicates the direction of the surface's normal.

The charge in a small area ΔA is $\Delta Q = \eta \Delta A$.

- For a continuous object we cannot look at every single charge individually. Instead define **linear charge density** and **surface charge density**

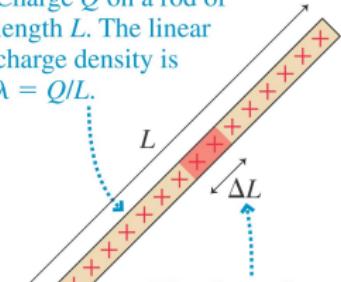
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- These definitions assume that the object is uniformly charged.

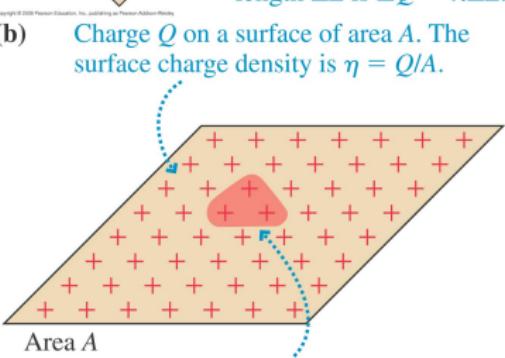
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(b) Charge Q on a surface of area A . The surface charge density is $\eta = Q/A$.



The diagram shows a flat, triangular-shaped surface with a uniform distribution of positive charges represented by red '+' symbols. A small area element ΔA is highlighted with a pink shaded region. The text 'The charge in a small area ΔA is $\Delta Q = \eta \Delta A$ ' is written below the surface.

- For a continuous object we cannot look at every single charge individually. Instead define **linear charge density** and **surface charge density**

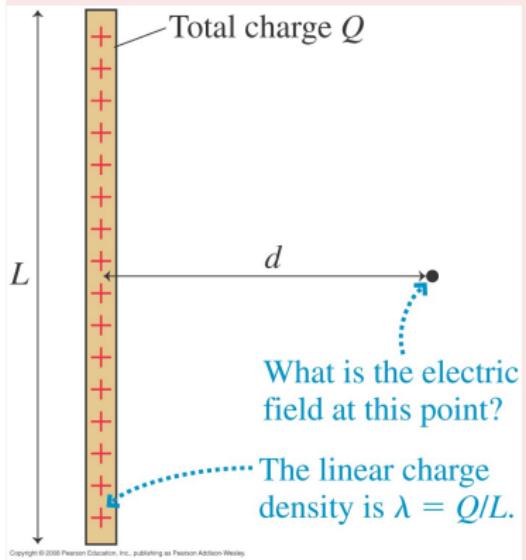
$$\lambda = \frac{Q}{L}, \quad \eta = \frac{Q}{A}$$

Where Q is the total charge on an object, not a single-particle charge.

- These definitions assume that the object is uniformly charged.
- We have some tricks to break the distributions into pieces, then build it back up again.

Example 27.3 - The Electric Field of a Line of Charge

Example 27.3

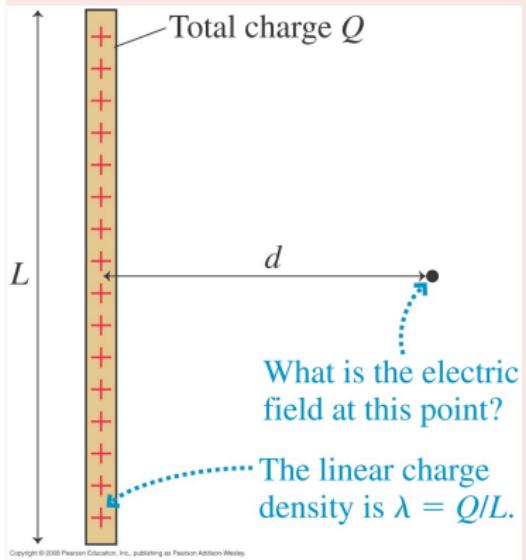


Find the electric field strength at a distance d in the plane that bisects a rod of length L and total charge q .

- The rod is thin, so assume the charge lies along a line

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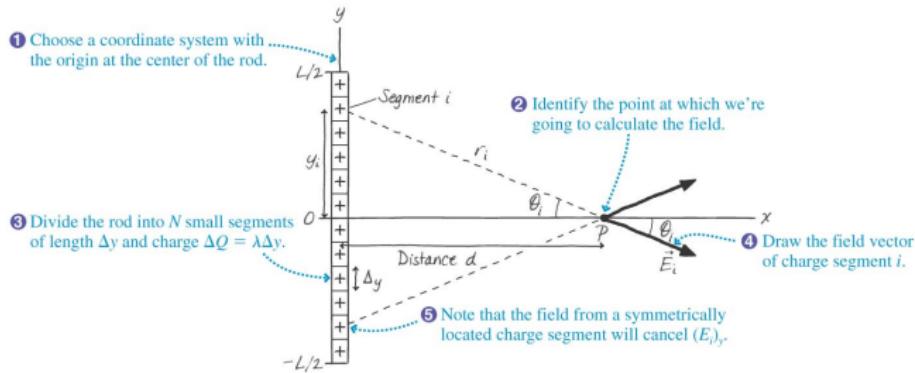
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Find the electric field strength at a distance d in the plane that bisects a rod of length L and total charge q .

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- The charge density of the line is

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Example 27.3

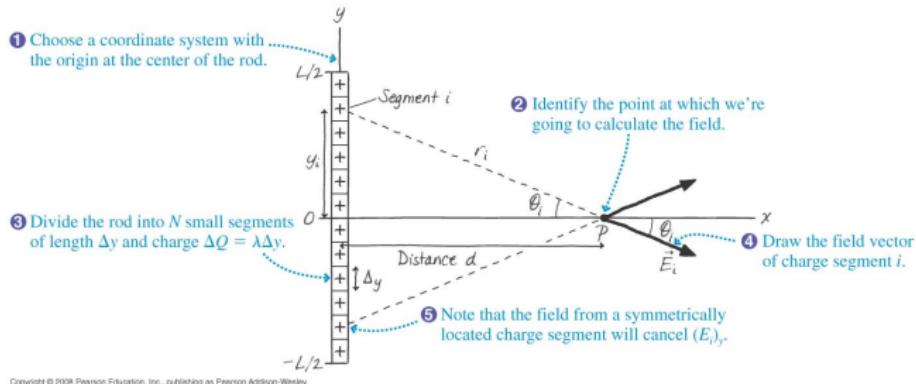


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- Model each little segment of charge (i) as a point charge

$$(E_i)_x = E_i \cos \theta_i = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i$$

Example 27.3



- Model each little segment of charge (i) as a point charge

$$(E_i)_x = E_i \cos \theta_i = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i$$

- We can express r_i^2 and $\cos \theta_i$ as

$$r_i = (y_i^2 + d^2)^{1/2}, \quad \cos \theta_i = \frac{d}{r} = \frac{d}{(y_i^2 + d^2)^{1/2}}$$

Example 27.3

- Plugging these into the electric field formula gives

$$(E_i)_x = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{y_i^2 + d^2} \frac{d}{\sqrt{y_i^2 + d^2}}$$

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- Now we can sum over all of the little segments

$$E_x = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{d\Delta Q}{(y_i^2 + d^2)^{3/2}}$$

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- Now we can sum over all of the little segments

$$E_x = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{d\Delta Q}{(y_i^2 + d^2)^{3/2}}$$

- Of course, the rod is not really in little segments. We should make those infinitely small and integrate.

Example 27.3

- The problem with trying to integrate this:

$$E_x = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{d\Delta Q}{(y_i^2 + d^2)^{3/2}}$$

is that we don't know how to integrate over Q .

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- We need to change the variable using the charge density.

$$\Delta Q = \lambda \Delta y = \frac{Q}{L} \Delta y$$

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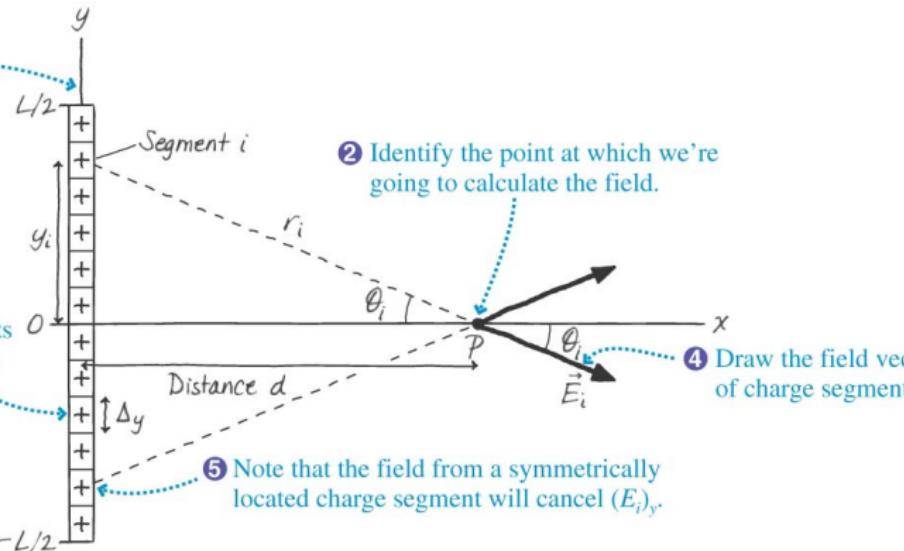
- Giving:

$$E_x = \frac{Q/L}{4\pi\epsilon_0} \sum_{i=1}^N \frac{d\Delta y}{(y_i^2 + d^2)^{3/2}}$$

Example 27.3

① Choose a coordinate system with the origin at the center of the rod.

③ Divide the rod into N small segments of length Δy and charge $\Delta Q = \lambda \Delta y$.



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$$E_x = \frac{Q/L}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{d}{(y_i^2 + d^2)^{3/2}} dy$$

Example 27.3

- We can actually do that integral:

$$E_x = \frac{Q/L}{4\pi\epsilon_0 d} \frac{y}{\sqrt{y_i^2 + d^2}} \Big|_{-L/2}^{L/2}$$

Example 27.3

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$$\begin{aligned} E_x &= \frac{Q/L}{4\pi\epsilon_0} \frac{y}{d\sqrt{y_i^2 + d^2}} \Big|_{-L/2}^{L/2} \\ &= \frac{Q/L}{4\pi\epsilon_0} \left[\frac{L/2}{d\sqrt{(L/2)^2 + d^2}} - \frac{-L/2}{d\sqrt{(-L/2)^2 + d^2}} \right] \end{aligned}$$

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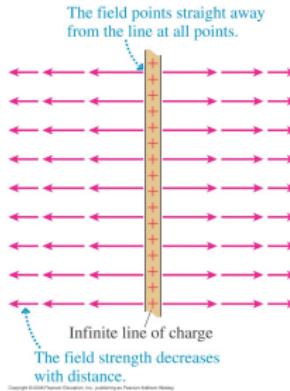
$$\begin{aligned} E_x &= \frac{Q/L}{4\pi\epsilon_0} \frac{y}{d\sqrt{y_i^2 + d^2}} \Big|_{-L/2}^{L/2} \\ &= \frac{Q/L}{4\pi\epsilon_0} \left[\frac{L/2}{d\sqrt{(L/2)^2 + d^2}} - \frac{-L/2}{d\sqrt{(-L/2)^2 + d^2}} \right] \\ &= \frac{1}{4\pi\epsilon_0} \frac{Q}{d\sqrt{d^2 + (L/2)^2}} \end{aligned}$$

- We should check this at the far-away limit, $d \gg L$

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{Q}{d^2}$$

Back to a point charge!!

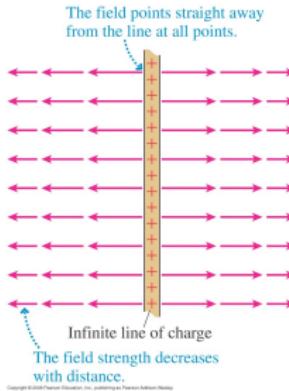
An Infinite Line of Charge



- Let's consider an infinitely long wire of the same charge density λ . We can use the formula for the wire in the extreme limit

$$E_{line} = \lim_{L \rightarrow \infty} \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r \sqrt{r^2 + (L/2)^2}} = \frac{1}{4\pi\epsilon_0} \frac{|Q|}{rL/2} = \frac{1}{4\pi\epsilon_0} \frac{2|\lambda|}{r}$$

An Infinite Line of Charge



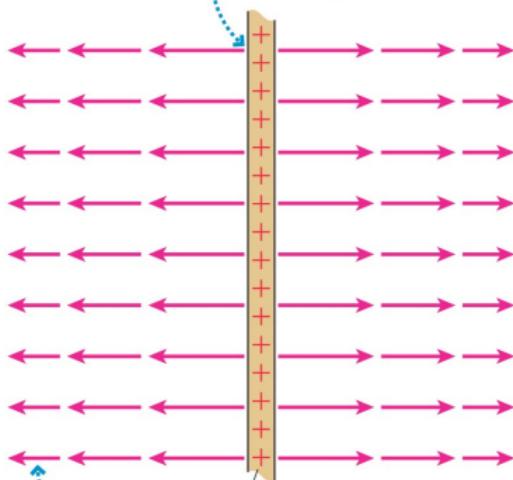
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- Notice that the field goes like $1/r$ instead of $1/r^2$

An Infinite Line of Charge

The field points straight away from the line at all points.

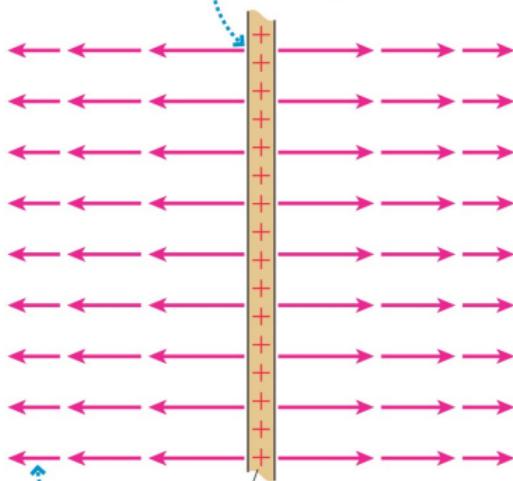


The field strength decreases with distance.

- Of course, no line of charge is really infinite.

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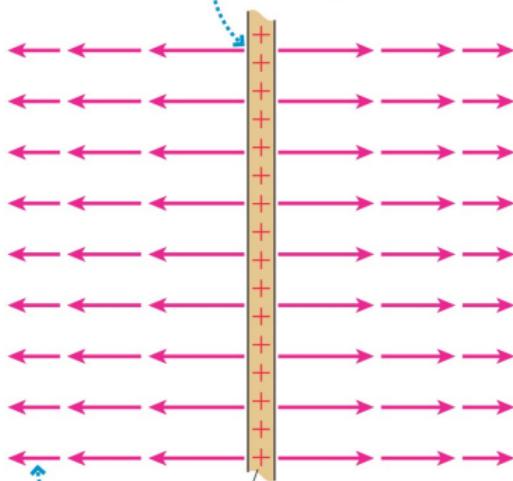


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- Of course, no line of charge is really infinite.
- The contributions from charges far down the wire are very small (like $1/r^2$), so a long wire exerts roughly the same force as an infinite one.

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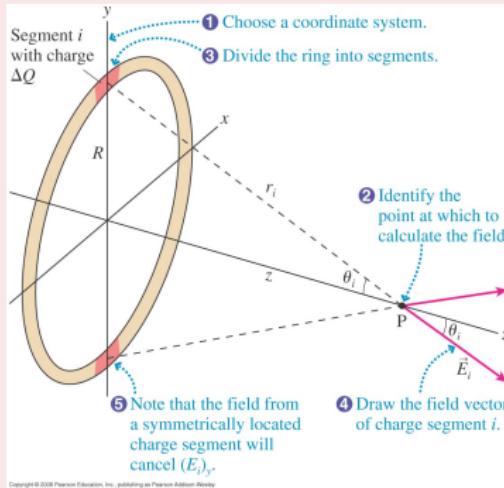


The field strength decreases with distance.

- Of course, no line of charge is really infinite.
- The contributions from charges far down the wire are very small ($1/r^2$), so a long wire exerts roughly the same force as an infinite one.
- There are problems with this close to the ends of the finite wire.

Rings, Disks, Planes and Spheres (27.4)

Example 27.5

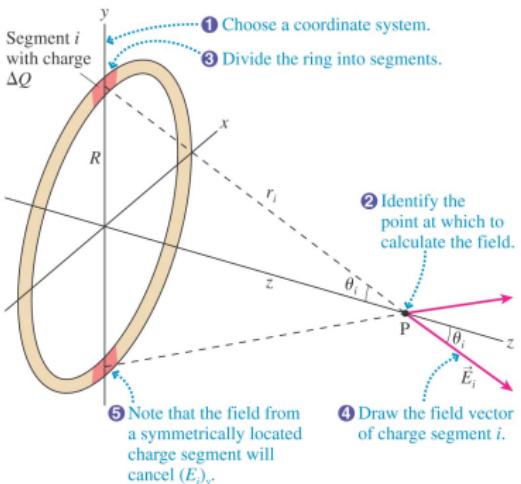


A thin ring of radius R is uniformly charged with total charge Q . Find the electric field at a point on the axis of the ring.

Electric Field from A Thin Ring

- The linear charge density along the ring is

$$\lambda = \frac{Q}{2\pi R}$$



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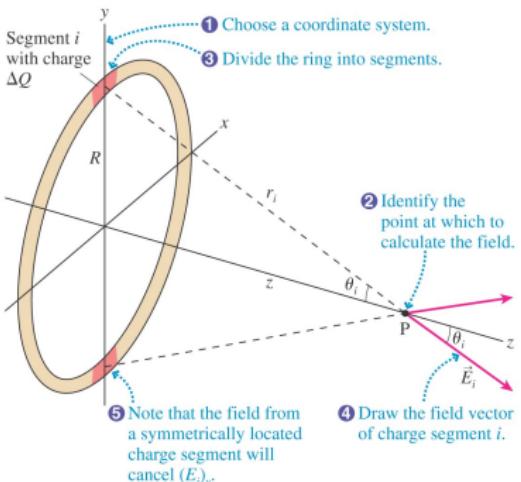
Electric Field from A Thin Ring

- The linear charge density along the ring is

$$\lambda = \frac{Q}{2\pi R}$$

- Divide the ring into N small segments and the z component of the i th segment is

$$(E_i)_z = E_i \cos \theta_i = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i$$



Electric Field from A Thin Ring

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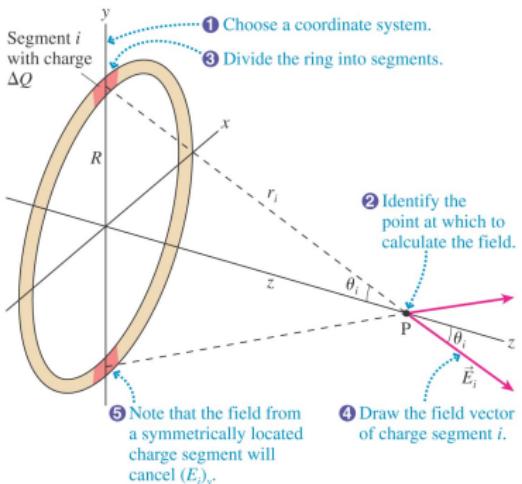
$$\lambda = \frac{Q}{2\pi R}$$

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- Every point on the ring is equidistant from the axis!

$$r_i = \sqrt{z^2 + R^2}$$



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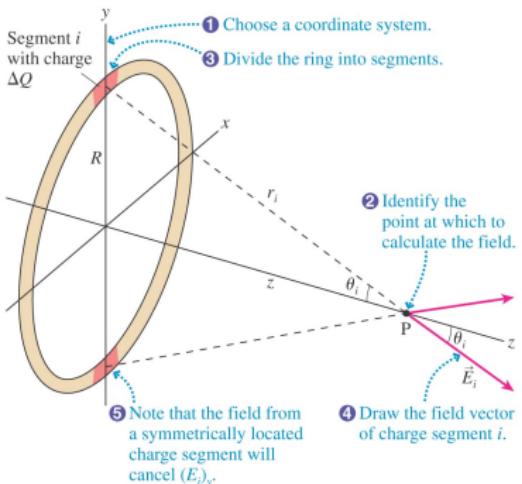
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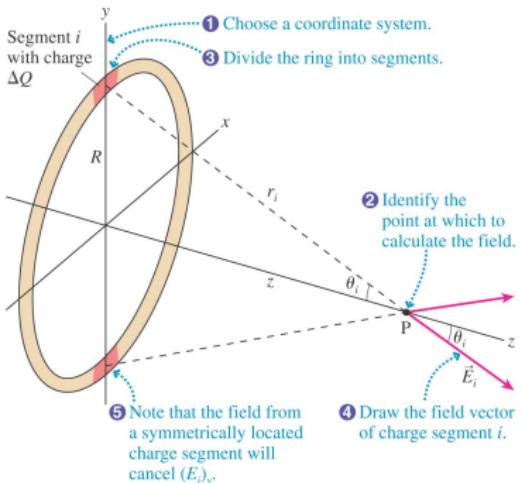
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$$\lambda = \frac{Q}{2\pi R}$$

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$$r_i = \sqrt{z^2 + R^2}$$
$$\cos \theta_i = \frac{z}{r_i} = \frac{z}{\sqrt{z^2 + R^2}}$$



Electric Field from A Thin Ring

- Substituting we have:

$$(E_i)_z = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i$$

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$$(E_i)_z = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{z^2 + R^2} \frac{z}{\sqrt{z^2 + R^2}}$$

Electric Field from A Thin Ring

- Substituting we have:

$$\begin{aligned}(E_i)_z &= \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i^2} \cos \theta_i \\ (E_i)_z &= \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{z^2 + R^2} \frac{z}{\sqrt{z^2 + R^2}} \\ &= \frac{1}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}} \Delta Q\end{aligned}$$

- This needs to be summed over all segments:

$$E_z = \frac{1}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}} \sum_{i=1}^N \Delta Q$$

Electric Field from A Thin Ring

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- Note that all points on the ring are the same distance from the axis. Who needs an integral??

$$E_z = \frac{1}{4\pi\epsilon_0} \frac{zQ}{(z^2 + R^2)^{3/2}}$$