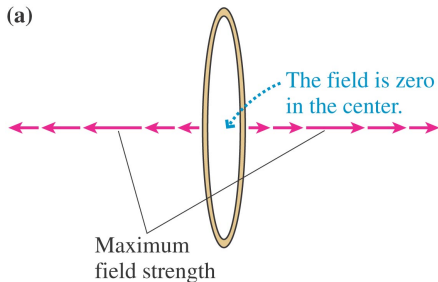
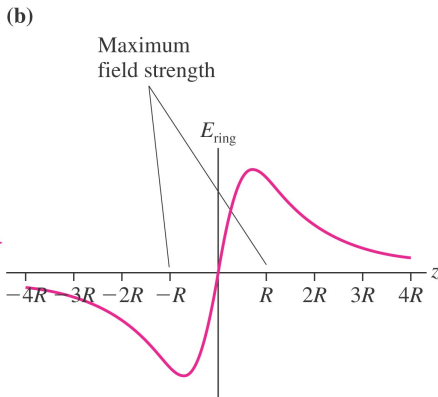


Electric Field from A Thin Ring

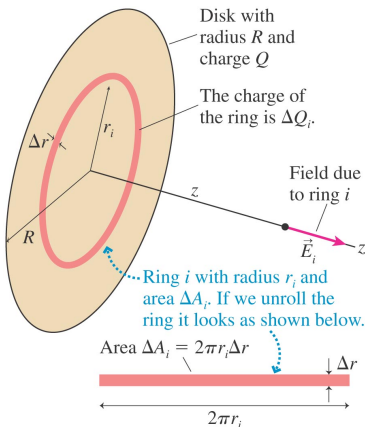


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A Disk of Charge



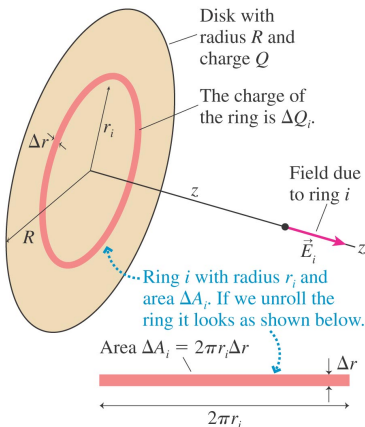
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- Now we will move from lines to surfaces. First, let's try a disk of zero thickness and charge density

$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

Again, we will look at an on-axis point.

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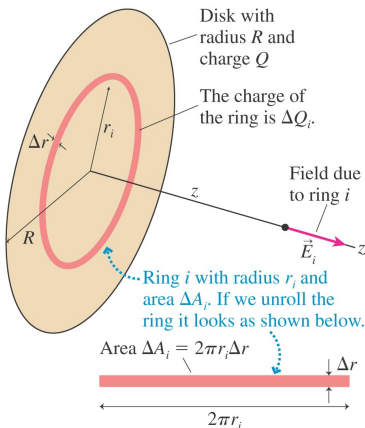
Again, we will look at an on-axis point.

- Now we know how to deal with a thin ring. So, we'll take a bunch of thin rings together and make them into a disk! Each ring gives

$$E_z = \frac{1}{4\pi\epsilon_0} \frac{z\Delta Q_i}{(z^2 + r_i^2)^{3/2}}$$

A Disk of Charge

- So, now we need to sum over many rings



$$E_z = \frac{z}{4\pi\epsilon_0} \sum_{i=1}^N \frac{\Delta Q_i}{(z^2 + r_i^2)^{3/2}}$$

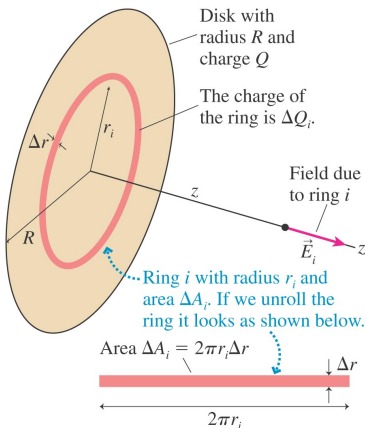
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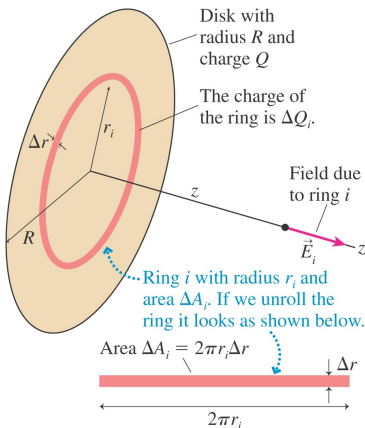
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- We do have to integrate this one. So, a key step is to figure out what variable we will integrate over.



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- We do have to integrate this one. So, a key step is to figure out what variable we will integrate over.
- Let's use the area of a ring and the surface charge density. The area of a ring is

$$\Delta A_i = 2\pi r_i \Delta r$$

and the charge is

$$\Delta Q_i = 2\pi\eta r_i \Delta r$$

A Disk of Charge

- Writing out the sum

$$E_z = \frac{\eta z}{2\epsilon_0} \sum_{i=1}^N \frac{r_i \Delta r}{(z^2 + r_i^2)^{3/2}}$$

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$$\begin{aligned} u &= z^2 + r^2 \\ du &= 2r dr \\ r dr &= \frac{1}{2} du \end{aligned}$$

- And the limits of integration change too...

A Disk of Charge

- The integral becomes

$$E_z = \frac{\eta z}{2\epsilon_0} \frac{1}{2} \int_{z^2}^{z^2+R^2} \frac{du}{u^{3/2}}$$

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A Disk of Charge

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- First factor the z out of the square root

$$E_z = \frac{\eta}{2\epsilon_0} \left[1 - \frac{1}{\sqrt{1 + R^2/z^2}} \right]$$

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- Now it is time for the **binomial approximation**:

$$(1 + x)^n \simeq 1 + nx, x \ll 1$$

So

$$1 - (1 + R^2/z^2)^{-1/2} \simeq 1 - \left(1 + \left(\frac{-1}{2} \right) \frac{R^2}{z^2} \right) = \frac{R^2}{2z^2}$$

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- Substituting this approximation into our field equation gives

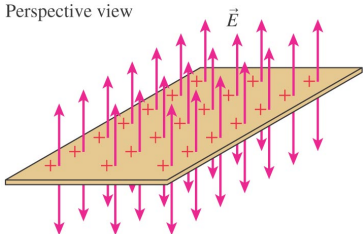
$$E_z = \frac{1}{4\pi\epsilon_0} \frac{Q}{z^2}$$

Hurray!!

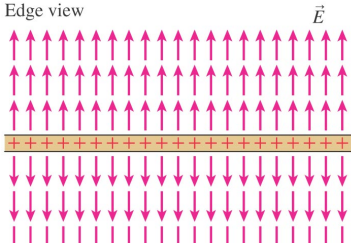
A Plane of Charge

- We model important electronic components (eg. electrodes) as planes of charge.

Perspective view

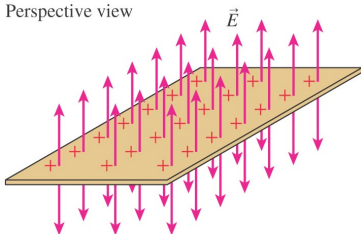


Edge view

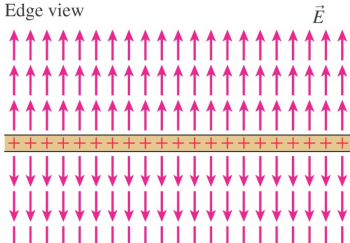


A Plane of Charge

Perspective view



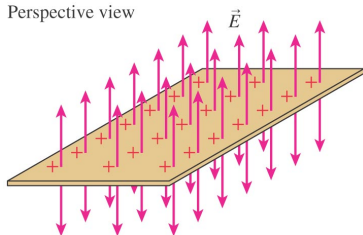
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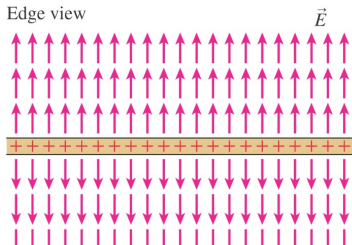
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- Plane are not really infinite but as long as distance to the plane is much smaller than distance to the edges, it is a good model.

A Plane of Charge

Perspective view



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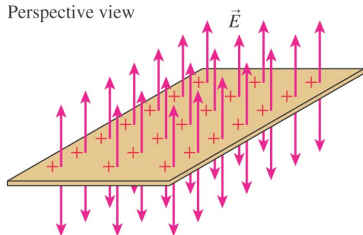


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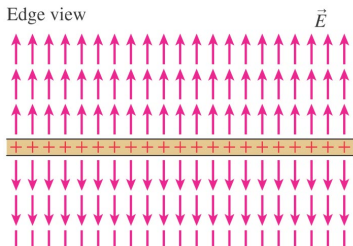
$$E_{\text{plane}} = \frac{\eta}{2\epsilon_0} = \text{constant}$$

A Plane of Charge

Perspective view



Edge view



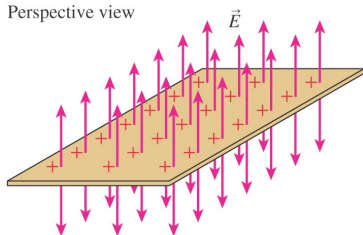
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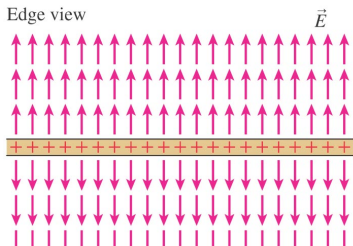
- It does not depend on r at all!!! Same field no matter how far away you are. Look at the parallel lines on the left...no divergence.

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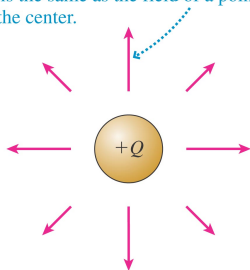
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A Sphere of Charge

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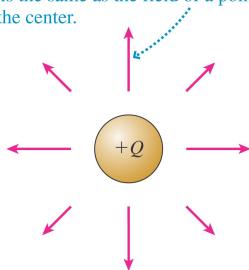
The electric field outside a sphere or spherical shell is the same as the field of a point charge Q at the center.



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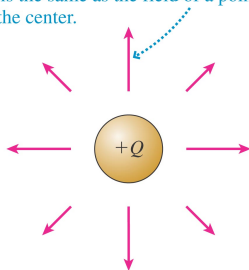


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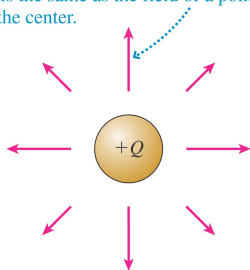
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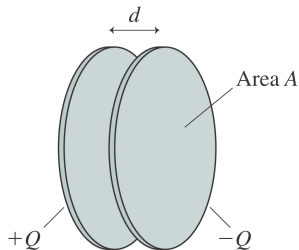
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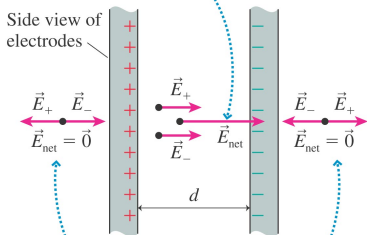
$$\vec{E}_{\text{sphere}} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}, \quad r \geq R$$

- What is the field **inside**?

A Parallel Plate Capacitor (27.5)



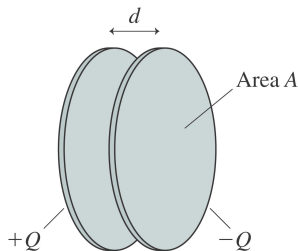
Inside the capacitor, \vec{E}_+ and \vec{E}_- are parallel, so the net field is large.



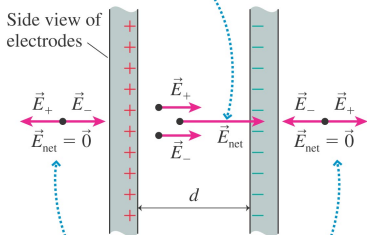
Outside the capacitor, \vec{E}_+ and \vec{E}_- are opposite, so the net field is zero.

- A parallel plate capacitor is formed from two large area plates (relative to their separation) of equal and opposite charge ($+Q$ and $-Q$).

A Parallel Plate Capacitor (27.5)



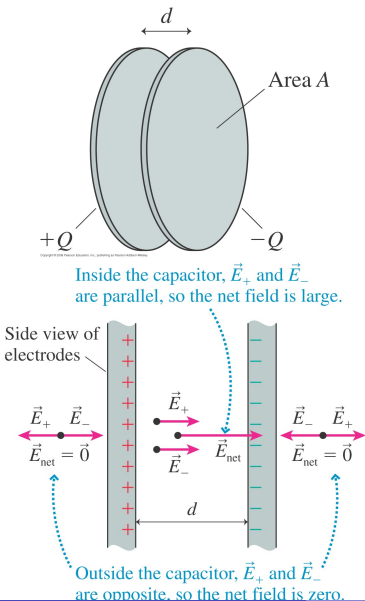
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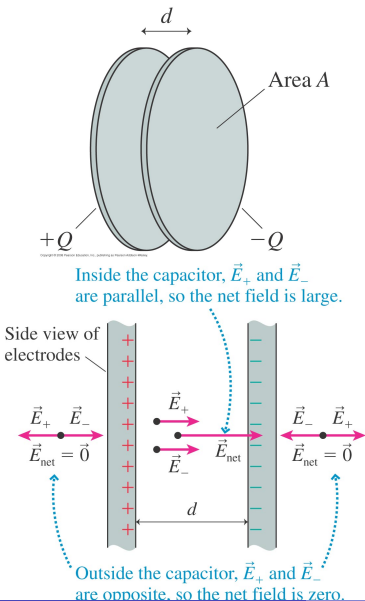
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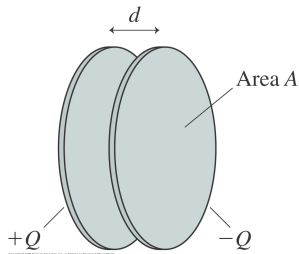
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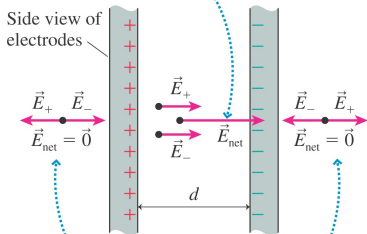


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- Opposite charges attract, so the extra charge sits on the inner surface of each plate.

A Parallel Plate Capacitor



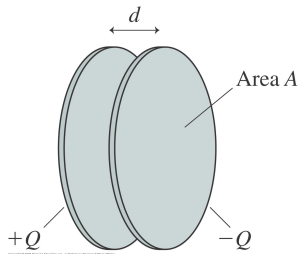
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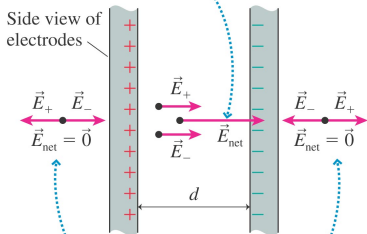
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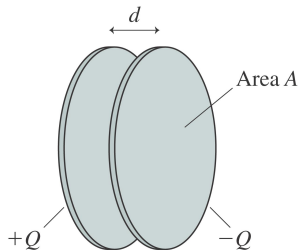


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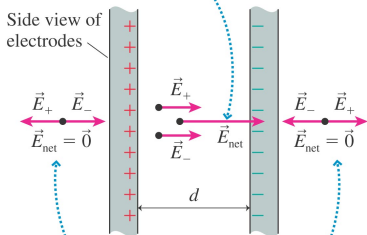
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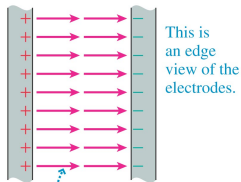
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- Outside the capacitor a test charge would see an opposite field from the positive and negative plate. Since the plates are “infinite” the fields would be identical strength...there is no field!!

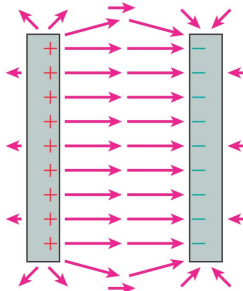
A Parallel Plate Capacitor

(a) Ideal capacitor



The field is constant, pointing from the positive to the negative electrode.

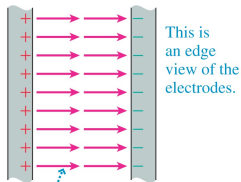
(b) Real capacitor



- An ideal capacitor has a strong electric field between the plates

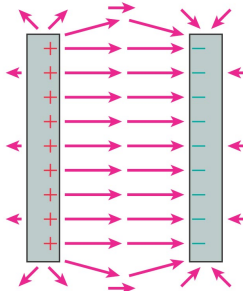
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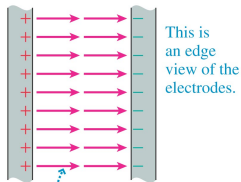
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- An ideal capacitor has a no electric field outside the plates

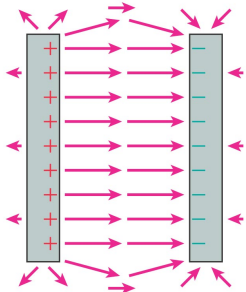
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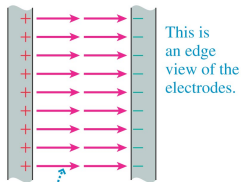
(b) Real capacitor



- An ideal capacitor has a strong electric field between the plates
- An ideal capacitor has a no electric field outside the plates
- A real-world capacitor has a **fringe field** outside the capacitor.

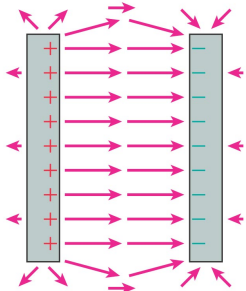
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(a) Ideal capacitor



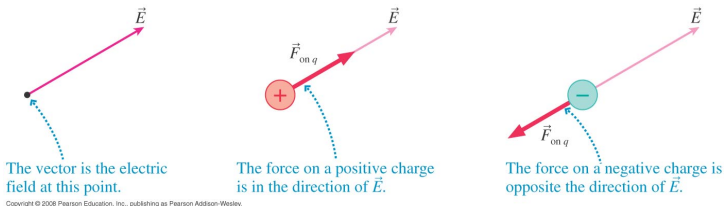
The field is constant, pointing from the positive to the negative electrode.

(b) Real capacitor



- An ideal capacitor has a strong electric field between the plates
- An ideal capacitor has no electric field outside the plates
- A real-world capacitor has a **fringe field** outside the capacitor.
- Capacitors create a **uniform electric field** which is highly useful in manipulating a charged particles.

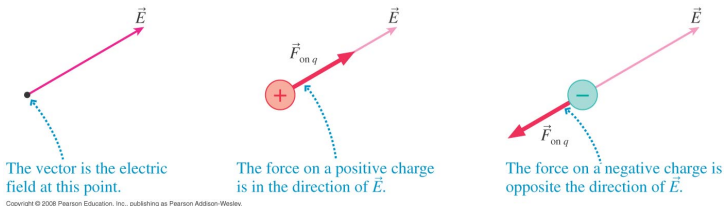
Motion of a Charged Particle in an Electric Field (27.6)



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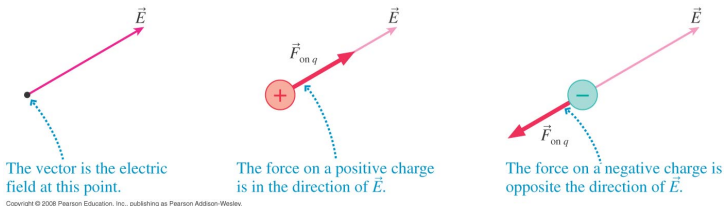


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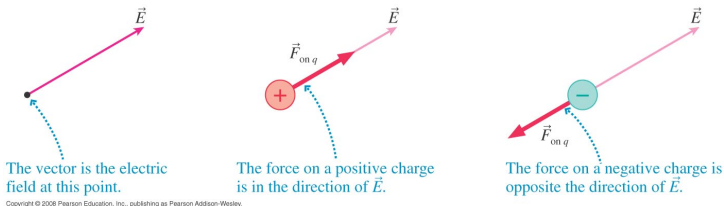
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- q/m is called the **charge-to-mass ratio**. Two equal charges with different mass experience the same force but different acceleration

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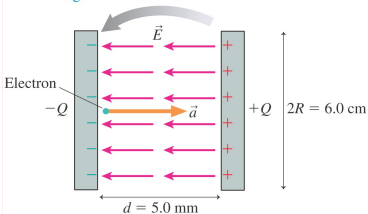
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- A constant acceleration should remind you of what happens to a particle in a gravitational field. All that mathematics you learned in P120 for projectiles will now come in handy for charged particles!
- We can use a parallel plate capacitor to make a beam of electrons and accelerate the particles.
- We could then use another capacitor to steer the beam.

Example 27.8 - an Electron Moving Across a Capacitor

Example 27.8

The capacitor was charged by transferring 10^{11} electrons from the right electrode to the left electrode.

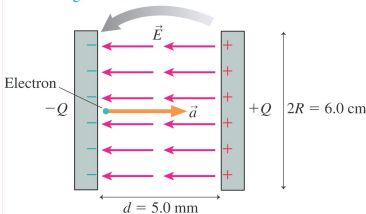


Two 6.0-cm-diameter electrodes are spaced 5.0 mm apart. They are charged by transferring 1.0×10^{11} electrons from one electrode (plate) to the other.

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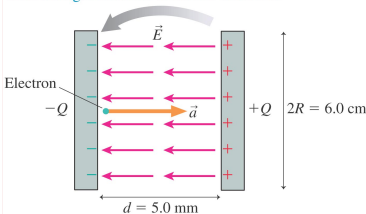


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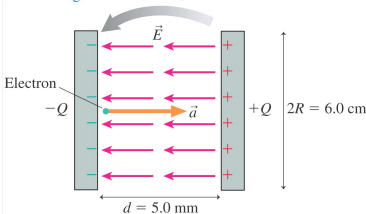


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Example 27.8

- The electric field inside the capacitor is

$$E = \frac{\eta}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = \frac{Ne}{\epsilon_0 \pi R^2} = 6.39 \times 10^5 \text{ N/C}$$

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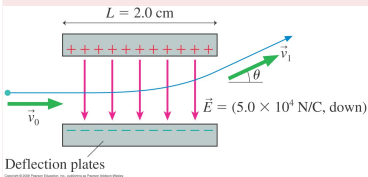
$$x_f = d = \frac{1}{2} a (\Delta t)^2$$
$$\Delta t = \sqrt{\frac{2d}{a}} = 3.0 \times 10^{-10} \text{ s} = 0.30 \text{ ns}$$

- The speed of the electron when it reaches the positive plate is

$$v = a \Delta t = 3.3 \times 10^7 \text{ m/s}$$

Example 27.9 - Deflecting an Electron Beam

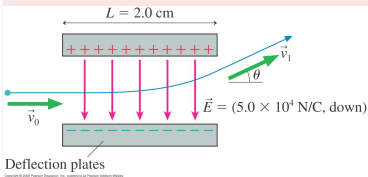
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An electron gun creates a beam of electrons moving horizontally with speed $3.3 \times 10^7 \text{ m/s}$. The electrons enter a 2.0-cm-long gap between two parallel electrodes where the electric field is $\vec{E} = (5.0 \times 10^4 \text{ N/C, down})$.

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- The electron enters with velocity $\vec{v}_0 = 3.3 \times 10^7 \hat{i}$ m/s and exits with $\vec{v}_1 = v_{1x}\hat{i} + v_{1y}\hat{j}$.

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- There is no horizontal force, so the x-component of velocity does not change. The time to cross the plates is then known:

$$\Delta t = \frac{L}{v_{0x}} = \frac{0.020\text{m}}{3.3 \times 10^7 \text{ m/s}} = 6.06 \times 10^{-10} \text{ s}$$

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- However, there is an upward acceleration of magnitude

$$a = \frac{eE}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(5.0 \times 10^4 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} = 8.78 \times 10^{15} \text{ m/s}^2$$

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- Giving a final vertical velocity of

$$v_{1y} = v_{0y} + a\Delta t = 5.3 \times 10^6 \text{ m/s}$$

Example 27.9 - Deflecting an Electron Beam

- So, the final velocity is

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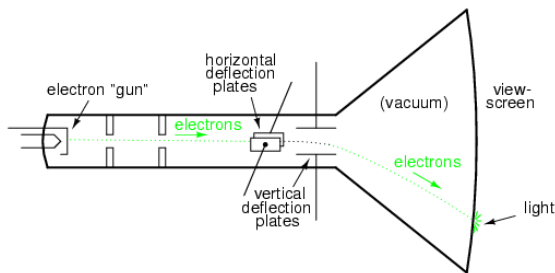
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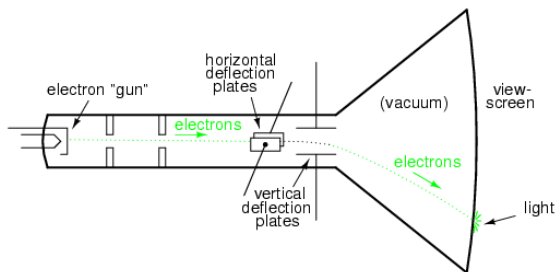
- We use the same mathematics as for mechanics/gravity...but the magnitudes of acceleration, velocity, etc. are much higher than what we are used to!
- In case you wondered why you bothered to do some special relativity last term...consider the speeds of these electrons!

Examples: Your Oscilloscope is a CRT



- Some oscilloscopes use an electron gun with vertical and horizontal deflection plates to control the beam.

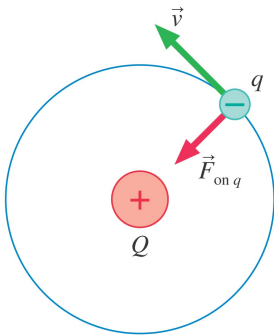
Examples: Your Oscilloscope is a CRT



- Some oscilloscopes use an electron gun with vertical and horizontal deflection plates to control the beam.
- The electron diffraction apparatus demonstrated in class used parallel charged plates to control the electron beam's direction.

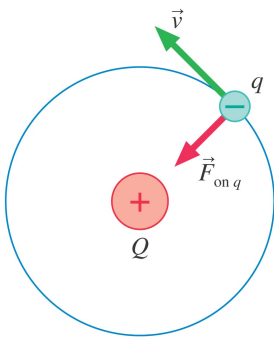
Motion in a Non-Uniform Field

- A non-uniform field is much more difficult to deal with.



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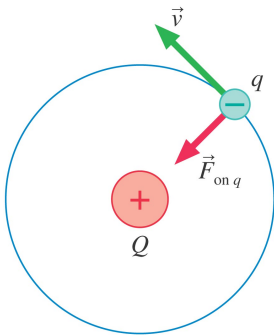
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Motion in a Non-Uniform Field



- A non-uniform field is much more difficult to deal with.
- However, one simple example is a negatively charged particle orbiting a positively charged one.
- Same mathematics as the moon orbiting the earth. The charge can move in a circular orbit if

$$|q|E = \frac{mv^2}{r}$$

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