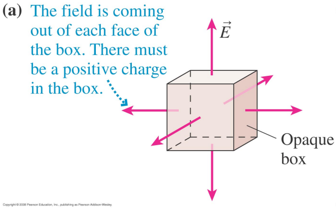
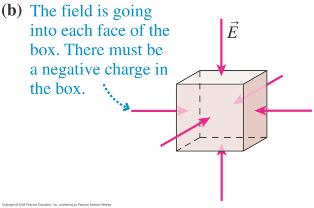
The Concept of Flux (28.2)





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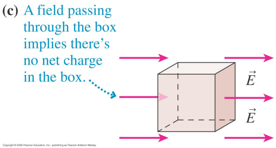

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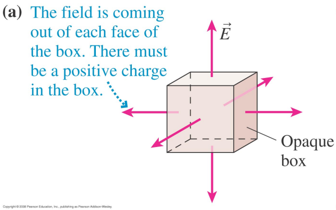

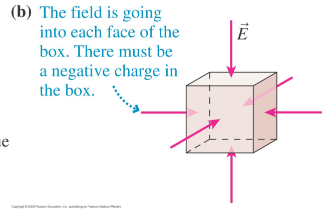

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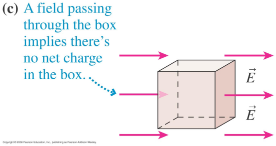

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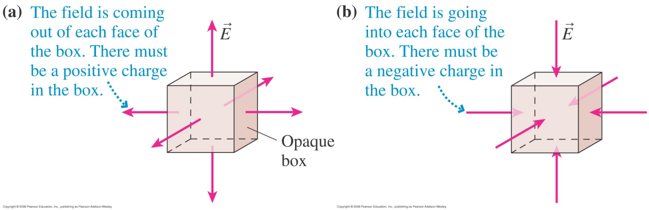

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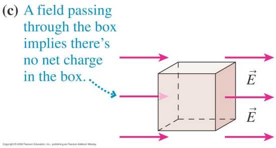

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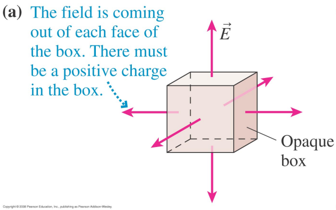
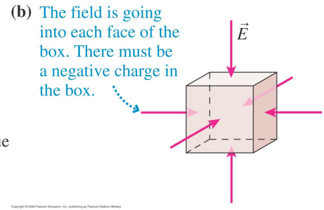
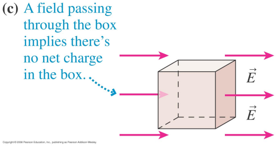
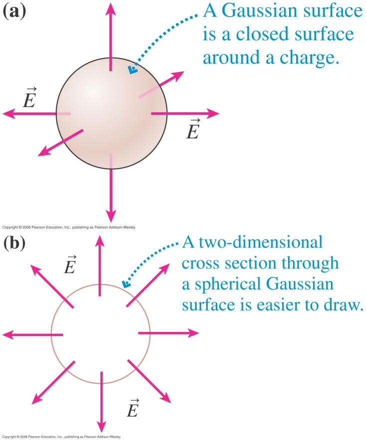




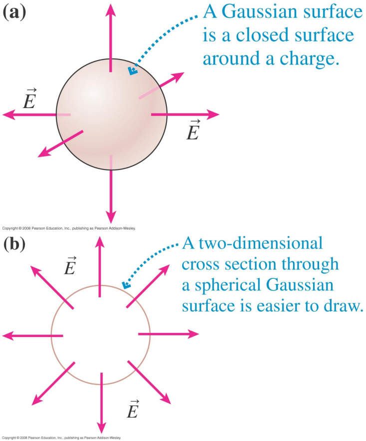
- We also need to define the concept of flux.
- By looking at the boxes above, can you tell what is in the box?
- The box on the right looks like it merely has an electric field flowing through the box....not a net charge inside.
- We need a better box. We will define a Gaussian surface surrounding a given charge distribution.



The Concept of Flux

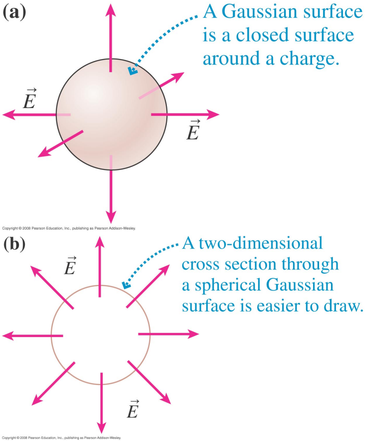


- A Gaussian surface is a mathematical surface which encloses a region of charge in 3D.



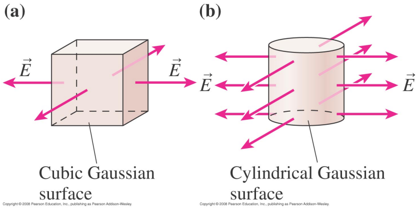
- A Gaussian surface is a mathematical surface which encloses a region of charge in 3D.
- We can use the spherical symmetry of the box on the left to determine that there is a net positive charge at the center of this box.

The Concept of Flux



- A Gaussian surface is a mathematical surface which encloses a region of charge in 3D.
- We can use the spherical symmetry of the box on the left to determine that there is a net positive charge at the center of this box.
- The electric field is everywhere perpendicular to the spherical surface and is the same magnitude everywhere on the surface.

The Concept of Flux



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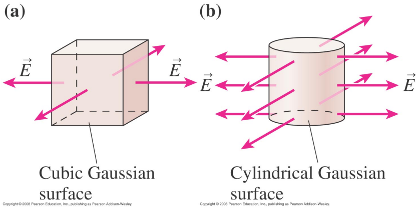
The Concept of Flux



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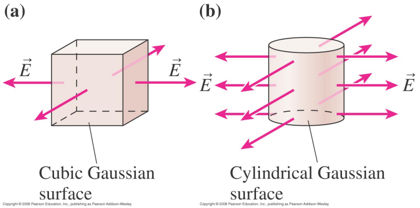

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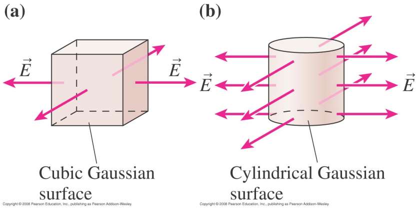

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- The box on the left has flux through 4/6 sides of the cube. So, what is inside??
- Choose a cylindrical box instead and it is clear...there is a cylindrical charge distribution inside.
- The lesson is: you need a closed surface of the right shape to solve the problem quickest.

Qualitative Summary of Flux

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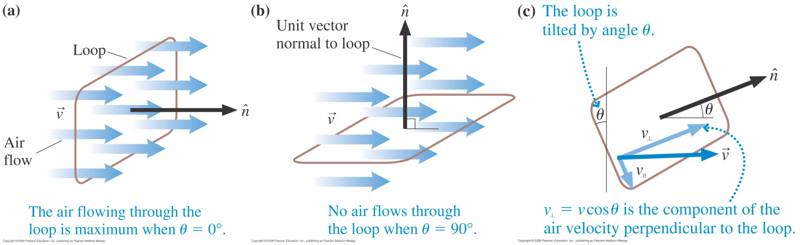
Qualitative Summary of Flux

- We enclose a charge distribution in a Gaussian surface
- Flux is modeled on fluid flow
- There is an outward flux through the surface from a positive enclosed charge.
- There is an inward flux through the surface from a negative enclosed charge.
- There is no **net** flux through a closed surface that does not enclose net charge.

Calculating Electric Flux (28.3)



- The figures above show air flowing through a loop. Clearly the rate of flow through the loop depends on the angle of the plane of the loop compared to the initial air velocity.

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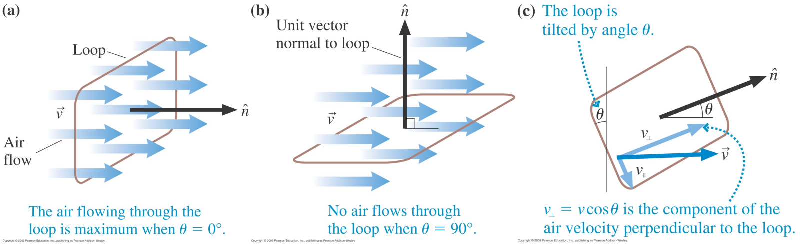

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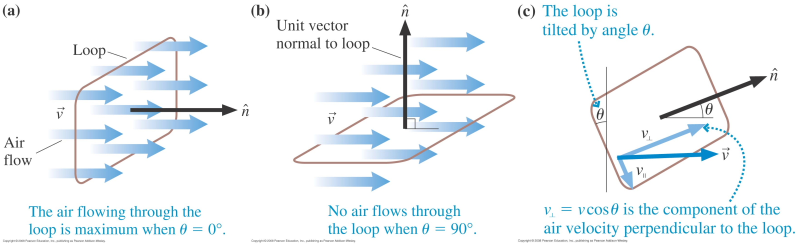
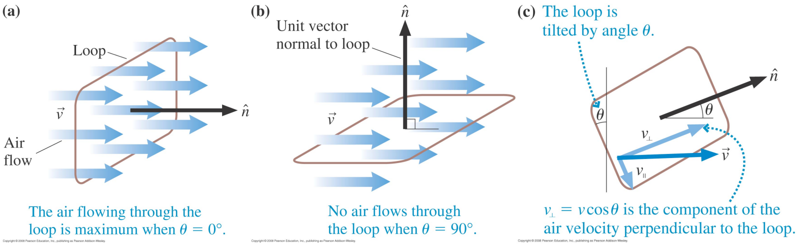


- The figures above show air flowing through a loop. Clearly the rate of flow through the loop depends on the angle of the plane of the loop compared to the initial air velocity.
- We define $\hat{n}$ as a unit vector perpendicular to the plane of the loop.
- We define θ as the angle between $\vec{v}$ and $\hat{n}$. Max flow is then at $\theta = 0°$ and min flow is at $\theta = 90°$

Calculating Electric Flux



- We can decompose $\vec{v}$ into two components...only the perpendicular one counts for the air flux. So, the volume of air per second through the loop is

$$V = v_\perp A = vA\cos\theta$$

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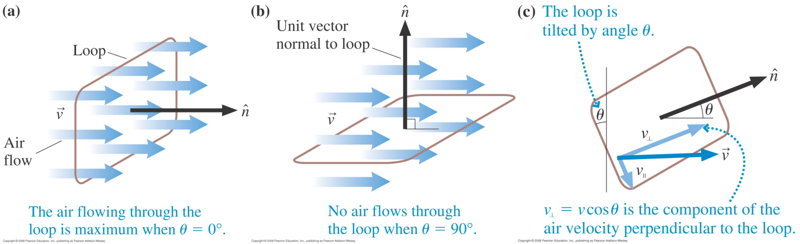

- We can decompose $\vec{v}$ into two components...only the perpendicular one counts for the air flux. So, the volume of air per second through the loop is

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- The same idea applies to electric fields. We define electric flux as

$$\Phi = E_\perp A = EA\cos\theta$$

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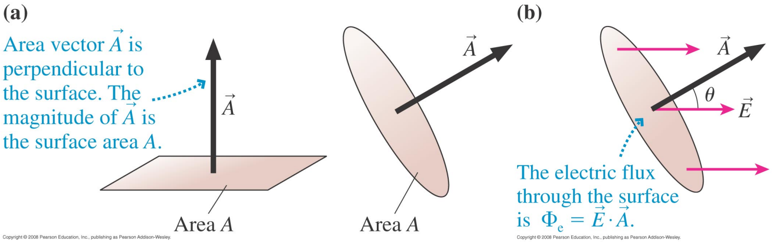

- That expression looks a lot like the magnitude of a dot product of 2 vectors:

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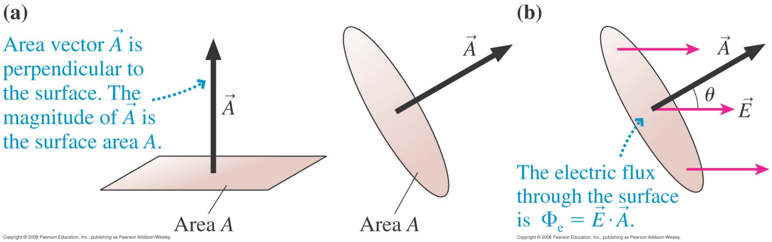

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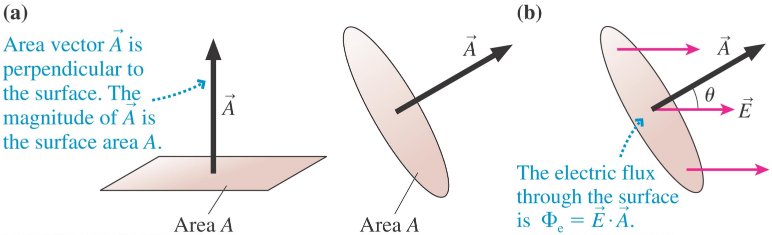

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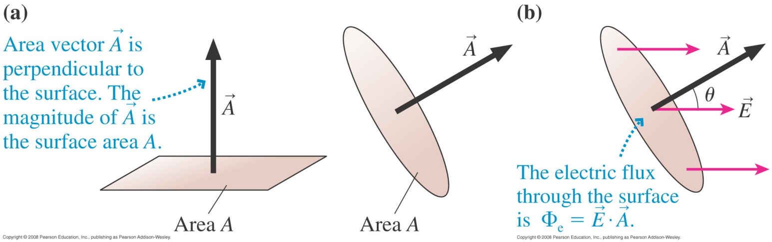

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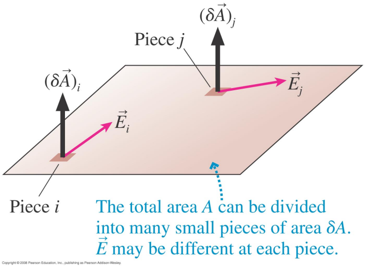
- Wait a minute! There is no such thing as an area vector! Well, now there is...we just made it up.
- The area vector is perpendicular to the surface with magnitude equal to the area of the surface. Makes for a convenient definition of Φ.

Electric Flux of a Nonuniform Electric Field



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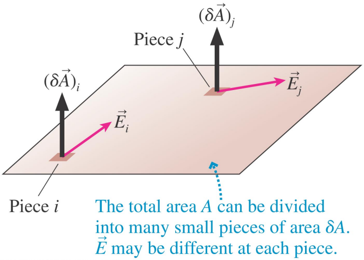

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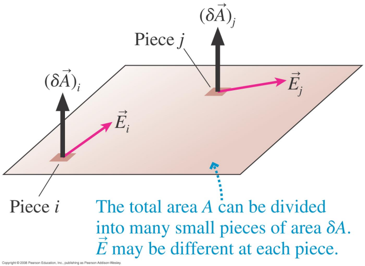

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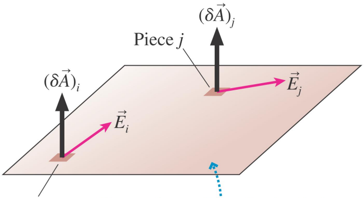
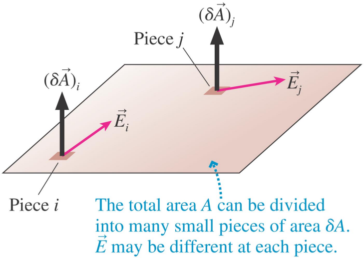


- We have so far been looking at fluxes which are constant over a surface. What if the flux were different at every point?
- Well, maybe we can divide the surface into little pieces over which the flux is constant!
- Because flux is a scalar, this is easier than adding fields.
- Consider a small area $\delta\vec{A}_i$ where the electric field is E_i:

$$\delta\phi_i = \vec{E}_i \cdot (\delta\vec{A})_i$$

Electric Flux of a Nonuniform Electric Field



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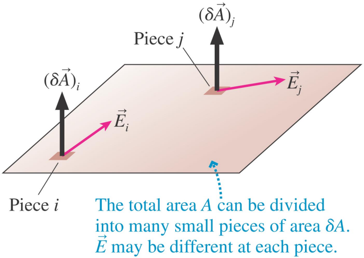

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- Now if we make δA_i infinitely small we have

$$\Phi_e = \int_{surface} \vec{E} \cdot d\vec{A}$$

this integral is known as a surface integral. (instead of dividing things into little 1D pieces, we divide into little multi-D pieces)

Electric Flux of a Nonuniform Electric Field

- Consider the special case in which $\vec{E}$ is the same everywhere

$$\Phi_e = \int_{surface} \vec{E} \cdot d\vec{A} = \int_{surface} E \cos\theta dA = E \cos\theta \int_{surface} dA$$

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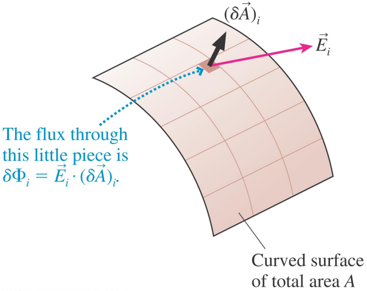
- Well, we know the integral of dA is just A, so

$$\Phi_e = EA \cos\theta$$

Good news! It makes sense in this limit....

The Flux Through a Curved Surface



- What if the surface is curved? Do the same thing!

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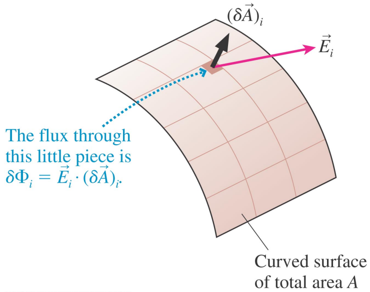
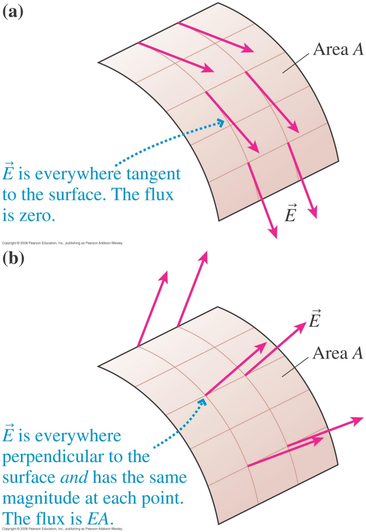


- What if the surface is curved? Do the same thing!
- Divide the surface into small areas δA and get the result

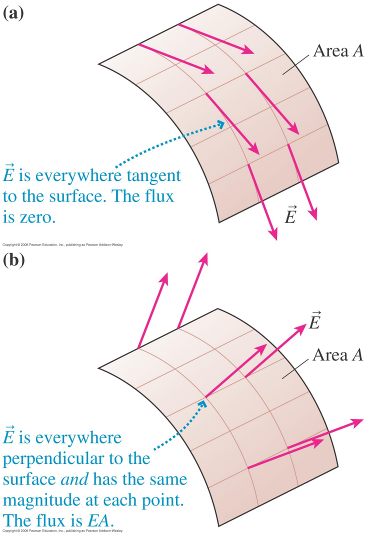
$$\Phi_e = \int_{surface} \vec{E} \cdot d\vec{A}$$

The Flux Through a Curved Surface



- There are a couple of special cases for which things are easy:

The Flux Through a Curved Surface



- There are a couple of special cases for which things are easy:
  1. If the electric field is everywhere tangent to the surface, the electric flux through the surface is zero.

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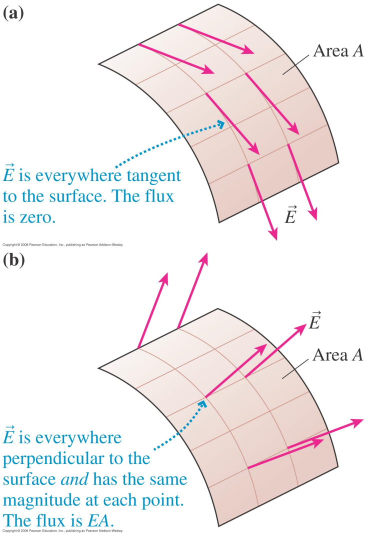

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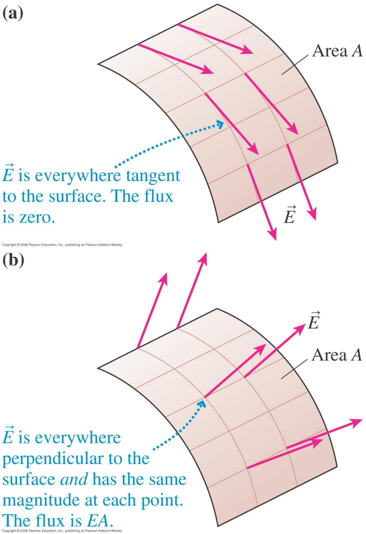

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- If the surface is closed we change notation a little

$$\Phi_e = \oint \vec{E} \cdot d\vec{A}$$

Example 28.2 - Flux through a closed cylinder

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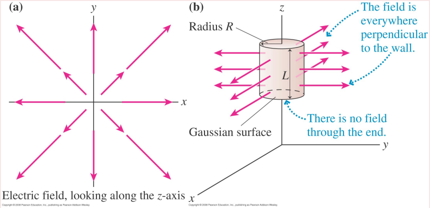

A cylindrical charge distribution has created the electric field

$$\vec{E} = \frac{E_0 r^2}{r_0^2}\hat{r}$$

where E_0 and r_0 are constants and where $\hat{r}$ lies in the x-y plane.

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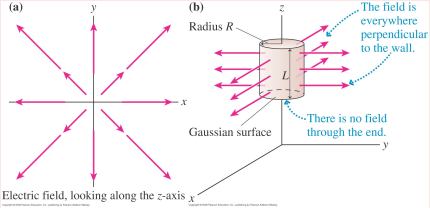

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where E_0 and r_0 are constants and where $\hat{r}$ lies in the x-y plane. Calculate the electric flux through a closed cylinder of length L and radius R that is centered along the Z-axis.

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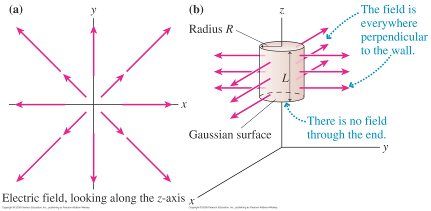

- We draw a cylindrical Gaussian surface with three surfaces: the top, the bottom, and the cylindrical wall.

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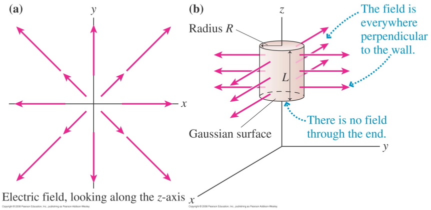

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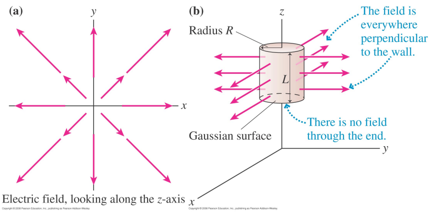

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- For the cylindrical wall, the electric field perpendicular and has the same strength at every point. Therefore the flux is

$$\Phi_{wall} = EA_{wall}$$

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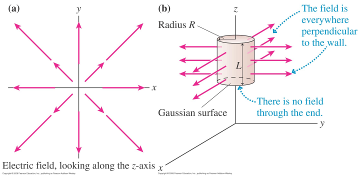

- To get the total flux we should add all the fluxes

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \Phi_{top} + \Phi_{bot} + \Phi_{wall} = 0 + 0 + EA_{wall} = EA_{wall}$$

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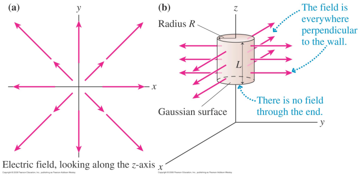

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- We have done a “surface integral” by just diving the surface into parts with tangent fields and perpendicular fields and exploiting known properties. Doing integrals without integrating!

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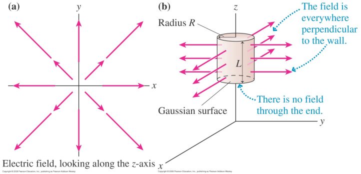
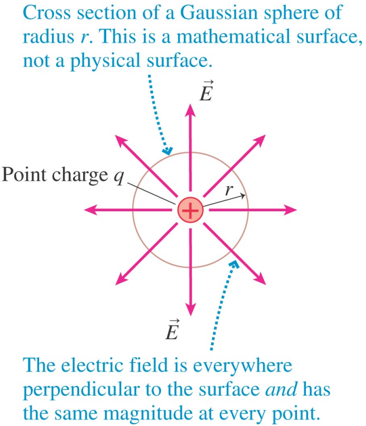


- To get the total flux we should add all the fluxes

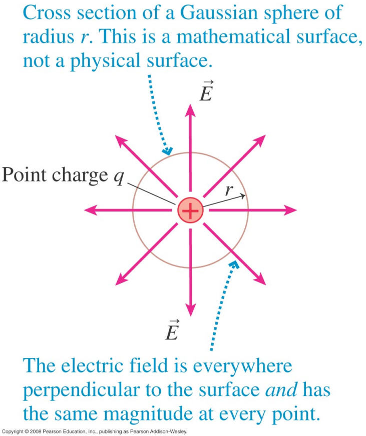
$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \Phi_{top} + \Phi_{bot} + \Phi_{wall} = 0 + 0 + EA_{wall} = EA_{wall}$$

- We have done a "surface integral" by just diving the surface into parts with tangent fields and perpendicular fields and exploiting known properties. Doing integrals without integrating!
- In this case we know both $\vec{E}$ and A_{wall}, so:

$$\Phi_e = \left(E_0 \frac{R^2}{r_0^2}\right)(2\pi RL) = \frac{2\pi LR^3}{r_0^2} E_0$$

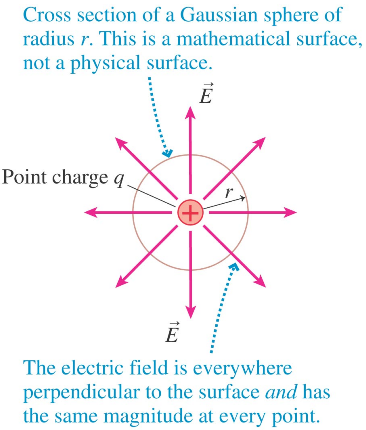


- Consider the electric field from a point charge passing through a surrounding Gaussian surface.



- Consider the electric field from a point charge passing through a surrounding Gaussian surface.
- By Coulomb's law we know the electric field everywhere on the sphere:

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$



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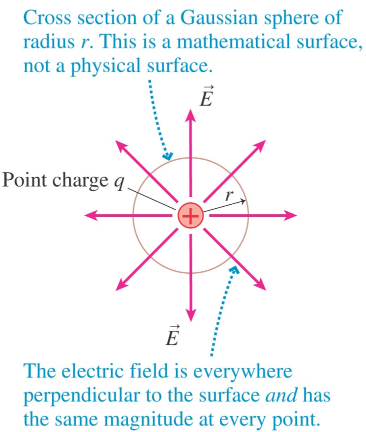
Gauss' Law (28.4)



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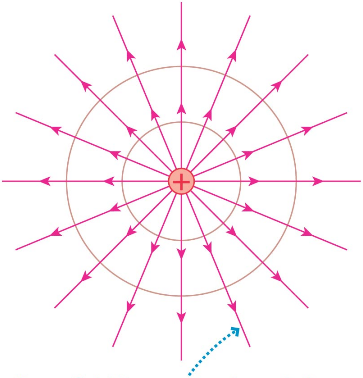
- The point-charge + sphere is pretty easy.

Electric Flux is Independent of Radius



Every field line passing through the smaller sphere also passes through the larger sphere. Hence the flux through the two spheres is the same.

- The electric flux through the whole surface does not depend on the radius of the sphere!

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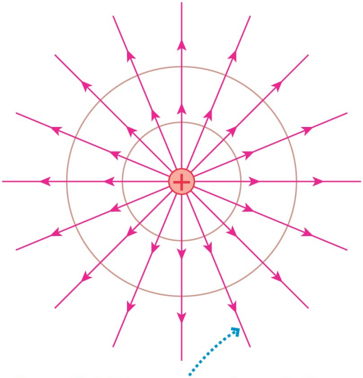

Every field line passing through the smaller sphere also passes through the larger sphere. Hence the flux through the two spheres is the same.

- The electric flux through the whole surface does not depend on the radius of the sphere!
- No matter how big the surface is, all radial lines still pass through it. However, the flux through a small piece of the surface will be different.