Traveling Waves

Leonardo Da Vinci (15th century)

“It often happens that the wave flees the place of its creation, while the water does not; like the waves made in a field of grain by the wind, where we see the waves running across the field while the grain remains in place.”
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So, a traveling wave is the propagation of a disturbance in a medium, but the medium itself does not propagate.
Traveling waves come in two types

Transverse (like light)

Up/down  Motion of wave at speed $v$
Traveling waves come in two types.

- **Transverse (like light)**
  - Up/down
  - Motion of wave at speed $v$

- **Longitudinal (like sound)**
  - Push/pull
  - Motion of wave at speed $v$
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Amplitude ($A$): Maximum value of a disturbance

- **Period ($T$):** Time for one complete wave cycle to pass a fixed point
- **Frequency ($f$):** Number of wave cycles passing a given point per unit time
  \[ f = \frac{1}{T} \]
- **Wave speed ($v$):** During one period a fixed observer sees one complete wavelength go by
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In other words, the speed of light does not depend on whether you use a lightbulb or a laser beam. However, the frequency does depend on the source....and so does the wavelength.
Sinusoidal Waves – Simulation

Simulation of transverse waves on a rope: http://phet.colorado.edu/
Mathematics of Sinusoidal Waves

We can define the displacement $D$ of a particle in a medium due to the passage of a sinusoidal wave (snapshot at $t = 0$):

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This equation is periodic with period $\lambda$ ($D(x) = D(x + \lambda)$). We can turn this into a moving wave by making the position depend on time: $x \rightarrow x - vt$

$$D(x, t) = A \sin \left(2\pi \frac{x - vt}{\lambda} + \phi_0\right) = A \sin \left(2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right) + \phi_0\right)$$
We can re-express this in different notation to clean things up a little. Let’s define the angular frequency ($\omega$) and wave number ($k$) as

$$\omega = 2\pi f = \frac{2\pi}{T}, \quad k = \frac{2\pi}{\lambda}$$

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Wave number is the spatial analogue of frequency, it describes the number of radians of wave cycle per unit distance. This allows us to write the equation from the previous page as

\[
D(x, t) = A \sin(kx - \omega t + \phi_0)
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Example: Waves on a String

In this case displacement is \( y \):

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This is referring to the particle speed.
Your text (page 613) then goes through a number of small algebraic steps looking at the net force on a small segment of string.

\[ v = \sqrt{\frac{T_s}{\mu}} \]

where \( T_s \) is the tension and \( \mu \) is the linear density. This defines the wave speed.

Please note that it depends only on properties of the medium and what that dependence is.
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Waves in 2 or 3 Dimensions

So far we have discussed waves in 1-D. What do they look like in more than one dimension?

Wave fronts are the crests of the wave. They are spaced one wavelength apart.

The circular wave fronts move outward from the source at speed $v$.

Very far away from the source, small sections of the wave fronts appear to be straight lines.
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Mathematically, little changes:

\[ D(r, t) = A(r) \sin(kr - \omega t + \phi_0) \]

(note amplitude change as wave spreads)
The phase of the wave is

\[(kx - \omega t + \phi_0)\]
Phase and Phase Difference

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Very soon the phase difference will be very important since we will talk about interference. The phase difference is

\[\Delta \phi = \phi_2 - \phi_1 = (kx_2 - \omega t + \phi_0) - (kx_1 - \omega t + \phi_0)\]
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Moving from one crest to another represents a phase shift of $2\pi$
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