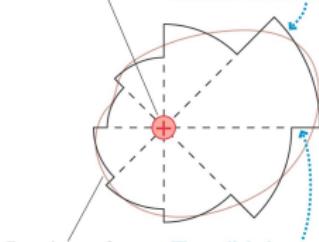


# Electric Flux is Independent of Surface Shape

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The spherical pieces are centered on the charge.



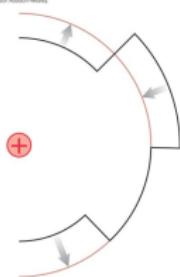
Gaussian surface of arbitrary shape

The radial pieces are along lines extending out from the charge. There's no flux through these.

The approximation with spherical and radial pieces can be as good as desired by letting the pieces become sufficiently small.

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(b)



The spherical pieces can slide in or out to form a complete sphere. Hence the flux through the pieces is the same as the flux through a sphere.

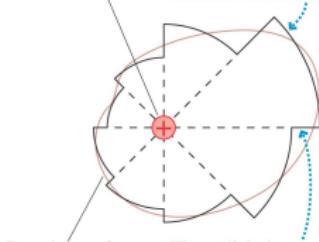
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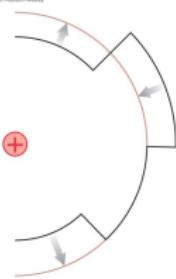
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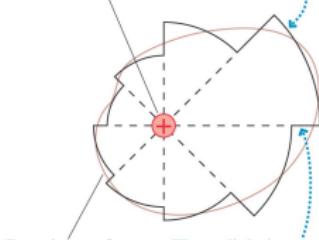
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- What about the flux through some arbitrary surface shape?
- Approximate the surface by a patchwork of radial and spherical pieces. Spherical pieces are centered on the charge. Radial pieces have 0 flux through them.

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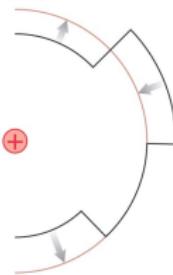
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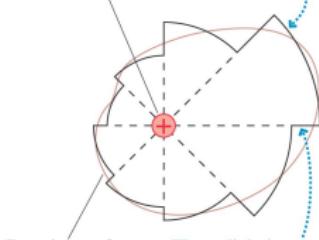
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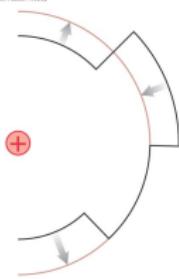
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- What about the flux through some arbitrary surface shape?
- Approximate the surface by a patchwork of radial and spherical pieces. Spherical pieces are centered on the charge. Radial pieces have 0 flux through them.
- We can make the pieces arbitrarily small to get the best model of the surface.
- We can slide the pieces around and make a sphere. So, the flux through any closed surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

# Charge Outside the Surface

- We have only been talking about charges enclosed by the surface.  
What about charges outside the surface?

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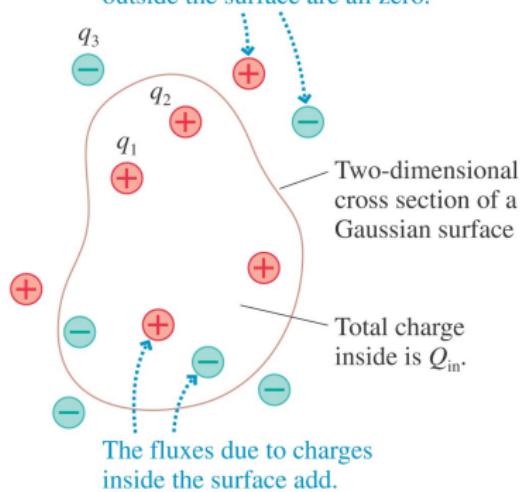
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What about charges outside the surface?
- Anything outside the surface will contribute nothing to the total flux!
- Any line entering the surface in one place will exit it in another....there is no net flux unless the charge is enclosed.

# Multiple Charges

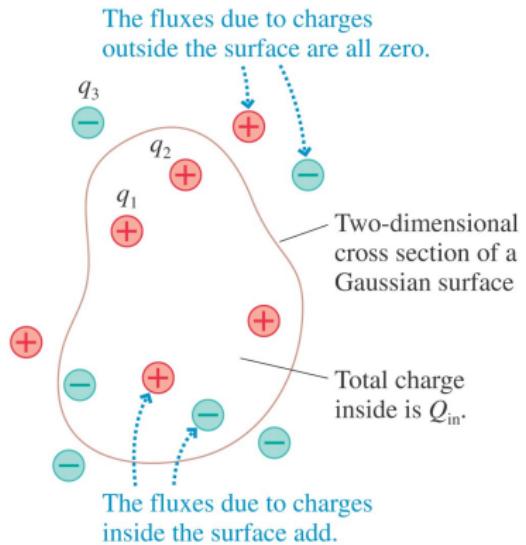
The fluxes due to charges outside the surface are all zero.



- What is the flux from a set of charges through a surface?

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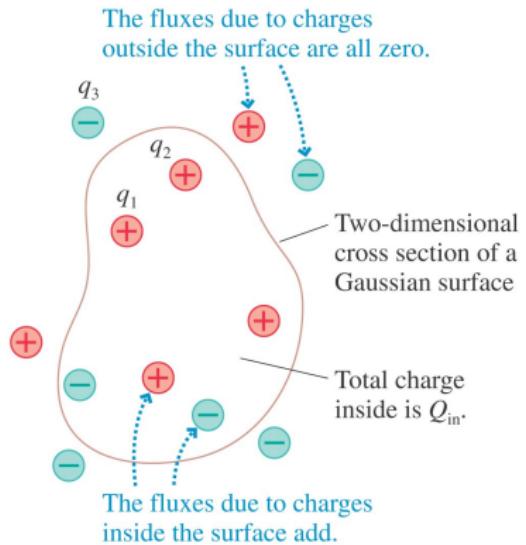
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- What is the flux from a set of charges through a surface?
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$$\Phi_e = \Phi_1 + \Phi_2 + \Phi_3 + \dots$$

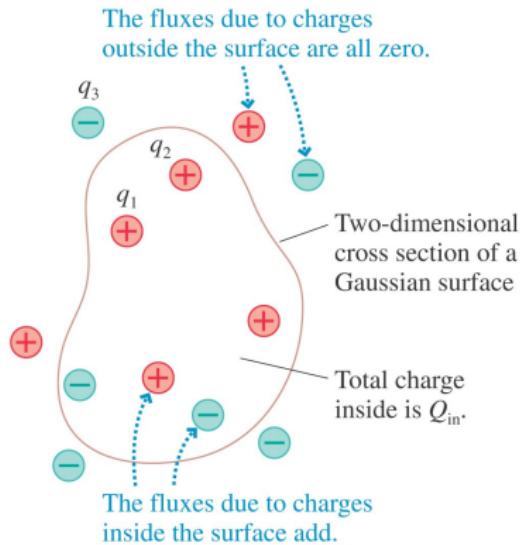
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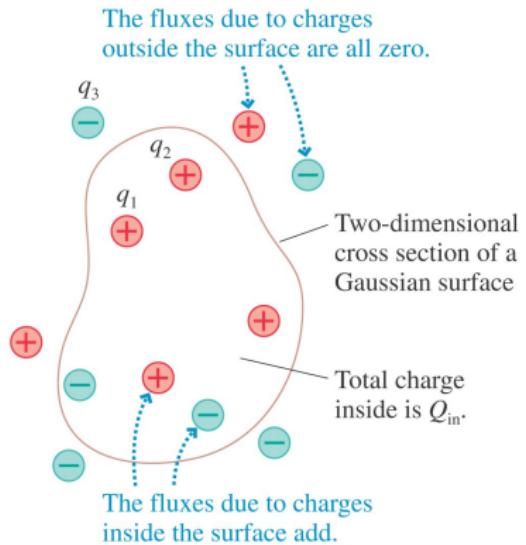
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$$\begin{aligned}\Phi_e &= \Phi_1 + \Phi_2 + \Phi_3 + \dots \\ \Phi_e &= \left( \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \dots \frac{q_i}{\epsilon_0} \right)_{inside} + (0)_{out}\end{aligned}$$

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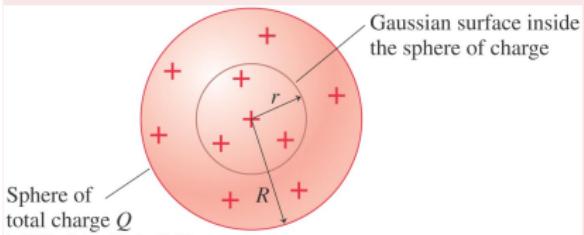
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where  $Q_{in} = q_1 + q_2 + \dots + q_i$

# Using Gauss' Law - Example 28.4

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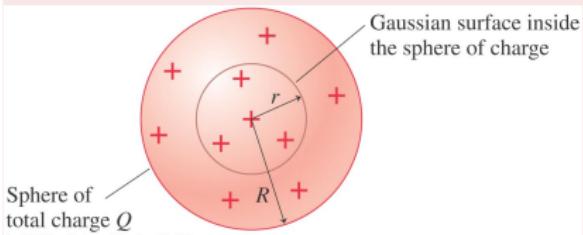


What is the electric field **inside** a uniformly charged sphere?

Solving this with Coulomb's Law and superposition would not be very fun.

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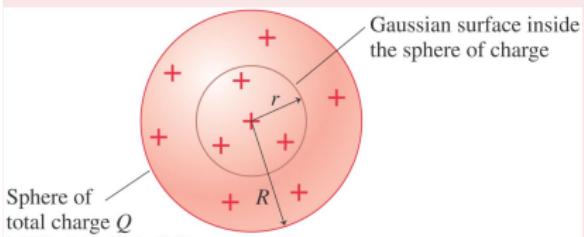


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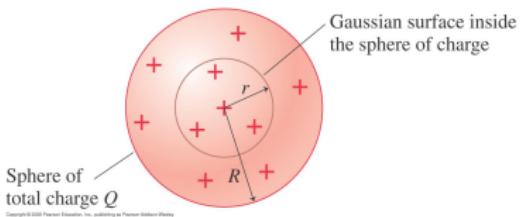
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Solving this with Coulomb's Law and superposition would not be very fun. However, there is a clear spherical symmetry in the problem. Let's try it with a spherical Gaussian surface inside the original sphere.

# Using Gauss' Law - Example 28.4

- $\vec{E}$  is perpendicular to the surface and has the same strength everywhere on the surface. The flux is therefore

$$\Phi_e = EA_{\text{sphere}} = 4\pi r^2 E = \frac{Q_{\text{in}}}{\epsilon_0}$$



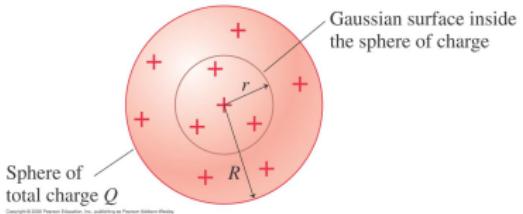
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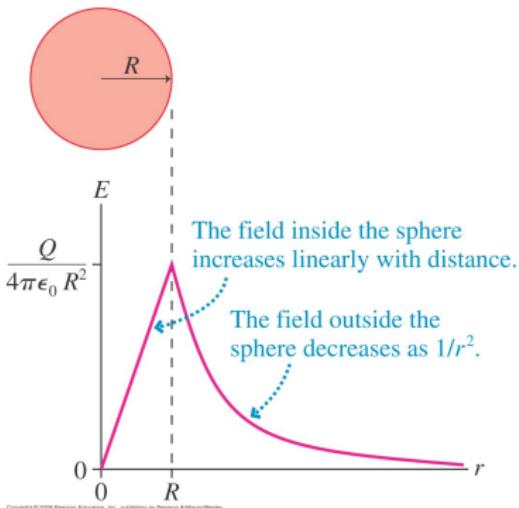
$$\Phi_e = EA_{\text{sphere}} = 4\pi r^2 E = \frac{Q_{in}}{\epsilon_0}$$

- $Q_{in}$  is the charge inside the Gaussian sphere (ie. the little one). The charge is uniform and the **volume charge density** is

$$\rho = \frac{Q}{V_R} = \frac{Q}{\frac{4}{3}\pi R^3}$$



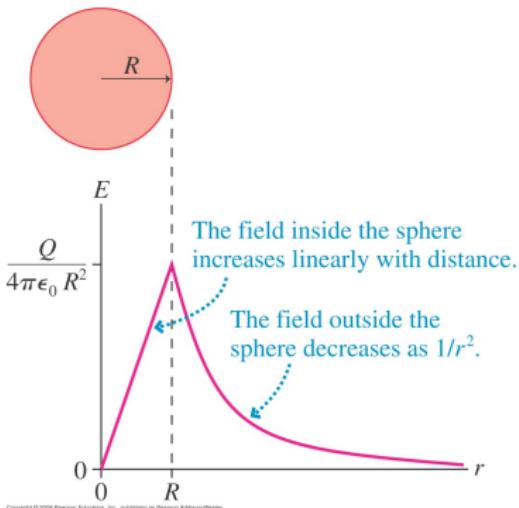
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$$Q_{in} = \rho V_r = \left( \frac{Q}{\frac{4}{3}\pi R^3} \right) \left( \frac{4}{3}\pi r^3 \right) = \frac{r^3}{R^3} Q$$

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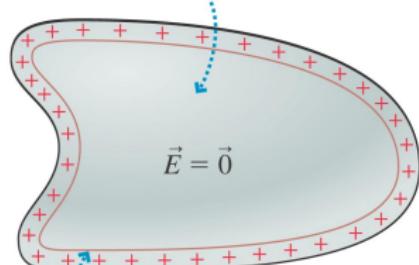
- Gauss' Law becomes

$$E = \frac{1}{4\pi r^2} \frac{Q_{in}}{\epsilon_0} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} r$$

# Conductors in Electrostatic Equilibrium (28.6)

- We know that a conductor has any excess charge on its surface and  $\vec{E}$  is zero anywhere on the inside (else charges would be moving).

The electric field inside the conductor is zero.



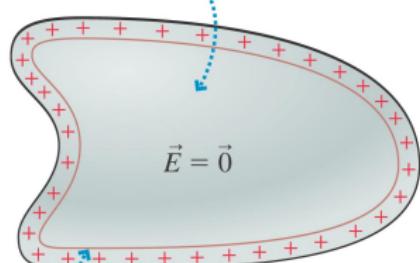
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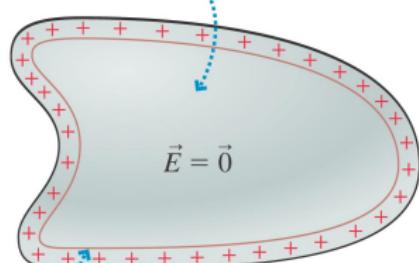
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- The electric field at the surface is perpendicular to the surface and has magnitude (see your text):

$$\vec{E}_{surface} = \frac{\eta}{\epsilon_0}$$

where  $\eta$  is surface charge density.

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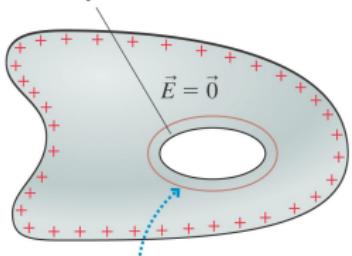


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# Conductors in Electrostatic Equilibrium

A hollow completely enclosed by the conductor

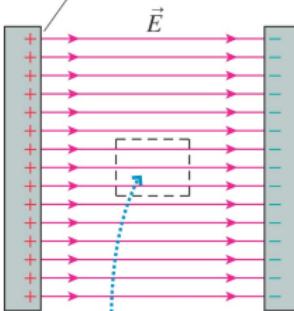


The flux through the Gaussian surface is zero. There's no net charge inside, hence no charge on this interior surface.

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(a)

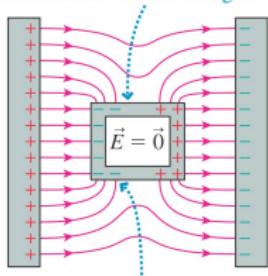
Parallel-plate capacitor



We want to exclude the electric field from this region.

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(b) The conducting box has been polarized and has induced surface charges.



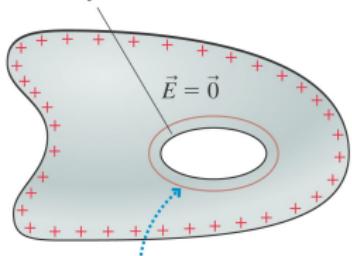
The electric field is perpendicular to all conducting surfaces.

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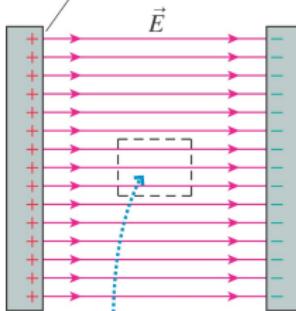


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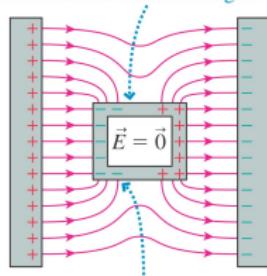
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- If we make a hole in the conductor we can analyze it with a Gaussian surface close to the hole. It is easy to conclude that  $Q_{in} = 0$ .
- Since  $\vec{E}$  is also zero inside the hole, this has practical applications: we can build a **Faraday cage**

# The Electric Potential (Chapter 29)

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- We are working our way towards really practical stuff like circuit-building.
- To get there, we need to talk about energy. Chapter 9 is all about electric potential.
- We have mostly focused on static charges so far. If we eventually want to understand the motion of charges we will need to understand electric potential.

# Mechanical Energy

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- Kinetic energy of a system is the sum of the kinetic energies of all of the particles in the system.
- Potential energy is the interaction energy of the system.

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- The component of force along the direction of motion does work on an object. Non-constant forces need the “chop-up and integrate” trick:

$$W = \sum_j (F_s)_j \Delta s_j \rightarrow \int_{s_i}^{s_f} F_s ds = \int_i^f \vec{F} \cdot d\vec{s}$$