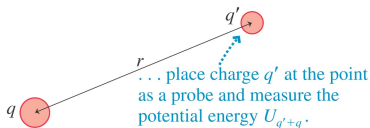# The Electric Potential of a Point Charge (29.6)

- We have only looked at a capacitor...let's not forget our old friend the point charge!
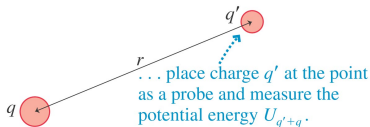
To determine the potential of q at this point . . .

q

q'

r

. . . place charge q' at the point as a probe and measure the potential energy $U_{q'+q}$.

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- Use a second charge (q') to probe the electric potential at a point in space. The potential energy will be

$$U_{q'+q} = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r}$$

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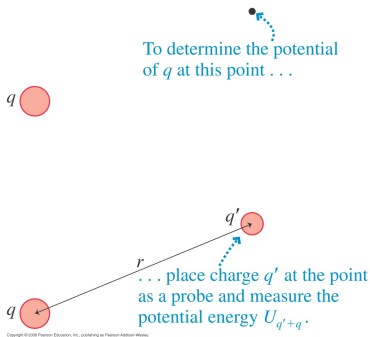
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- The potential is then

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

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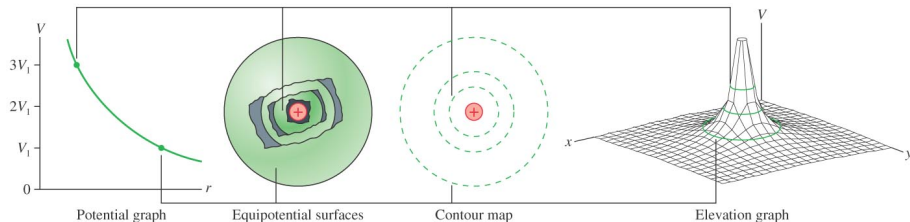- Use a second charge (q') to probe the electric potential at a point in space. The potential energy will be

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- This extends through all of space with $V = 0$ at infinity.

Visualizing the Potential of a Point Charge

Potential graph   Equipotential surfaces   Contour map   Elevation graph

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The Electric Potential of a Charged Sphere

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$$V_0 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

- A sphere of radius R charged to V_0 has total charge $Q = 4\pi\epsilon_0 R V_0$. Substituting this in gives

$$V = \frac{R}{r}V_0$$

- The electric potential, like the electric field, obeys the principle of superposition:

$$V = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i}$$

where r_i is the distance from the charge q_i to the point in space at which the potential is being calculated.

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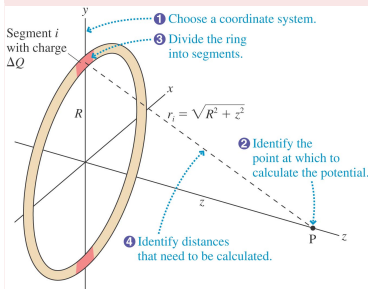
$$V = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i}$$

  where r_i is the distance from the charge q_i to the point in space at which the potential is being calculated.

- As we did before, we will extend our calculation to a continuous charge distribution. Often this involves an integration.

The Potential of a Ring of Charge
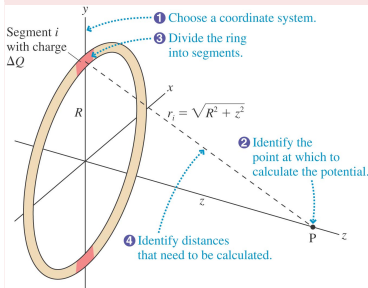
The Potential of a Ring of Charge (Example 29.11)

A thin, uniformly charged ring of radius R has total charge Q. Find the potential at distance z on the axis of the ring.

- The distance r_i is

$$r_i = \sqrt{R^2 + z^2}$$

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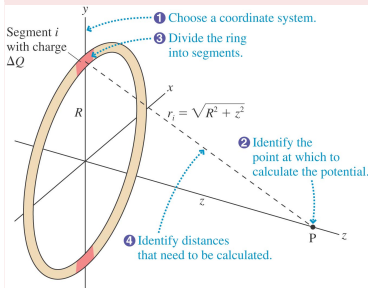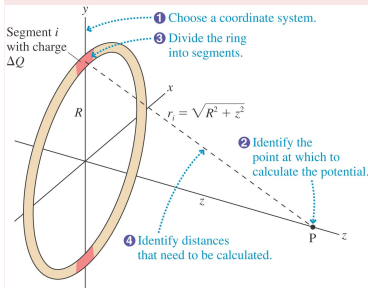
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$$V = \sum_{i=1}^{N} \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i} = \frac{1}{4\pi\epsilon_0} \frac{1}{\sqrt{R^2 + z^2}} \sum_{i=1}^{N} \Delta Q$$

The Potential of a Ring of Charge

The Potential of a Ring of Charge (Example 29.11)

❶ Choose a coordinate system.

❸ Divide the ring into segments.

Segment i with charge ΔQ

$r_i = \sqrt{R^2 + z^2}$

❷ Identify the point at which to calculate the potential.

❹ Identify distances that need to be calculated.

A thin, uniformly charged ring of radius R has total charge Q. Find the potential at distance z on the axis of the ring.
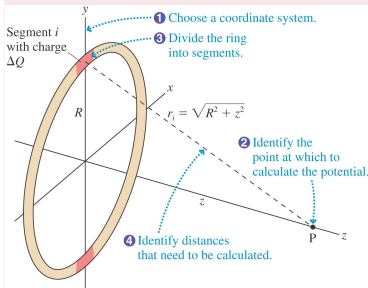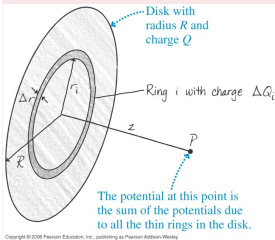
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$$V = \sum_{i=1}^{N} \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i} = \frac{1}{4\pi\epsilon_0} \frac{1}{\sqrt{R^2 + z^2}} \sum_{i=1}^{N} \Delta Q = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{R^2 + z^2}}$$

The Potential of a Disk of Charge

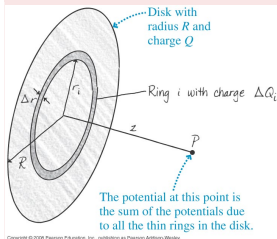## The Potential of a Disk of Charge (Example 29.12)

A thin, uniformly charged disk of radius R has total charge Q. Find the potential at distance z on the axis of the disk.

- The disk has uniform charge density

$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

The Potential of a Disk of Charge

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- We know the potential due to any ring

$$V_i = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q_i}{\sqrt{r_i^2 + z^2}}$$

The Potential of a Disk of Charge

- Now sum over rings

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{\Delta Q_i}{\sqrt{r_i^2 + z^2}}$$

Disk with radius R and charge Q

Ring i with charge ΔQ_i

The potential at this point is the sum of the potentials due to all the thin rings in the disk.

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$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{\Delta Q_i}{\sqrt{r_i^2 + z^2}}$$

- To turn this into an integral we need to substitute for ΔQ_i

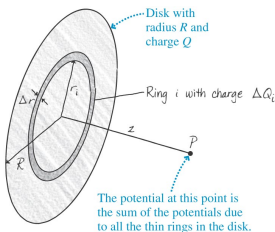$$\Delta Q_i = \eta \Delta A_i = \frac{Q}{\pi R^2} 2\pi r \Delta r = \frac{2Q}{R^2} r_i \Delta r$$

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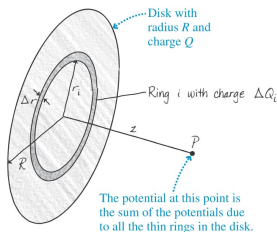$$\Delta Q_i = \eta \Delta A_i = \frac{Q}{\pi R^2} 2\pi r \Delta r = \frac{2Q}{R^2} r_i \Delta r$$
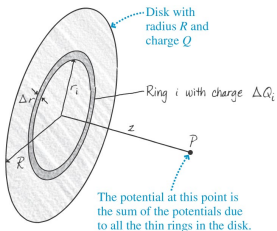
- The potential at P is then

$$V = \frac{Q}{2\pi\epsilon_0 R^2} \int_0^R \frac{r dr}{\sqrt{r^2 + z^2}}$$

- You can do this integral by substitution

$$u \;=\; r^2 + z^2$$

Disk with radius R and charge Q

Ring i with charge ΔQ_i

z

P

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The Potential of a Disk of Charge

Disk with radius R and charge Q

Ring i with charge ΔQ_i

z

P

Δr

r_i

R

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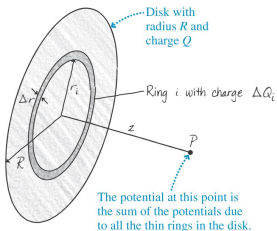
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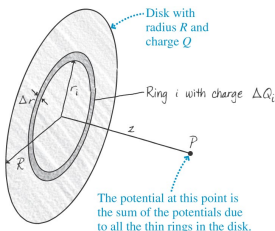
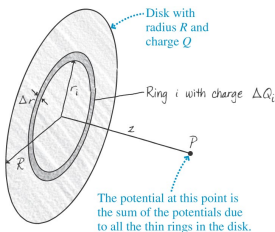- You can do this integral by substitution

$$u = r^2 + z^2$$
$$r\,dr = \frac{1}{2}du$$
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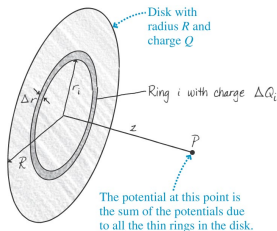
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&= \frac{Q}{2\pi\epsilon_0 R^2}\left(\sqrt{R^2+z^2}-|z|\right)
\end{aligned}
$$

- Now we start Chapter 30 of your text.

Potential and Field (Chapter 30)

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- We will learn how to use capacitors.

Finding Potential from Electric Field

- Given the electric field in some region of space, how do we calculate the potential?

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Finding Potential from Electric Field

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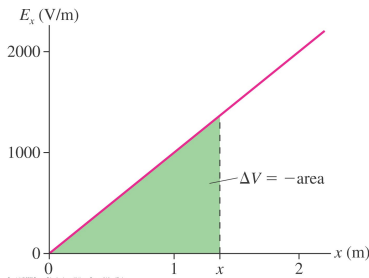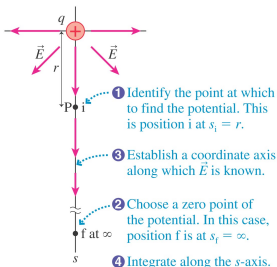- The potential difference is

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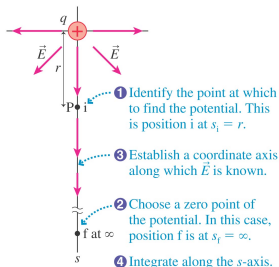
- We know that an integral is the area under a curve, so

● Let's do a practical example with a zero of potential chosen at infinity

Finding Potential from Electric Field

① Identify the point at which to find the potential. This is position i at $s_i = r$.

③ Establish a coordinate axis along which $\vec{E}$ is known.

② Choose a zero point of the potential. In this case, position f is at $s_f = \infty$.

④ Integrate along the s-axis.
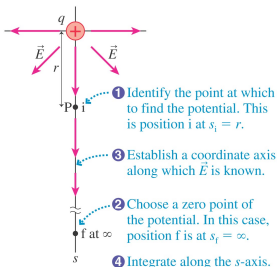
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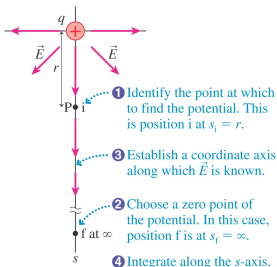
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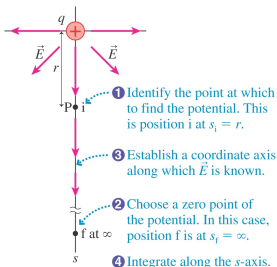$$\Delta V = V(\infty) - V(r) = -\int_r^\infty E_s ds$$

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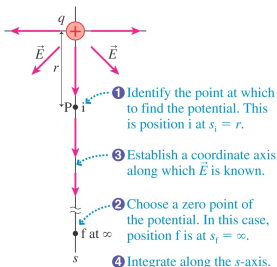$$\Delta V = V(\infty) - V(r) = -\int_r^\infty E_s ds$$

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$$V(r) = V(\infty) + \frac{q}{4\pi\epsilon_0}\int_r^\infty \frac{ds}{s^2} = \frac{q}{4\pi\epsilon_0}\frac{-1}{s}\bigg|_r^\infty$$

Finding Potential from Electric Field

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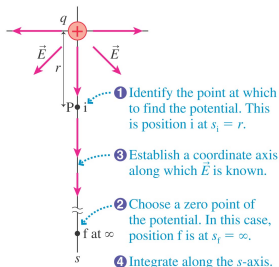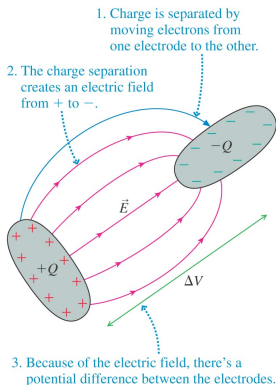$$\Delta V = V(\infty) - V(r) = -\int_r^\infty E_s \, ds$$

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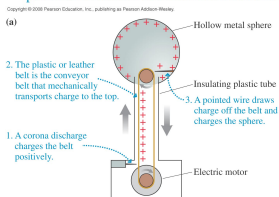$$E_s = \frac{1}{4\pi\epsilon_0} \frac{q}{s^2}$$

- Thus the potential at r is

$$V(r) = V(\infty) + \frac{q}{4\pi\epsilon_0} \int_r^\infty \frac{ds}{s^2} = \frac{q}{4\pi\epsilon_0} \left. \frac{-1}{s} \right|_r^\infty = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

1. Charge is separated by moving electrons from one electrode to the other.

2. The charge separation creates an electric field from $+$ to $-$.

$-Q$

$\vec{E}$

$+Q$

ΔV

3. Because of the electric field, there's a potential difference between the electrodes.

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(a)

Hollow metal sphere

2. The plastic or leather belt is the conveyor belt that mechanically transports charge to the top.

Insulating plastic tube

3. A pointed wire draws charge off the belt and charges the sphere.

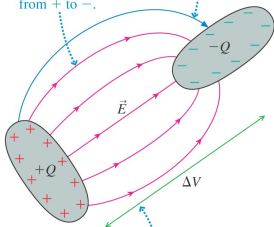1. A corona discharge charges the belt positively.

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- Any charge separation causes a potential difference.

Sources of Electric Potential (30.2)

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Sources of Electric Potential (30.2)

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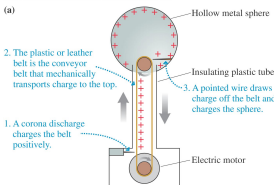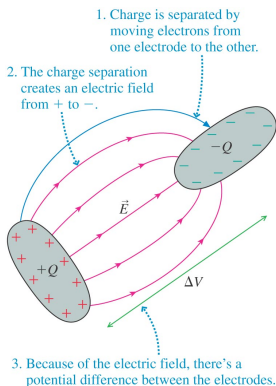2. The charge separation creates an electric field from + to −.

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2. The plastic or leather belt is the conveyor belt that mechanically transports charge to the top.

Insulating plastic tube

3. A pointed wire draws charge off the belt and charges the sphere.

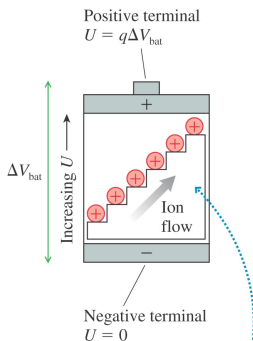1. A corona discharge charges the belt positively.

Electric motor

- Any charge separation causes a potential difference.
- There is a potential difference between electrodes (1st picture) of

$$\Delta V = V_{pos} - V_{neg} = -\int_{neg}^{pos} E_s \, ds$$

- In class we have seen a mechanical charge separator - a Van de Graaff generator.

Positive terminal
$U = q\Delta V_{bat}$

ΔV_{bat}

Increasing $U \longrightarrow$

Ion flow

$\vec{F}$

Negative terminal
$U = 0$
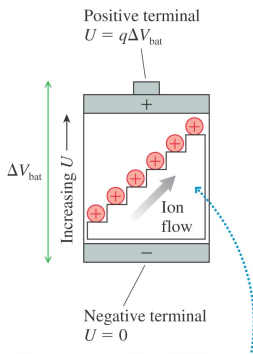
The charge escalator "lifts" charge from the negative side to the positive side. Charge q gains energy $\Delta U = q\Delta V_{bat}$.

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- We are more familiar with a chemical charge separator - a battery

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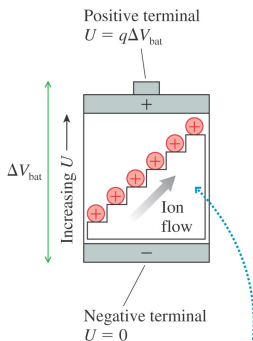
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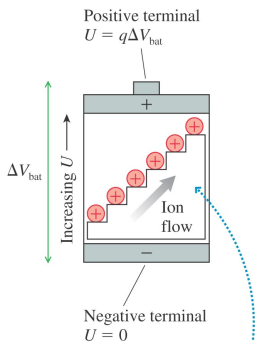
Batteries and emf

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Batteries and emf

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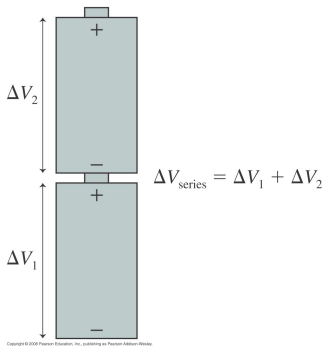Increasing U

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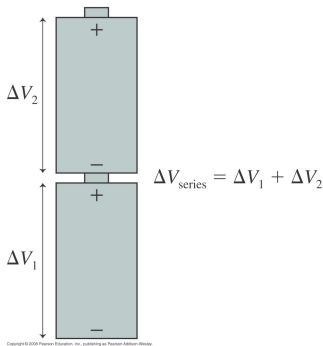
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- We will ignore the chemistry and discuss the battery as a charge escalator which lifts positive charges from the negative terminal to the positive terminal.
- Doing this requires work...energy from chemical reactions. In an ideal battery $\Delta U = W_{chem}$
- The work done per charge in a battery is known as its emf ($\mathcal{E}$). For example $\mathcal{E} = 1.5V, 9V, etc$

ΔV_2

ΔV_1

$\Delta V_{\text{series}} = \Delta V_1 + \Delta V_2$

- The voltage between the terminals is called the terminal voltage and is just slightly less than $\mathcal{E}$.

ΔV_2

ΔV_1
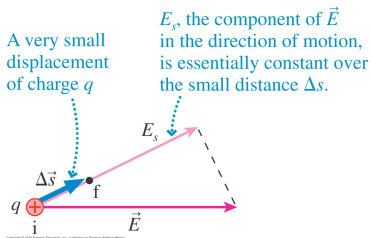
$\Delta V_{\text{series}} = \Delta V_1 + \Delta V_2$

- The voltage between the terminals is called the terminal voltage and is just slightly less than $\mathcal{E}$.
- If you put multiple batteries in series (see left) the combined voltage is simply the sum of the two individual voltages.
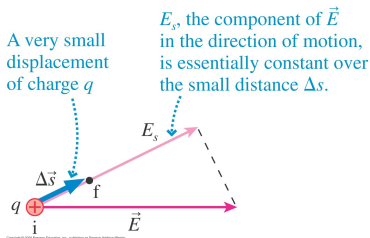
$$\Delta V_{series} = \Delta V_1 + \Delta V_2 + \cdots$$

A very small displacement of charge q

E_s, the component of $\vec{E}$ in the direction of motion, is essentially constant over the small distance Δs.

E_s

$\Delta \vec{s}$

f

q

i

$\vec{E}$

- The work done by the electric field as q moves through a small distance Δs is

$$W = F_s \Delta s = q E_s \Delta s$$

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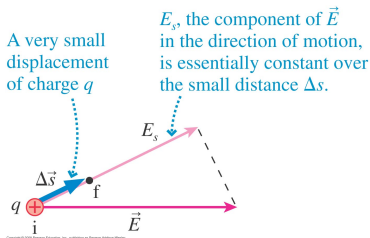
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- The potential difference is

$$\Delta V = \frac{\Delta U_{q+sources}}{q} = \frac{-W}{q} = -E_s \Delta s$$

Finding the Electric Field from the Potential (30.3)

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- Rearranging and making Δs infinitely small gives

$$E_s = -\frac{dV}{ds}$$

- Let's use a point charge as an example

Finding the Electric Field from the Potential

- Let's use a point charge as an example
- Choose the *s* axis to be in the radial direction, parallel to $\vec{E}$ and we see

$$E_r = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{q}{4\pi\epsilon_0 r}\right) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

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- Similarly, we could have started with potential and derived the $\vec{E}$ of a charged ring or disk with a simple derivative!

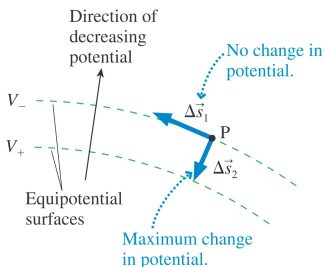# Finding the Electric Field from the Potential

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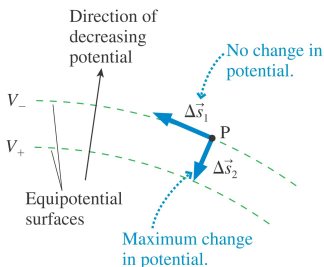- The geometric interpretation is that the $\vec{E}$ is the slope of the *V* vs. *s* graph.

The Geometry of Potential and Field

- The figure shows 2 equipotential surfaces, one at positive potential with respect to the other.
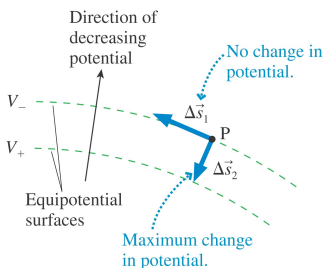

Direction of decreasing potential

No change in potential.

$\Delta \vec{s}_1$

P

$\Delta \vec{s}_2$

V_-

V_+

Equipotential surfaces

Maximum change in potential.

The Geometry of Potential and Field

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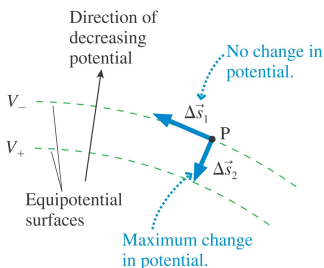- Consider 2 displacements $\Delta \vec{s}_1$ and $\Delta \vec{s}_2$.

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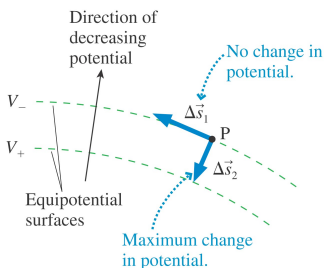
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$$E_\perp = -\frac{dV}{ds} \approx -\frac{V_+ - V_-}{\Delta s}$$

The Geometry of Potential and Field

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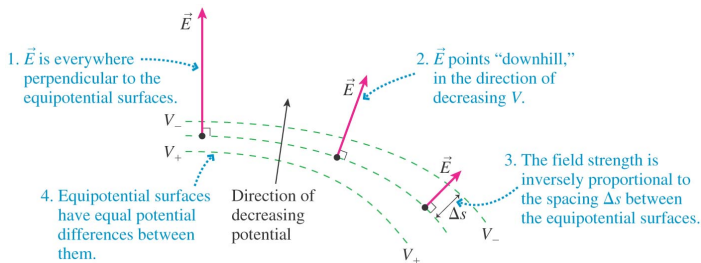
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- $\vec{E}$ is perpendicular to equipotential surfaces and points "downhill".
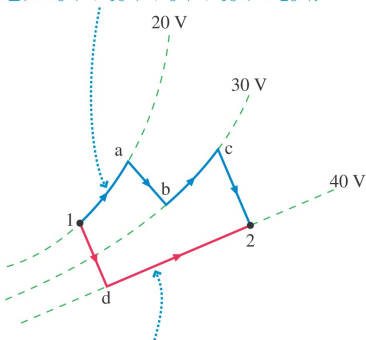
The Geometry of Potential and Field

1. $\vec{E}$ is everywhere perpendicular to the equipotential surfaces.

2. $\vec{E}$ points "downhill," in the direction of decreasing V.

3. The field strength is inversely proportional to the spacing Δs between the equipotential surfaces.

4. Equipotential surfaces have equal potential differences between them.

Direction of decreasing potential

V_-
V_+

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$$\vec{E} = E_x \hat{\imath} + E_y \hat{\jmath} + E_z \hat{k} = -\left(\frac{\partial V}{\partial x} \hat{\imath} + \frac{\partial V}{\partial y} \hat{\jmath} + \frac{\partial V}{\partial z} \hat{k} \right)$$

$$\vec{E} = -\nabla V$$

The potential difference along path 1-a-b-c-2 is
ΔV = 0 V + 10 V + 0 V + 10 V = 20 V.

20 V

30 V

a

c

b

40 V

1

2

d

The potential difference along path
1-d-2 is ΔV = 20 V + 0 V = 20 V.

- The work done moving a charge from point 1 to point 2 is independent of the path.

The Geometry of Potential and Field

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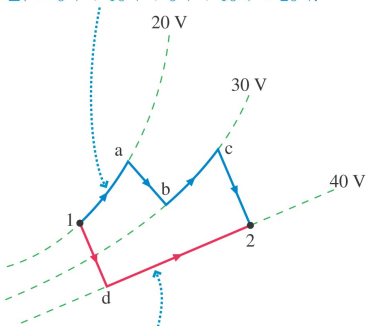The potential difference along path
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The Geometry of Potential and Field

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- The work done moving a charge from point 1 to point 2 is independent of the path.
- Moving from point 1 to 2 on the blue path leads to a potential increase of 20V. Moving from 2 to 1 on the red path leads to a decrease of 20V.
- Kirchhoff's Loop Law says that the potential differences encountered while moving around a loop is zero

$$\Delta V_{loop} = \sum_{i} (\Delta V)_i = 0$$