Doppler Effect

The Doppler effect is the change in frequency of a wave detected by an observer moving relative to the source of the wave.

- It is named after Austrian physicist Christian Doppler who described it in 1842. Simulation: http://astro.unl.edu/classaction/animations/light/dopplershift.html

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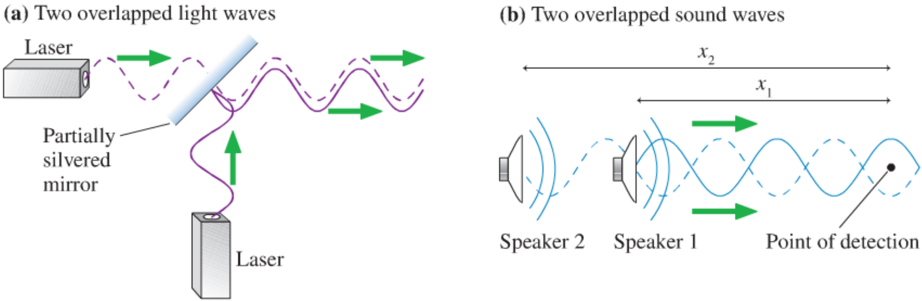
- Applications of the Doppler effect range through sirens, weather radar and determining the speed of stars.

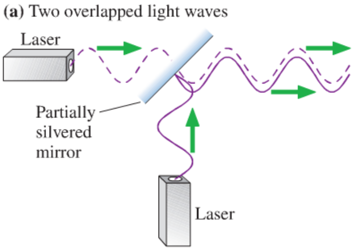
Interference in 1-D



We assume sinusoidal waves of the same frequency and amplitude traveling to the right along the x-axis. The displacements are

$$
\begin{aligned}
D_1(x_1, t) &= a \sin(kx_1 - \omega t + \phi_{10}) = a \sin \phi_1 \\
D_2(x_2, t) &= a \sin(kx_2 - \omega t + \phi_{20}) = a \sin \phi_2
\end{aligned}
$$

Interference in 1-D - Constructive Interference



These two waves are in phase and will give constructive interference. If they are perfectly in phase and of equal amplitude a, this will lead to a combined amplitude $A = 2a$.

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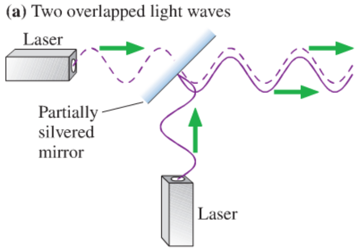

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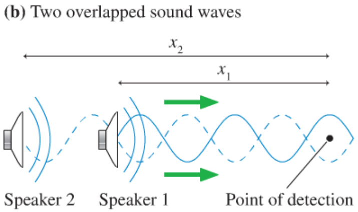

These two waves are out of phase and will give destructive interference. If they are 180° out of phase and of equal amplitude a, this will lead to a combined amplitude $A = a - a = 0$.

Interference in 1-D - Phase Differences

Remember our mathematical description of the two waves:

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Let's now concentrate on the phases (arguments of the sin)

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The phase difference is then

$$\Delta\phi = \phi_2 - \phi_1$$

Interference in 1-D - Phase Differences

Let's express the phase difference another way

$$\Delta\phi = (kx_2 - \omega t + \phi_{20}) - (kx_1 - \omega t + \phi_{10})$$

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There are two distinct contributions: the path length difference (Δx term) and the inherent phase difference ($\Delta\phi_0$ term).

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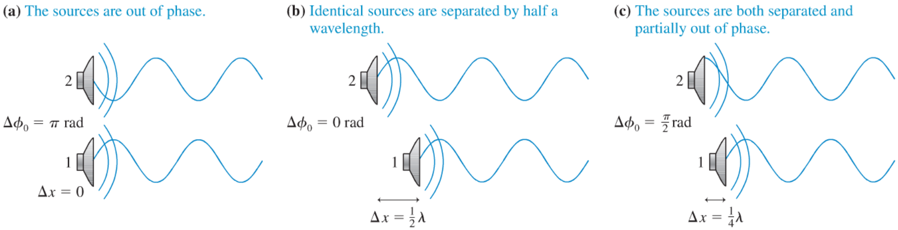
Maximum Constructive Interference

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \phi_0 = m \cdot 2\pi \text{ rad}, m = 0, 1, 2, 3, \ldots$$

Maximum Destructive Interference

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \phi_0 = \left(m + \frac{1}{2}\right) \cdot 2\pi \text{ rad}, m = 0, 1, 2, 3, \ldots$$

Interference in 1-D - Phase Differences



Mathematics of Interference in 1-D

- The displacement resulting from two waves is

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$$D = \underbrace{\left[2a\cos\frac{\Delta\phi}{2}\right]}\sin(kx_{ave} - \omega t + (\phi_0)_{ave})$$

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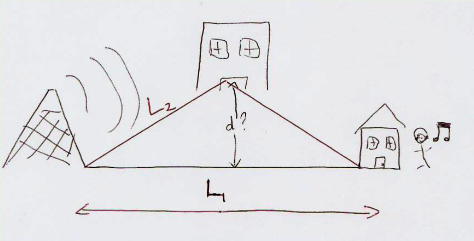
(gives us back constructive and destructive phase-differences found earlier)

Example of EM Interference

Listening to AM Radio

Suppose you are listening to AM650 (650kHz) and you live 23km from the radio tower. There is another building halfway between you and the tower and radio waves are bouncing off of that building. How far off to the side is the building if destructive interference occurs between the direct and reflected waves? (assume equal amplitudes and no phase shift on reflection)



(i.e., What is *d*??)

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$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \phi_0 = \left(m + \frac{1}{2}\right) \cdot 2\pi \text{ rad}, m = 0, 1, 2, 3, \ldots$$

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- Take the minimum d as $m = 0$

$$\frac{\lambda}{2} = \sqrt{L_1^2 + 4d^2} - L_1$$

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$$\begin{aligned}\frac{\lambda}{2} &= \sqrt{L_1^2+4d^2} - L_1 \\ L_1 + \frac{\lambda}{2} &= \sqrt{L_1^2+4d^2} \\ L_1^2 + L_1\lambda + \frac{\lambda^2}{4} &= L_1^2 + 4d^2\end{aligned}$$

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- Plugging in the numbers:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{650000 \text{ Hz}} = 462 \text{ m}$$

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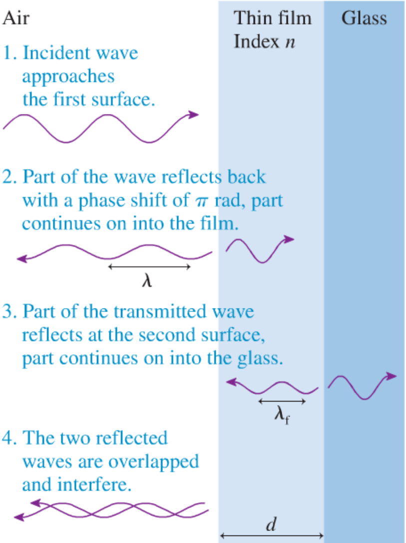
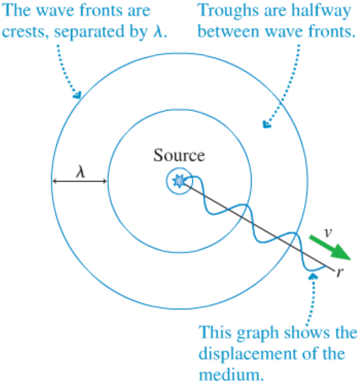
Application: anti-reflective coating



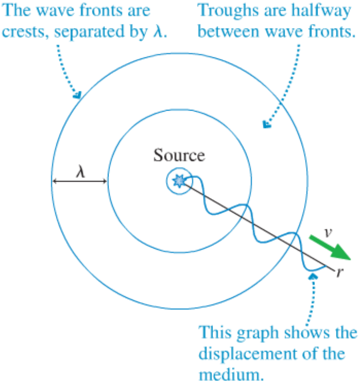
Interference in 2 or 3 Dimensions



Working in 2 or 3 dimensions is not very different from working in 1-D:

$$D(r,t) = a\sin(kr - \omega t + \phi_0)$$

where r is the distance measured outwards from the source.



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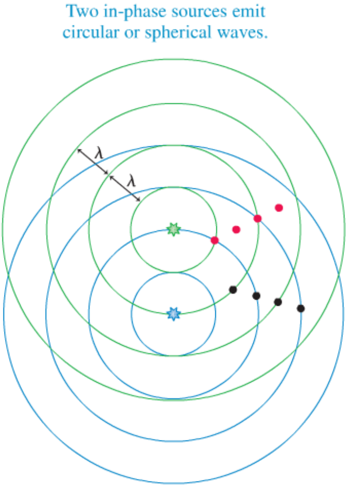
where r is the distance measured outwards from the source. Essentially we just replace x everywhere with a radial coordinate r...



- Points of constructive interference. A crest is aligned with a crest, or a trough with a trough.
- Points of destructive interference. A crest is aligned with a trough of another wave.

Interference also occurs with spherical waves. Again we look for places where crests or troughs align.

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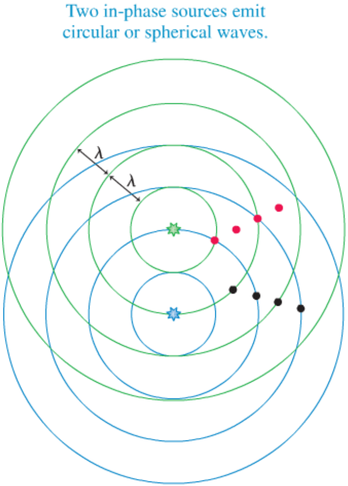

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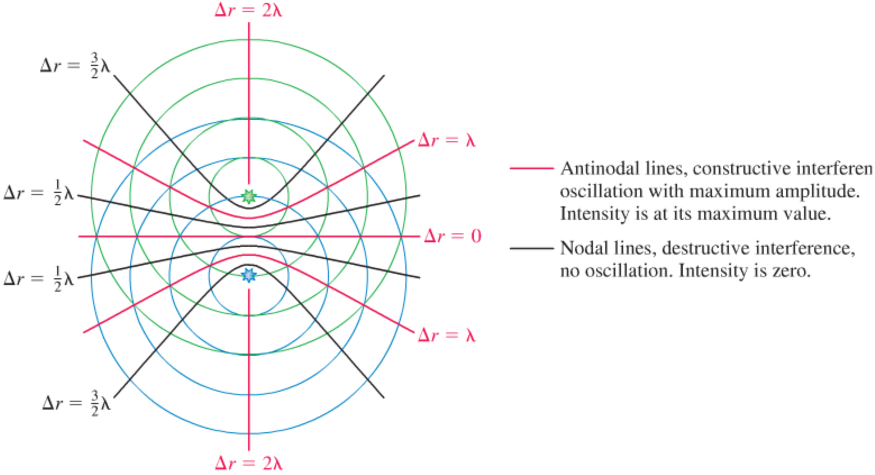
Nodal and Anti-Nodal Lines



Simulation: http://phet.colorado.edu/sims/wave-interference