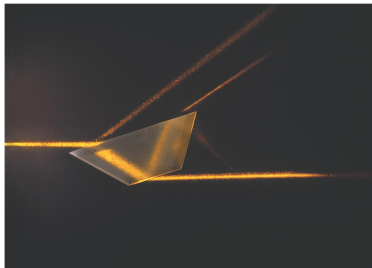


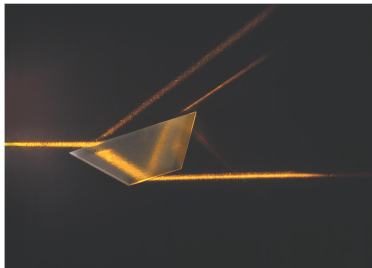
# Refraction (23.3)



Two things happen when light hits the boundary between transparent materials

- 1 Part of the light reflects from the surface

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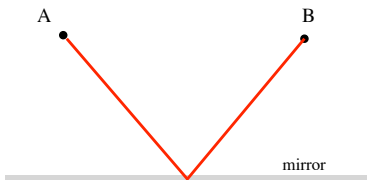


Two things happen when light hits the boundary between transparent materials

- 1 Part of the light reflects from the surface
- 2 Part of the light is transmitted through the second medium with a change of direction. This is called **refraction**

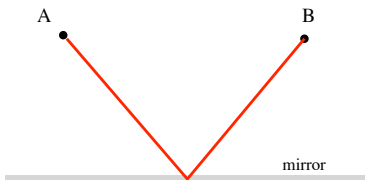
# Fermat's Principle for Reflection

- When light travels from A to B what path will it take?



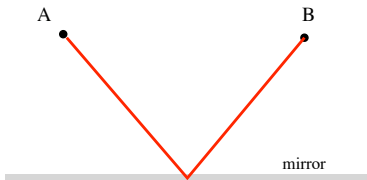
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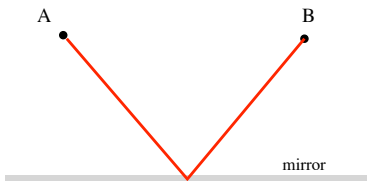


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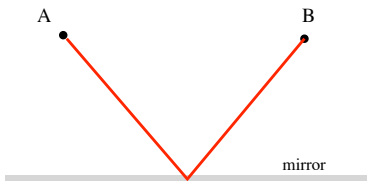


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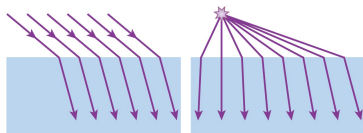
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- When light travels from A to B what path will it take?
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- Fermat's principle says: **Light travelling between two points takes the path of least time.**

# Refraction

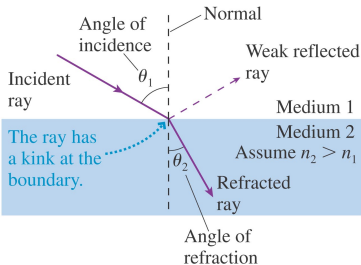
(a)



Refraction of a parallel beam of light  
and of rays from a point source

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

(b)



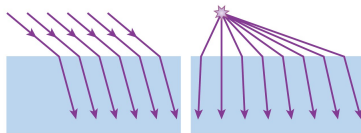
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

- Simplify by drawing a single ray



# Refraction

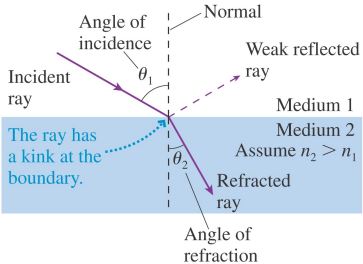
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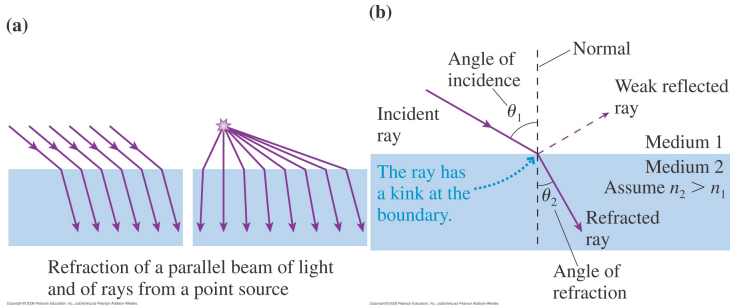
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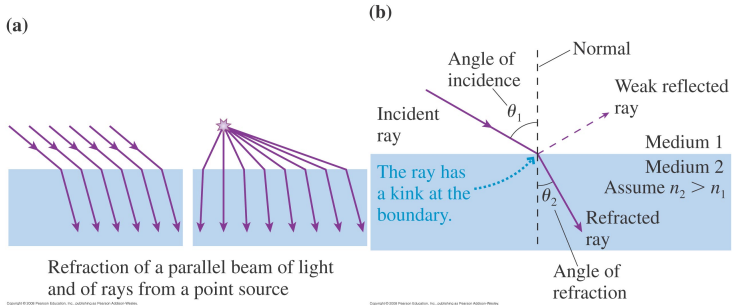
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- Snell's Law tell us that

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

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We have already mentioned the **index of refraction** a couple of times in the course...but a quick reminder:

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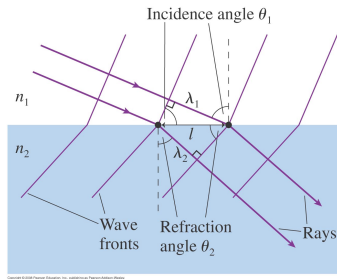
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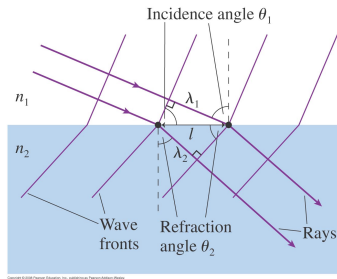
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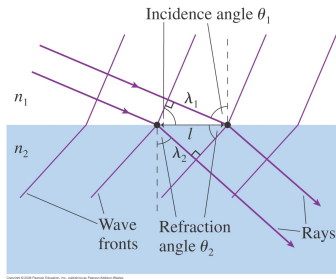
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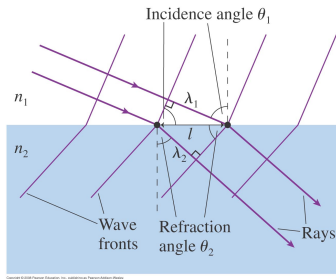


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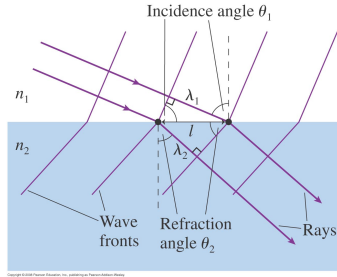


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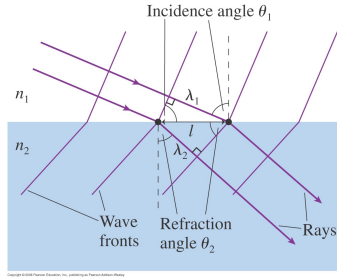
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- In each medium the wave fronts are parallel to each other.

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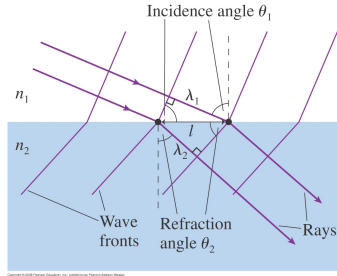
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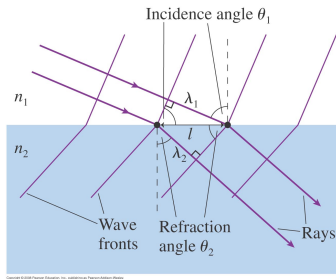
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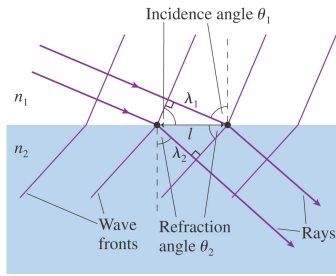


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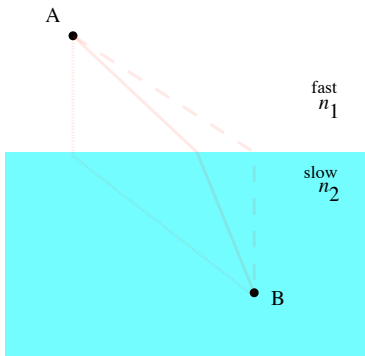
$$l = \frac{\lambda_2}{\sin \theta_2}$$

- Setting these equal to each other and using  $\lambda_1 = \lambda_0/n_1$ ,  $\lambda_2 = \lambda_0/n_2$  gives

$$\frac{\lambda_0}{n_1 \sin \theta_1} = \frac{\lambda_0}{n_2 \sin \theta_2}$$
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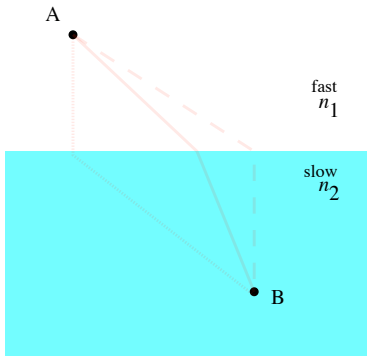


# Fermat's Principle for Refraction



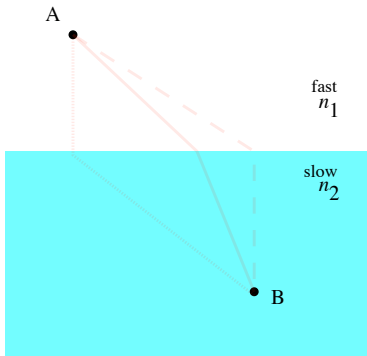
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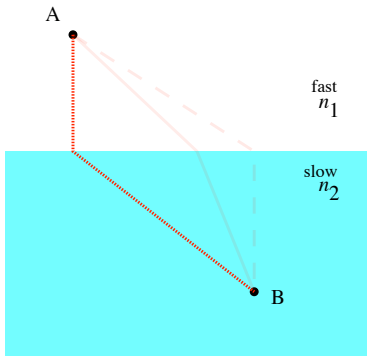
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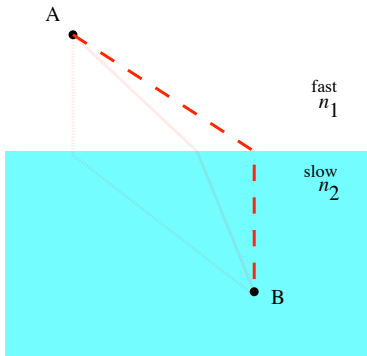
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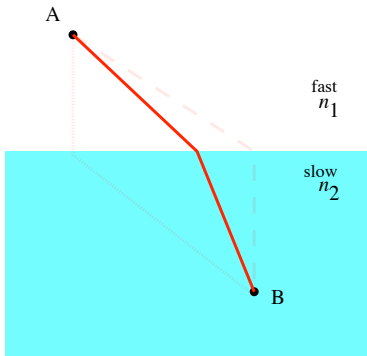
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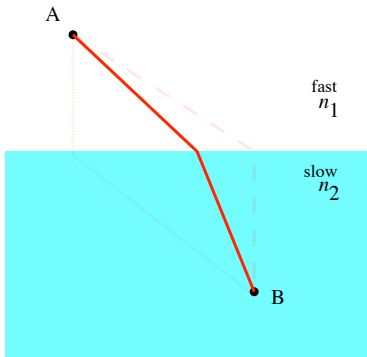
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# Fermat's Principle for Refraction



- It turns out that Fermat's Principle is consistent with Snell's law.
- See problem CP23.80 in the textbook.

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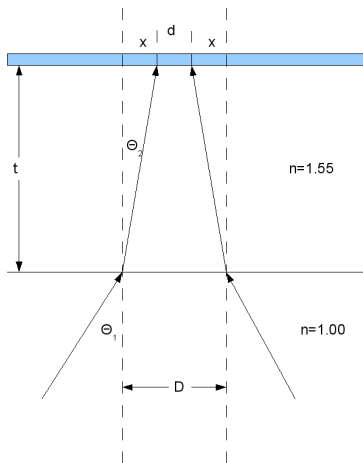
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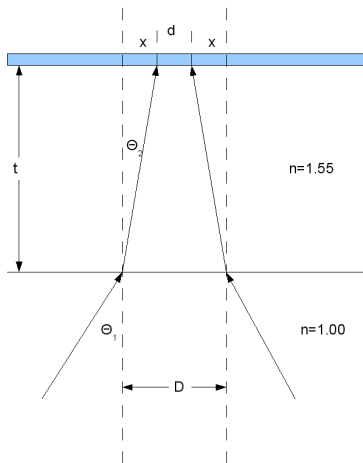
# Example: Refraction and CDs



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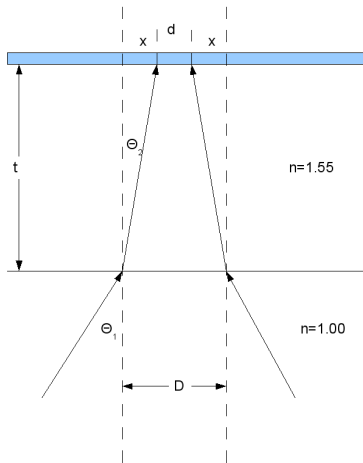


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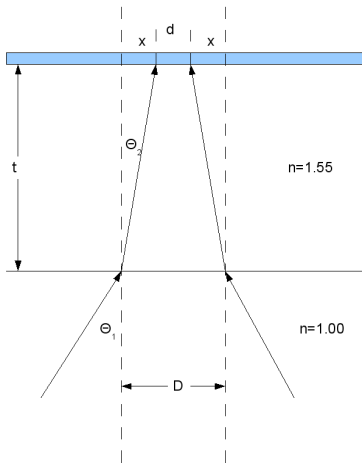


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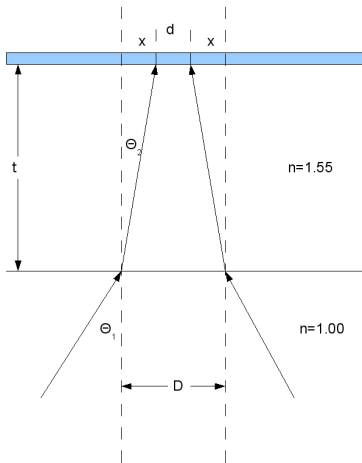
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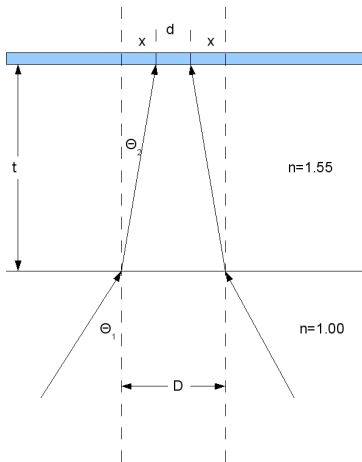
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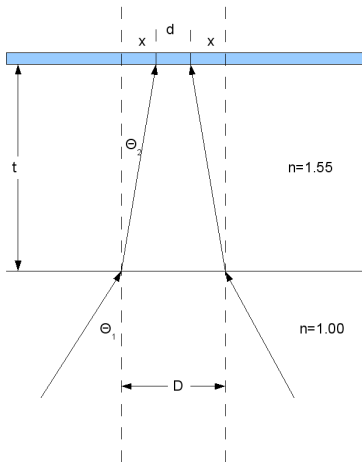
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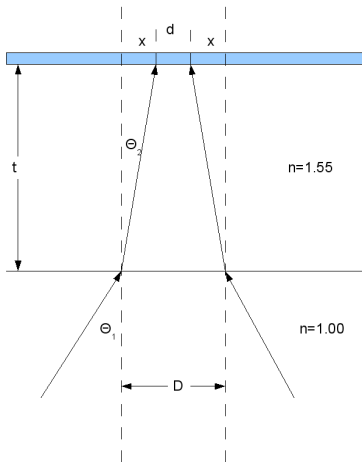
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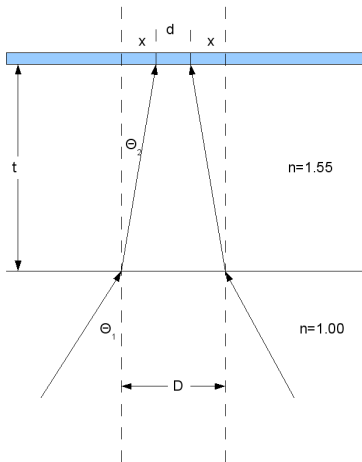
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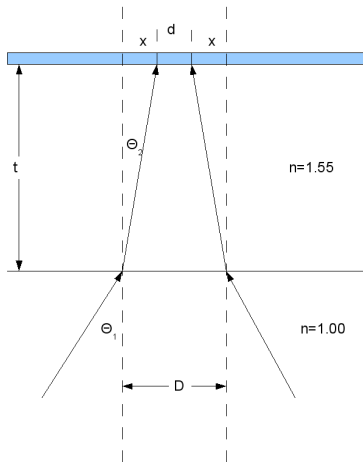
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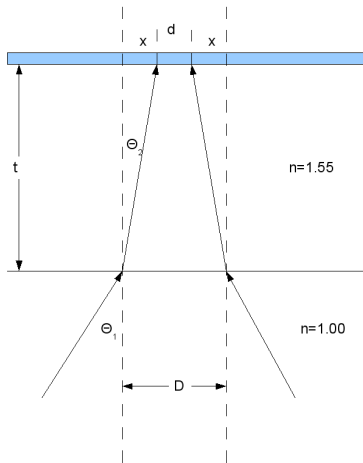


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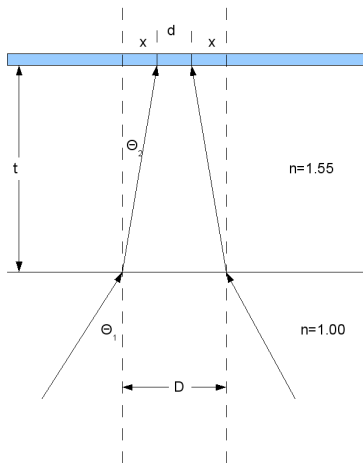
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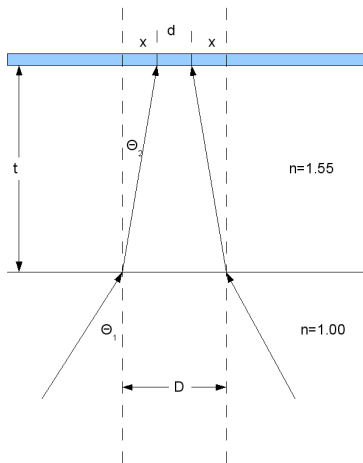
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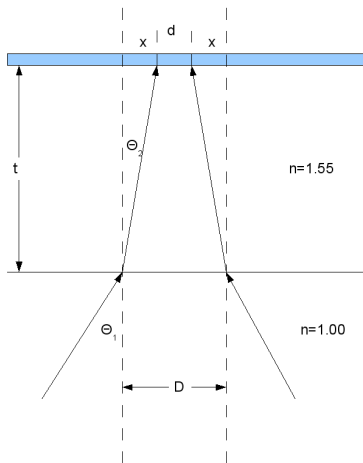
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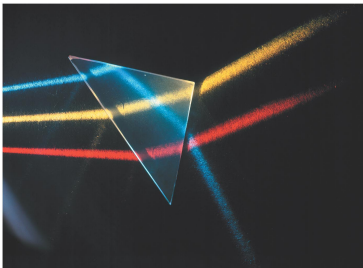
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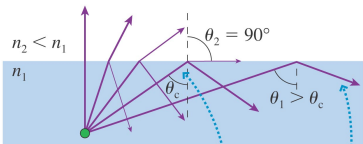
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# Total Internal Reflection (23.3)



- If light strikes a boundary in which it transitions from a high index of refraction to a lower one, it can undergo **Total Internal Reflection (TIR)**.

The angle of incidence is increasing.  
Transmission is getting weaker. →

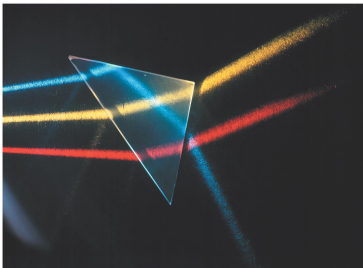


Critical angle when  $\theta_2 = 90^\circ$

Reflection is getting stronger. →

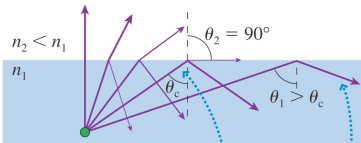
Total internal reflection  
occurs when  $\theta_1 \geq \theta_c$ .

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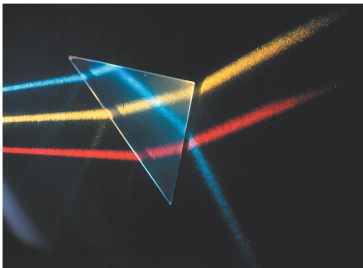
Critical angle when  $\theta_2 = 90^\circ$

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Total internal reflection  
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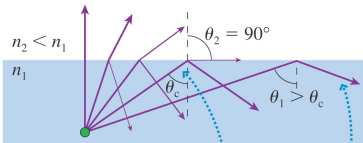
- If light strikes a boundary in which it transitions from a high index of refraction to a lower one, it can undergo **Total Internal Reflection (TIR)**.
- The figure on the left shows several rays leaving a source inside a high- $n$  medium. As the angle of incidence gets larger the angle of refraction gets closer and closer to  $90^\circ$ .

# Total Internal Reflection (23.3)



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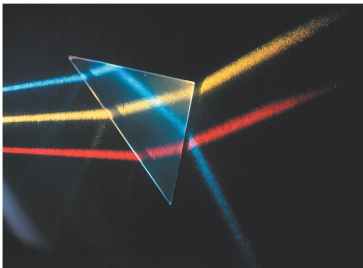
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- When the angle of refraction ( $\theta_2$ ) is exactly  $90$  degrees we reach the **critical angle**. Above the critical angle there is no transmitted light.



# Total Internal Reflection (23.3)

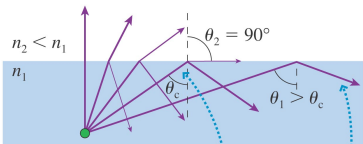


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$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

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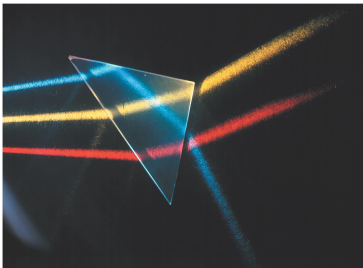


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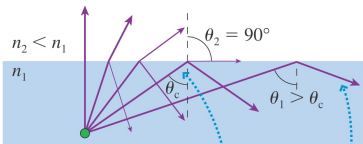
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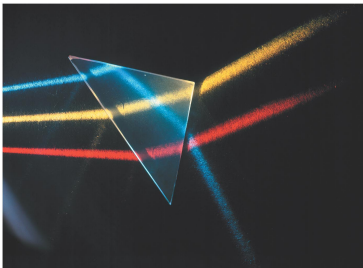
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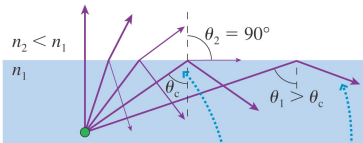
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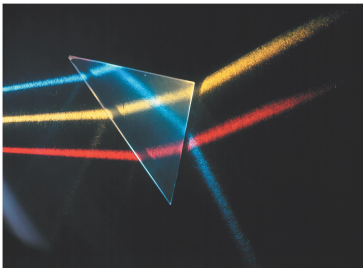
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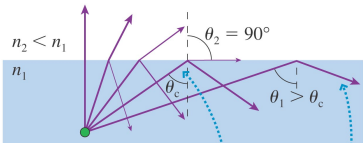
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$$\theta_c = \sin^{-1} \left( \frac{1.00}{1.50} \right) = 42^\circ$$

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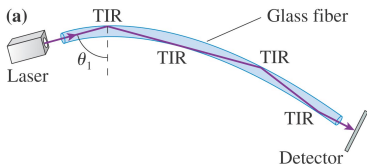
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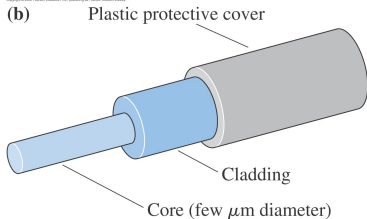
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- There is no TIR if  $n_2 > n_1$

# Fibre Optics



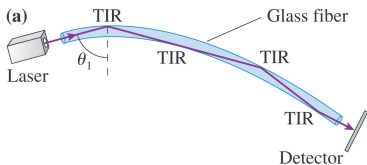
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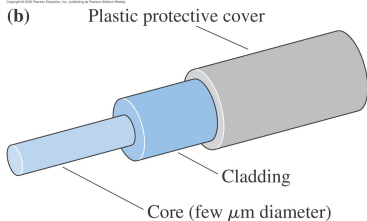
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- Fiber optics is an important application of TIR

# Fibre Optics



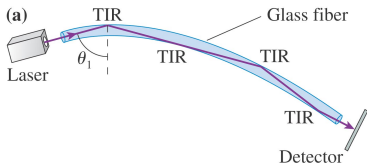
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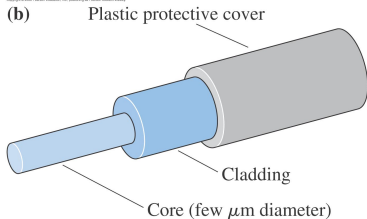
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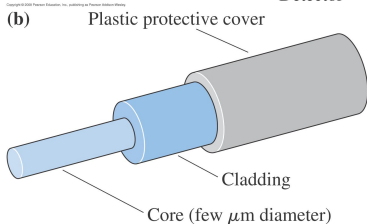
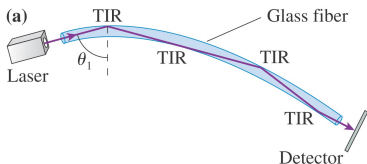
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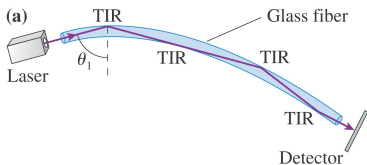
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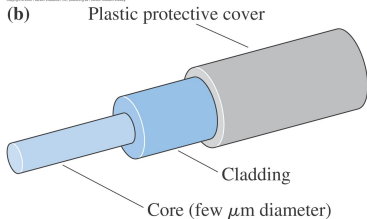
- Fiber optics is an important application of TIR
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- They are covered in lower-index cladding to prevent light leakage (e.g., scratches).



# Fibre Optics



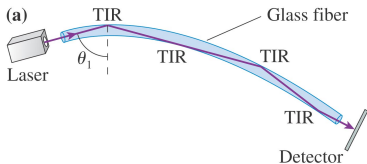
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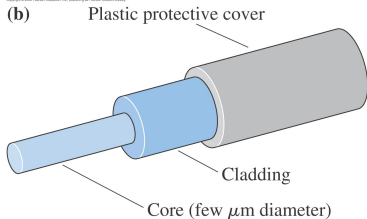
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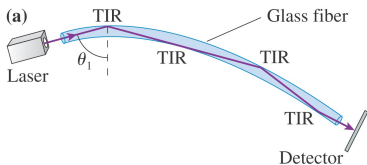
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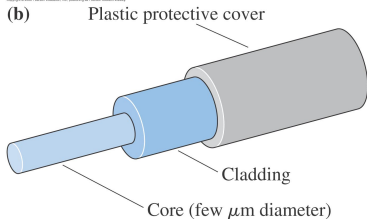
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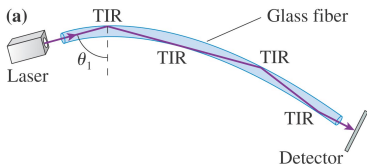
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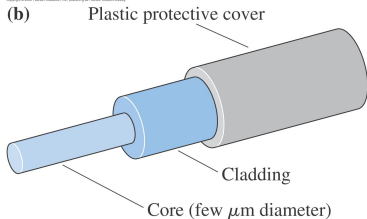
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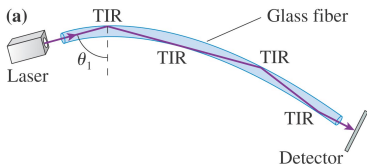
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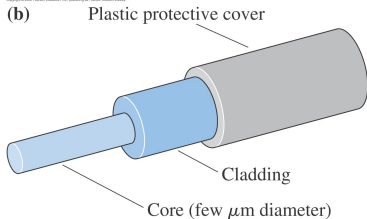
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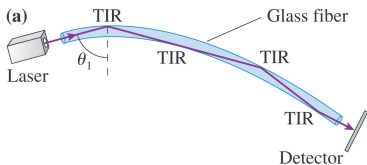
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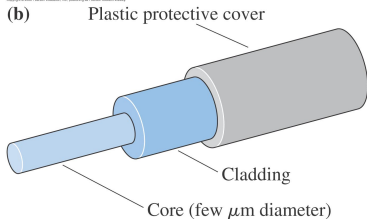
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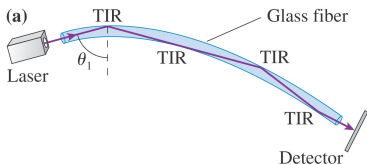
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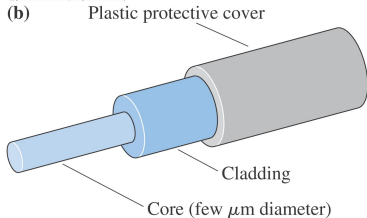
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