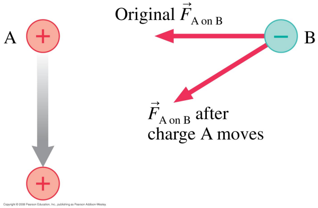
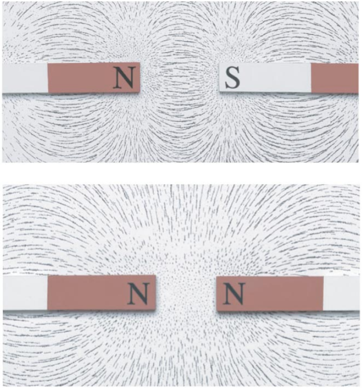
The Field Model (26.5)

- How does the electric force get propagated from one particle to another?
- Newton's theories were not time-dependent - instantaneous action at a distance
- Instantaneous action at a distance is a bit hard to believe!
- What if the two particles below were 100 light-years apart??



The Field Concept



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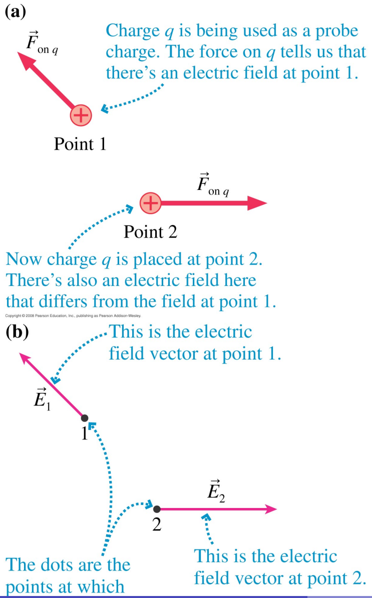
- Faraday suggested that the space around a charged object was altered. Other charges then interacted with that altered space.
- The iron filings were reacting to the altered space close to the magnet...they were reacting to the magnetic field.
- The field exists everywhere in space. Electric fields, magnetic fields, gravitational fields are some examples.
- We talked about light being a "self-sustaining oscillation of the EM field"

The Electric Field: video

The video shown in today's class can be found at
`http://www.learner.org/resources/series42.html`
it is episode 29.

The Electric Field



- We will describe a field model of electric interactions.
- Source charges alter the space around them creating an electric field $\vec{E}$
- A separate charge placed in the field experiences a force $\vec{F}$ exerted on it by the field.
- The field is defined as

$$\vec{E}(x,y,z) \equiv \frac{\vec{F}_{on\ q} \text{ at } (x,y,z)}{q}$$

The magnitude of the field is known as the electric field strength.

The Electric Field

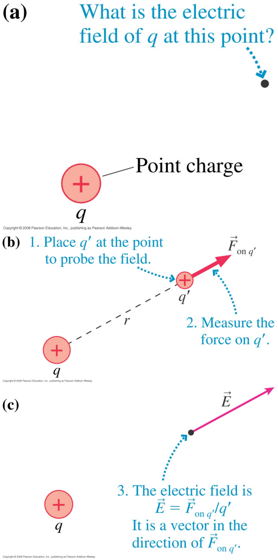
- We are using q as a test-charge or a probe of the field. You can make a field map by moving the charge around.
- The field is the agent that exerts a force on our probe.
- This is a vector field. That means that we assign a vector to every point in space.
- If q is positive, the electric field vector points in the same direction as the force on the charge.
- The electric field does not depend on the size of q. There is a q in both the numerator and denominator of

$$\vec{E}(x,y,z) \equiv \frac{\vec{F}_{on\ q}\mathrm{at}(x,y,z)}{q}$$

which cancel out.
- Often we want to calculate the force on a test charge like

$$\vec{F}_{on\ q} = q\vec{E}$$

The Electric Field of a Point Charge



- Assuming both charges are positive, q' will be repelled from q according to Coulomb's Law

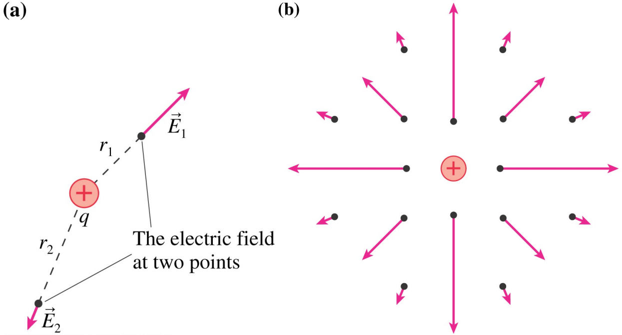
$$F_{on\ q'} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r^2}$$

- So, the electric field is pointing away from q as well and is:

$$E(x,y,z) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

These equations represent the magnitudes of the electric force and electric field respectively.

The Electric Field of a Point Charge



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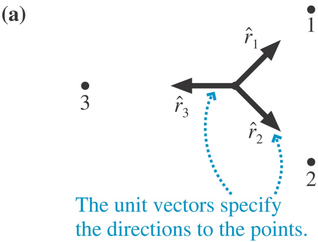
- The field strength goes like $1/r^2$
- So, if we draw the $\vec{E}$ at each point in space the lengths of the vectors will be very different from each other.

Unit Vector Notation



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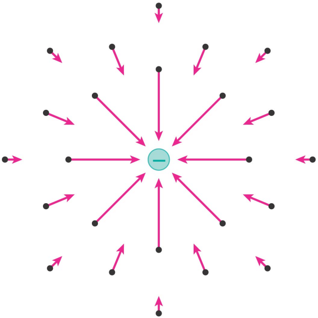

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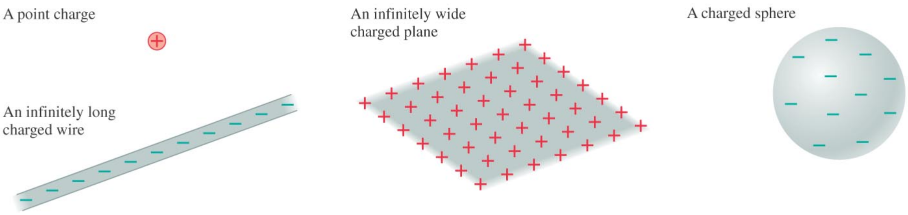
- We need a mathematical way to specify the direction of the field
- We will use a unit vector in the radial direction
- Define $\hat{r}$ to be a vector of length 1 from the origin to the point of interest.
- The vector electric field can be written as

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

The Electric Field (Chapter 27)

- Electric fields are everywhere: natural and manipulated.
- So far we have been drawing electric fields resulting from a single charge. What about complex objects?
- Chapter 27 is mainly about calculating electric fields from complex objects containing many charges.
- In other words, we will try to calculate realistic electric fields...with some simplifications of course ;-)

Electric Field Models (27.1)



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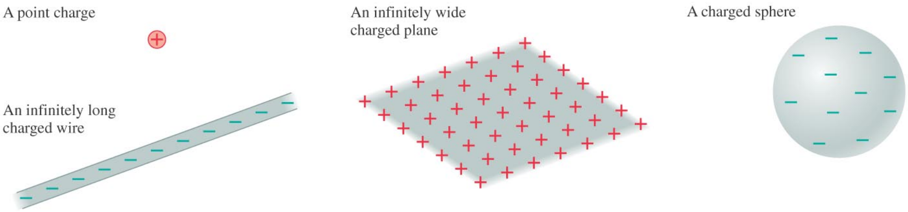
1. A point charge
2. An infinitely long wire
3. An infinitely wide charged plane
4. A charged sphere

Electric Field Models



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- Small objects (or far-away objects) can often be modeled as points or spheres
- Wires or planes can be often modeled as infinite, even if they aren't.
- Everything starts from a point:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Point Charges and Superposition

- So, isn't any distribution of charges just a whole bunch of:

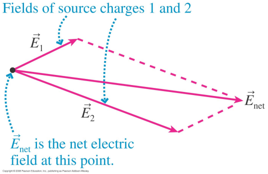
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

?

- Actually, yes. As we noted earlier, the electric force obeys the principle of superposition

$$\vec{E}_{net} = \frac{\vec{F}_{on\ q}}{q} = \frac{\vec{F}_{1\ on\ q}}{q} + \frac{\vec{F}_{2\ on\ q}}{q} + \cdots = \sum_i \vec{E}_i$$

The net electric field is the vector sum of the electric fields due to each charge



Limiting Cases and Typical Field Strength

- Your text emphasizes using limiting cases to get an understanding of the effects of a given charge distribution. A very common thing to do in physics!
- Limiting cases often allow for a simpler treatment (eg. some terms in equations just disappear) and/or allow the physical picture to be seen more clearly.
- An example: a charge distribution should look like a point charge when viewed from a great distance. If this is not the case, you probably have the wrong description!

The Electric Field of Multiple Point Charges (27.2)

- We already noted that the electric field is a vector field, and superposition is a vector sum. So:

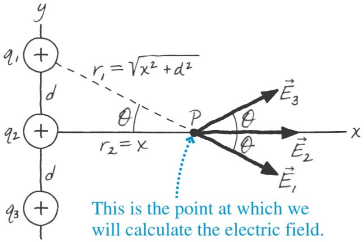
$$
\begin{aligned}
(E_{net})_x &= (E_1)_x + (E_2)_x + \cdots = \sum (E_i)_x \\
(E_{net})_y &= (E_1)_y + (E_2)_y + \cdots = \sum (E_i)_y \\
(E_{net})_z &= (E_1)_z + (E_2)_z + \cdots = \sum (E_i)_z
\end{aligned}
$$

- Sometimes it is useful to write this as

$$
\vec{E}_{net} = (E_{net})_x \hat{\imath} + (E_{net})_y \hat{\jmath} + (E_{net})_z \hat{k}
$$

Example 27.1: The Electric Field of 3 Equal Point Charges

Example 27.1



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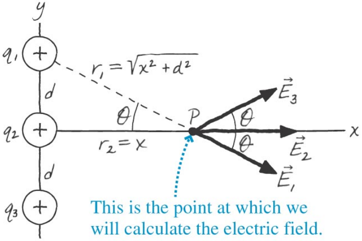
Three equal point charges q are located on the y-axis at $y = 0$ and $y = \pm d$. What is the electric field at a point on the x-axis?

- There are some clear simplifications - we do not care about the z direction at all. The y components cancel out.
- The x components add like

$$(E_{net})_x = (E_1)_x + (E_2)_x + (E_3)_x = 2(E_1)_x + (E_2)_x$$

Example 27.1: The Electric Field of 3 Equal Point Charges



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$$(E_2)_x = E_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{x^2}$$

$$(E_1)_x = E_1\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1^2}\cos\theta$$

- But r_1 and θ vary with x. We should express E_1 in terms of x

$$\cos\theta = \frac{x}{r_1} = \frac{x}{(x^2+d^2)^{1/2}}$$

$$(E_1)_x = \frac{1}{4\pi\epsilon_0}\frac{q}{x^2+d^2}\frac{x}{(x^2+d^2)^{1/2}} = \frac{1}{4\pi\epsilon_0}\frac{xq}{(x^2+d^2)^{3/2}}$$

Example 27.1: The Electric Field of 3 Equal Point Charges

Combining the expressions gives

$$\begin{aligned}(E_{net})_x &= 2(E_1)_x + (E_2)_x = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{x^2} + \frac{2x}{(x^2+d^2)^{3/2}}\right] \\ \vec{E}_{net} &= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{x^2} + \frac{2x}{(x^2+d^2)^{3/2}}\right]\hat{\imath}\end{aligned}$$

- Notice as $x \to 0$ the second term vanishes

$$\vec{E}_{net} = \frac{1}{4\pi\epsilon_0}\frac{q}{x^2}\hat{\imath}$$

- As x gets very large, d becomes insignificant compared to x.

$$\vec{E}_{net} = \frac{1}{4\pi\epsilon_0}\frac{(3q)}{x^2}\hat{\imath}$$