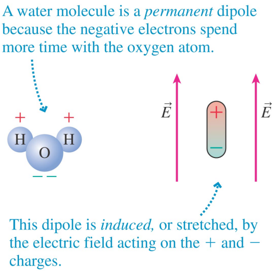
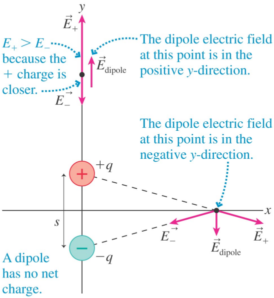
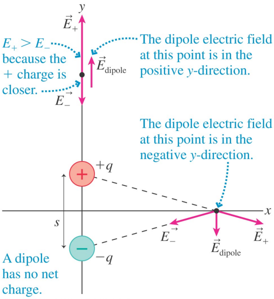
The Electric Field of a Dipole





- We have already seen an induced electric dipole. Natural dipoles also exist. What kind of electric field do they produce?
- Overall the dipole is neutral.
- But, the test charge (left) is closer to the positive charge than it is to the negative. A force results.

The Electric Field of a Dipole



- Let's calculate the electric field at the point on the y axis with

$$\begin{aligned} r_+ &= y - \frac{s}{2} \\ r_- &= y + \frac{s}{2} \end{aligned}$$

- The sum of the fields is

$$\begin{aligned} (E_{\text{dipole}})_y &= \frac{1}{4\pi\epsilon_0}\frac{q}{(y-\frac{1}{2}s)^2} + \frac{1}{4\pi\epsilon_0}\frac{(-q)}{(y+\frac{1}{2}s)^2} \\ &= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(y-\frac{1}{2}s)^2} - \frac{1}{(y+\frac{1}{2}s)^2}\right] \\ &= \frac{q}{4\pi\epsilon_0}\left[\frac{2ys}{(y-\frac{1}{2}s)^2(y+\frac{1}{2}s)^2}\right] \end{aligned}$$

The Electric Field of a Dipole

$$(E_{\text{dipole}})_y = \frac{q}{4\pi\epsilon_0}\left[\frac{2ys}{(y-\frac{1}{2}s)^2(y+\frac{1}{2}s)^2}\right]$$

- For distances much larger than the charge separation ($y \gg s$) the $y - \frac{1}{2}s$ is just y and

$$(E_{\text{dipole}})_y = \frac{1}{4\pi\epsilon_0}\frac{2qs}{y^3}$$

- We define the dipole moment as $\vec{p} = qs$ (direction negative to positive) $\vec{p} = qs\hat{\jmath}$ in this case, so that

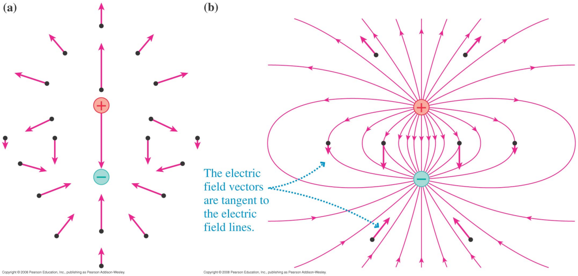
$$\vec{E}_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}, \text{(on axis of dipole)}$$

- In the perpendicular plane that bisects the dipole we can also show

$$\vec{E}_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{(-\vec{p})}{r^3}, \text{(perpendicular to dipole)}$$

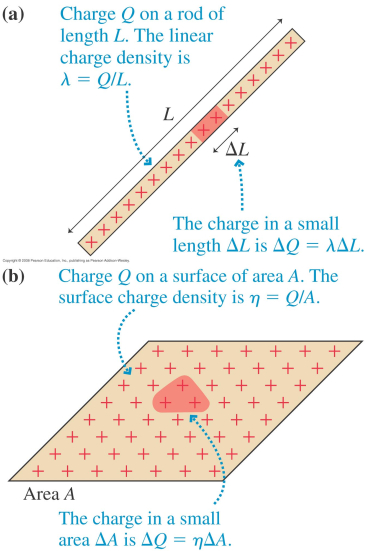
Picturing the Electric Field

We have a couple of different ways to represent an electric field:



- Electric field lines are continuous curves tangent to electric field vectors
- Closely spaced field lines represent larger field strength
- Electric field lines never cross
- Electric field lines start on positive charges and end on negative charges.

The Electric Field of a Continuous Charge Distribution (27.3)



- For a continuous object we cannot look at every single charge individually. Instead define linear charge density and surface charge density

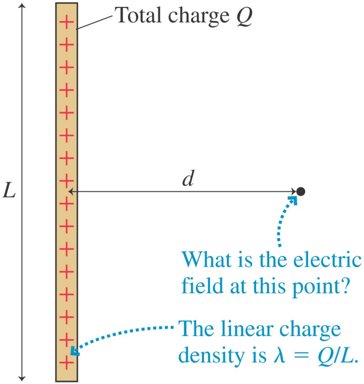
$$\lambda = \frac{Q}{L}, \quad \eta = \frac{Q}{A}$$

Where Q is the total charge on an object, not a single-particle charge.

- These definitions assume that the object is uniformly charged.
- We have some tricks to break the distributions into pieces, then build it back up again.

Example 27.3 - The Electric Field of a Line of Charge

Example 27.3

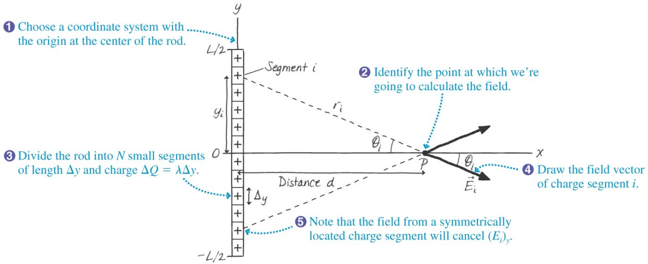


Find the electric field strength at a distance d in the plane that bisects a rod of length L and total charge q.

- The rod is thin, so assume the charge lies along a line
- The charge density of the line is

$$\lambda = \frac{Q}{L}$$

Example 27.3



- Model each little segment of charge (i) as a point charge

$$(E_i)_x = E_i \cos\theta_i = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{r_i^2}\cos\theta_i$$

- We can express r_i^2 and $\cos\theta_i$ as

$$r_i = (y_i^2 + d^2)^{1/2}, \quad \cos\theta_i = \frac{d}{r} = \frac{d}{(y_i^2 + d^2)^{1/2}}$$

Example 27.3

- Plugging these into the electric field formula gives

$$
\begin{aligned}
(E_i)_x &= \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{y_i^2 + d^2} \frac{d}{\sqrt{y_i^2 + d^2}} \\
&= \frac{1}{4\pi\epsilon_0} \frac{d\Delta Q}{(y_i^2 + d^2)^{3/2}}
\end{aligned}
$$

- Now we can sum over all of the little segments

$$
E_x = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{d\Delta Q}{(y_i^2 + d^2)^{3/2}}
$$

- Of course, the rod is not really in little segments. We should make those infinitely small and integrate.

Example 27.3

- The problem with trying to integrate this:

$$E_x = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{d\Delta Q}{(y_i^2 + d^2)^{3/2}}$$

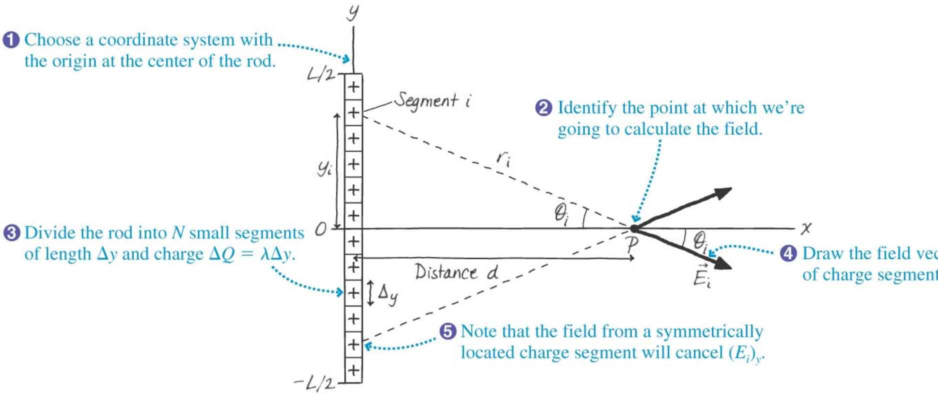
  is that we don't know how to integrate over Q.
- We need to change the variable using the charge density.

$$\Delta Q = \lambda \Delta y = \frac{Q}{L} \Delta y$$

- Giving:

$$E_x = \frac{Q/L}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{d\Delta y}{(y_i^2 + d^2)^{3/2}}$$

Example 27.3



$$E_x = \frac{Q/L}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{d}{(y_i^2 + d^2)^{3/2}} dy$$

Example 27.3

- We can actually do that integral:

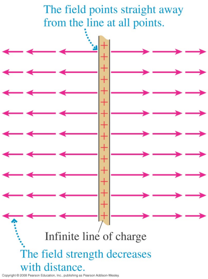
$$
\begin{aligned}
E_x &= \frac{Q/L}{4\pi\epsilon_0} \frac{y}{d\sqrt{y_i^2 + d^2}} \Bigg|_{-L/2}^{L/2} \\
&= \frac{Q/L}{4\pi\epsilon_0} \left[\frac{L/2}{d\sqrt{(L/2)^2 + d^2}} - \frac{-L/2}{d\sqrt{(-L/2) + d^2}} \right] \\
&= \frac{1}{4\pi\epsilon_0} \frac{Q}{d\sqrt{d^2 + (L/2)^2}}
\end{aligned}
$$

- We should check this at the far-away limit, $d \gg L$

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{Q}{d^2}$$

Back to a point charge!!

An Infinite Line of Charge

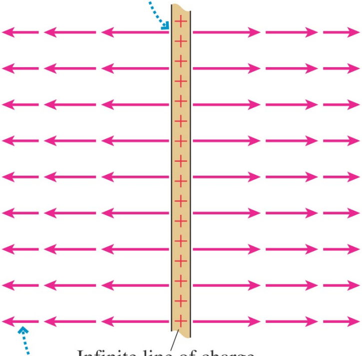


- Let's consider an infinitely long wire of the same charge density λ. We can use the formula for the wire in the extreme limit

$$E_{line} = \lim_{L\to\infty} \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r\sqrt{r^2 + (L/2)^2}} = \frac{1}{4\pi\epsilon_0} \frac{|Q|}{rL/2} = \frac{1}{4\pi\epsilon_0} \frac{2|\lambda|}{r}$$

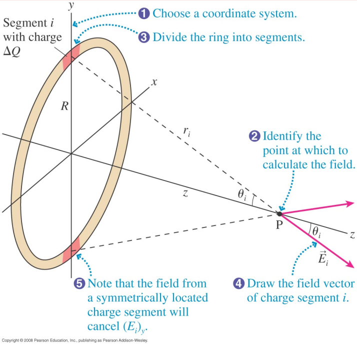
- Notice that the field goes like $1/r$ instead of $1/r^2$

An Infinite Line of Charge



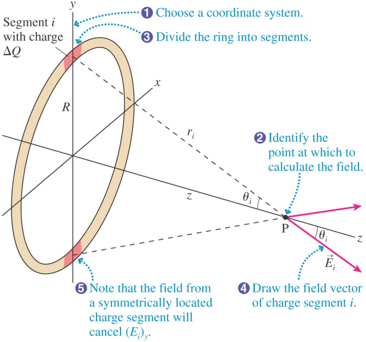
- Of course, no line of charge is really infinite.
- The contributions from charges far down the wire are very small ($like 1/r^2$), so a long wire exerts roughly the same force as an infinite one.
- There are problems with this close to the ends of the finite wire.

Example 27.5



A thin ring of radius R is uniformly charged with total charge Q. Find the electric field at a point on the axis of the ring.

Electric Field from A Thin Ring



- The linear charge density along the ring is

$$\lambda = \frac{Q}{2\pi R}$$

- Divide the ring into N small segments and the z component of the *ith* segment is

$$(E_i)_z = E_i \cos\theta_i = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{r_i^2}\cos\theta_i$$

- Every point on the ring is equidistant from the axis!

$$\begin{aligned} r_i &= \sqrt{z^2 + R^2} \\ \cos\theta_i &= \frac{z}{r_i} = \frac{z}{\sqrt{z^2+R^2}} \end{aligned}$$

Electric Field from A Thin Ring

- Substituting we have:

$$
\begin{aligned}
(E_i)_z &= \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{r_i^2}\cos\theta_i \\
(E_i)_z &= \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{z^2+R^2}\frac{z}{\sqrt{z^2+R^2}} \\
&= \frac{1}{4\pi\epsilon_0}\frac{z}{(z^2+R^2)^{3/2}}\Delta Q
\end{aligned}
$$

- This needs to be summed over all segments:

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{z}{(z^2+R^2)^{3/2}}\sum_{i=1}^{N}\Delta Q$$

- Note that all points on the ring are the same distance from the axis. Who needs an integral??

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}$$