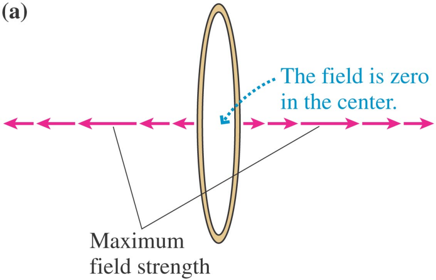
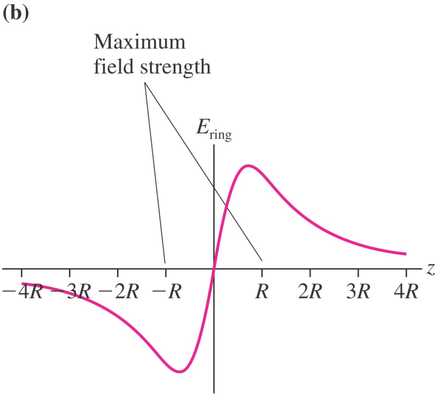
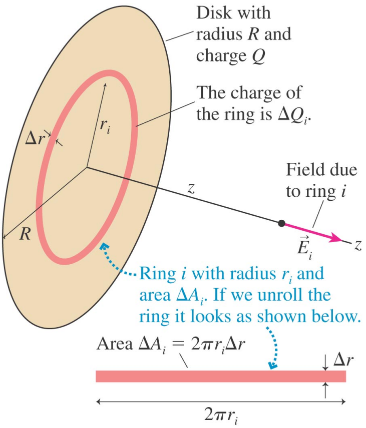
Electric Field from A Thin Ring





A Disk of Charge



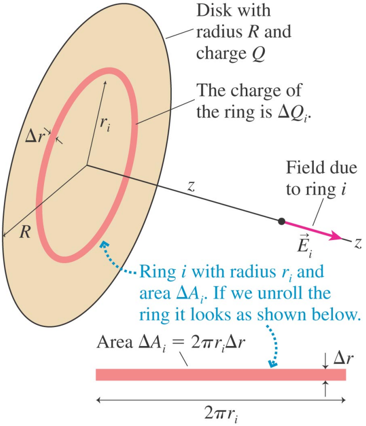
- Now we will move from lines to surfaces. First, let's try a disk of zero thickness and charge density

$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

Again, we will look at an on-axis point.

- Now we know how to deal with a thin ring. So, we'll take a bunch of thin rings together and make then into a disk! Each ring gives

$$E_z = \frac{1}{4\pi\epsilon_0} \frac{z \Delta Q_i}{(z^2 + r_i^2)^{3/2}}$$



- So, now we need to sum over many rings

$$E_z = \frac{z}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{\Delta Q_i}{(z^2 + r_i^2)^{3/2}}$$

- We do have to integrate this one. So, a key step is to figure out what variable we will integrate over.
- Let's use the area of a ring and the surface charge density. The area of a ring is

$$\Delta A_i = 2\pi r_i \Delta r$$

and the charge is

$$\Delta Q_i = 2\pi\eta r_i \Delta r$$

A Disk of Charge

- Writing out the sum

$$E_z = \frac{\eta z}{2\epsilon_0} \sum_{i=1}^{N} \frac{r_i \Delta r}{(z^2 + r_i^2)^{3/2}}$$

- The integral is

$$E_z = \frac{\eta z}{2\epsilon_0} \int_0^R \frac{rdr}{(z^2 + r_i^2)^{3/2}}$$

- Now we just need to do the integral. This is made easy with a variable change

$$\begin{aligned} u &= z^2 + r^2 \\ du &= 2rdr \\ rdr &= \frac{1}{2} du \end{aligned}$$

- And the limits of integration change too...

A Disk of Charge

- The integral becomes

$$
\begin{aligned}
E_z &= \frac{\eta z}{2\epsilon_0}\frac{1}{2}\int_{z^2}^{z^2+R^2}\frac{du}{u^{3/2}} \\
&= \frac{\eta z}{4\epsilon_0}\frac{-2}{u^{1/2}}\bigg|_{z^2}^{z^2+R^2} \\
&= \frac{\eta z}{2\epsilon_0}\left[\frac{1}{z}-\frac{1}{\sqrt{z^2+R^2}}\right] \\
E_z &= \frac{\eta}{2\epsilon_0}\left[1-\frac{z}{\sqrt{z^2+R^2}}\right]
\end{aligned}
$$

A Disk of Charge

- What about checking this answer in the limiting case $z \gg R$?
- First factor the z out of the square root

$$E_z = \frac{\eta}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{1 + R^2/z^2}}\right]$$

  and we know that R^2/z^2 will be tiny in the limiting case.
- Now it is time for the binomial approximation:

$$(1 + x)^n \simeq 1 + nx, x \ll 1$$

  So

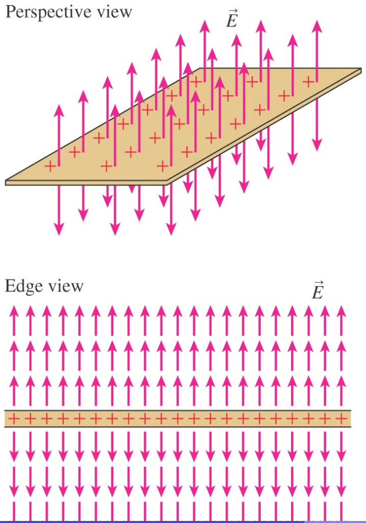
$$1 - (1 + R^2/z^2)^{-1/2} \simeq 1 - \left[1 + \left(\frac{-1}{2}\right)\frac{R^2}{z^2}\right] = \frac{R^2}{2z^2}$$

- Substituting this approximation into our field equation gives

$$E_z = \frac{1}{4\pi\epsilon_0}\frac{Q}{z^2}$$

  Hurray!!

A Plane of Charge

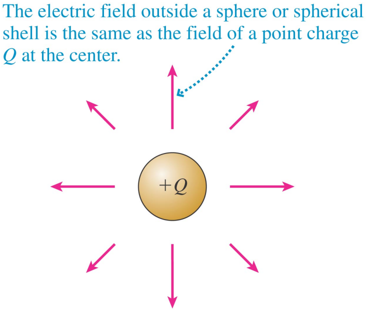


- We model important electronic components (eg. electrodes) as planes of charge.
- Plane are not really infinite but as long as distance to the plane is much smaller than distance to the edges, it is a good model.
- Take the charged disk formula and let $R \to \infty$:

$$E_{plane} = \frac{\eta}{2\epsilon_0} = \text{constant}$$

- It does not depend on r at all!!! Same field no matter how far away you are. Look at the parallel lines on the left...no divergence.

A Sphere of Charge

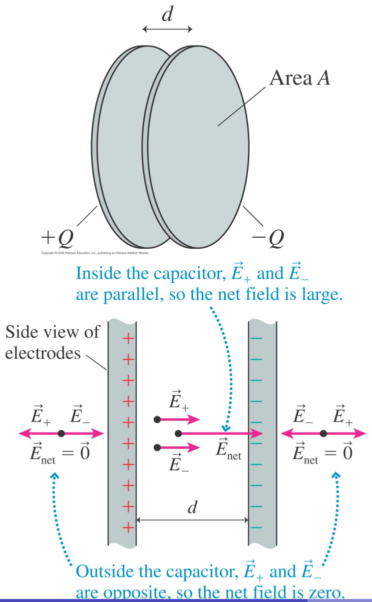


- A sphere of charge is another interesting case.
- This is very much analogous to gravity of a sphere. In that case pretend all of the mass is at the center. In this case, pretend all charge is at the center.
- The field is (without proof):

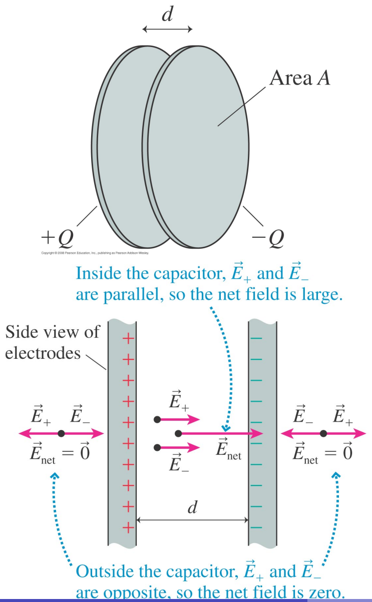
$$\vec{E}_{sphere} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}, \ r \geq R$$

- What is the field inside?

A Parallel Plate Capacitor (27.5)



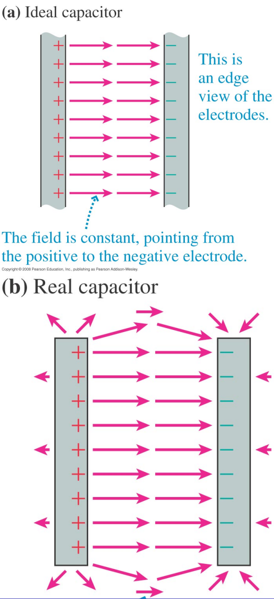
- A parallel plate capacitor is formed from two large area plates (relative to their separation) of equal and opposite charge ($+Q$ and $-Q$).
- Capacitors play important roles in many electronic circuits. Those taking Physics 131 will have a chance to "play" with capacitors.
- Note that its net charge is zero but some charge has been transferred from one plate to the other.
- Opposite charges atract, so the extra charge sits on the inner surface of each plate.



- The electric fields from the positive plate point in the same direction as those from the negative plate - towards the negative plate.
- The net electric field inside the capacitor is then

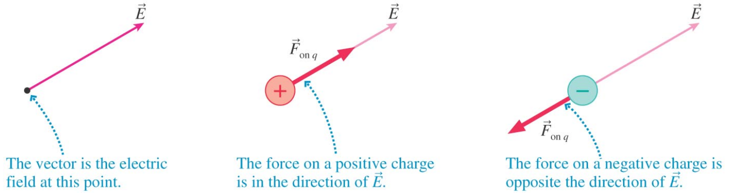
$$\vec{E}_{capacitor} = \vec{E}_+ + \vec{E}_- = \frac{\eta}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

- Outside the capacitor a test charge would see an opposite field from the positive and negative plate. Since the plates are “infinite” the fields would be identical strength...there is no field!!



- An ideal capacitor has a strong electric field between the plates
- An ideal capacitor has a no electric field outside the plates
- A real-world capacitor has a fringe field outside the capacitor.
- Capacitors create a uniform electric field which is highly useful in manipulating a charged particles.

Motion of a Charged Particle in an Electric Field (27.6)



- The force on a charged particle in an electric field is

$$\vec{F}_{on\ q} = q\vec{E}$$

- The sign of the force is determined by the charge of the particle.
- This force will cause the charge to accelerate according to

$$\vec{a} = \frac{\vec{F}_{on\ q}}{m} = \frac{q}{m}\vec{E}$$

- q/m is called the charge-to-mass ratio. Two equal charges with different mass experience the same force but different acceleration

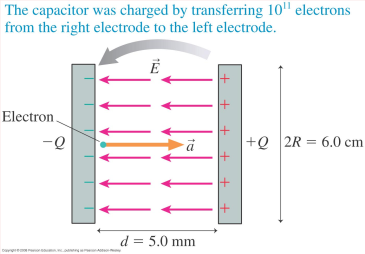
Motion of a Charged Particle in a Uniform Field

- Using a capacitor to generate a uniform field has many important applications. A charged particle will move with constant acceleration in a uniform field:

$$a = \frac{qE}{m} = \text{constant}$$

- A constant acceleration should remind you of what happens to a particle in a gravitational field. All that mathematics you learned in P120 for projectiles will now come in handy for charged particles!
- We can use a parallel plate capacitor to make a beam of electrons and accelerate the particles.
- We could then use another capacitor to steer the beam.

Example 27.8



Two 6.0-cm-diameter electrodes are spaced 5.0 mm apart. They are charged by transferring 1.0×10^{11} electrons from one electrode (plate) to the other. An electron is released from rest at the surface of the negative electrode. How long does it take the electron to cross to the positive electrode? What is its speed as it collides with the positive electrode? (assume the space between the electrodes is a vacuum)

Example 27.8

- The electric field inside the capacitor is

$$E = \frac{\eta}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = \frac{Ne}{\epsilon_0 \pi R^2} = 6.39 \times 10^5 \text{ N/C}$$

- The electron's acceleration is then

$$a = \frac{eE}{m} = 1.1 \times 10^{17} \text{ m/s}^2$$

  where $m = 9.11 \times 10^{-31}$kg
- The time to cross the capacitor is then given by a familiar set of formulae

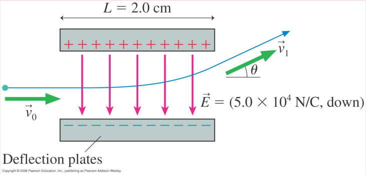
$$\begin{aligned} x_f = d &= \frac{1}{2} a (\Delta t)^2 \\ \Delta t &= \sqrt{\frac{2d}{a}} = 3.0 \times 10^{-10} \text{ s} = 0.30 \text{ ns} \end{aligned}$$

- The speed of the electron when it reaches the positive plate is

$$v = a\Delta t = 3.3 \times 10^7 \text{ m/s}$$

Example 27.9 - Deflecting an Electron Beam

Example 27.9



An electron gun creates a beam of electrons moving horizontally with speed 3.3×10^7 m/s. The electrons enter a 2.0-cm-long gap between two parallel electrodes where the electric field is $\vec{E} = (5.0 \times 10^4 \text{ N/C}, \mathrm{down})$. In which direction, and by what angle, is the electron beam deflected by these electrodes?

Example 27.9 - Deflecting an Electron Beam

- The electron enters with velocity $\vec{v}_0 = 3.3 \times 10^7 \hat{\imath}$ m/s and exits with $\vec{v}_1 = v_{1x}\hat{\imath} + v_{1y}\hat{\jmath}$.
- The deflection angle is

$$\theta = \tan^{-1}\left(\frac{v_{1y}}{v_{1x}}\right)$$

- There is no horizontal force, so the x-component of velocity does not change. The time to cross the plates is then known:

$$\Delta t = \frac{L}{v_{0x}} = \frac{0.020 \text{ m}}{3.3 \times 10^7 \text{ m/s}} = 6.06 \times 10^{-10} \text{ s}$$

- However, there is an upward acceleration of magnitude

$$a = \frac{eE}{m} = \frac{(1.60 \times 10^{-19} C)(5.0 \times 10^4 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} = 8.78 \times 10^{15} \text{ m/s}^2$$

- Giving a final vertical velocity of

$$v_{1y} = v_{0y} + a\Delta t = 5.3 \times 10^6 \text{ m/s}$$

Example 27.9 - Deflecting an Electron Beam

- So, the final velocity is

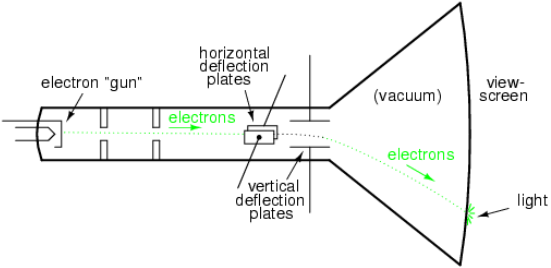
$$\vec{v} = (3.3 \times 10^7 \hat{\imath} + 5.3 \times 10^6 \hat{\jmath}) \text{ m/s}$$

- The final angle is

$$\theta = \tan^{-1}\left(\frac{v_{1y}}{v_{1x}}\right) = 9.1^\circ$$

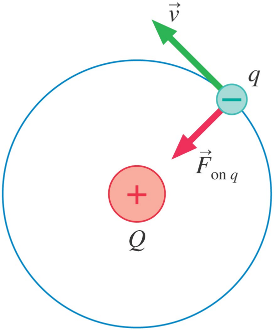
- We use the same mathematics as for mechanics/gravity...but the magnitudes of acceleration, velocity, etc. are much higher than what we are used to!
- In case you wondered why you bothered to do some special relativity last term...consider the speeds of these electrons!

Examples: Your Oscilloscope is a CRT



- Some oscilloscopes use an electron gun with vertical and horizontal deflection plates to control the beam.
- The electron diffraction apparatus demonstrated in class used parallel charged plates to control the electron beam's direction.

Motion in a Non-Uniform Field



- A non-uniform field is much more difficult to deal with.
- However, one simple example is a negatively charged particle orbiting a positively charged one.
- Same mathematics as the moon orbiting the earth. The charge can move in a circular orbit if

$$|q|E = \frac{mv^2}{r}$$