# Traveling Waves

**Leonardo Da Vinci (15th century)**

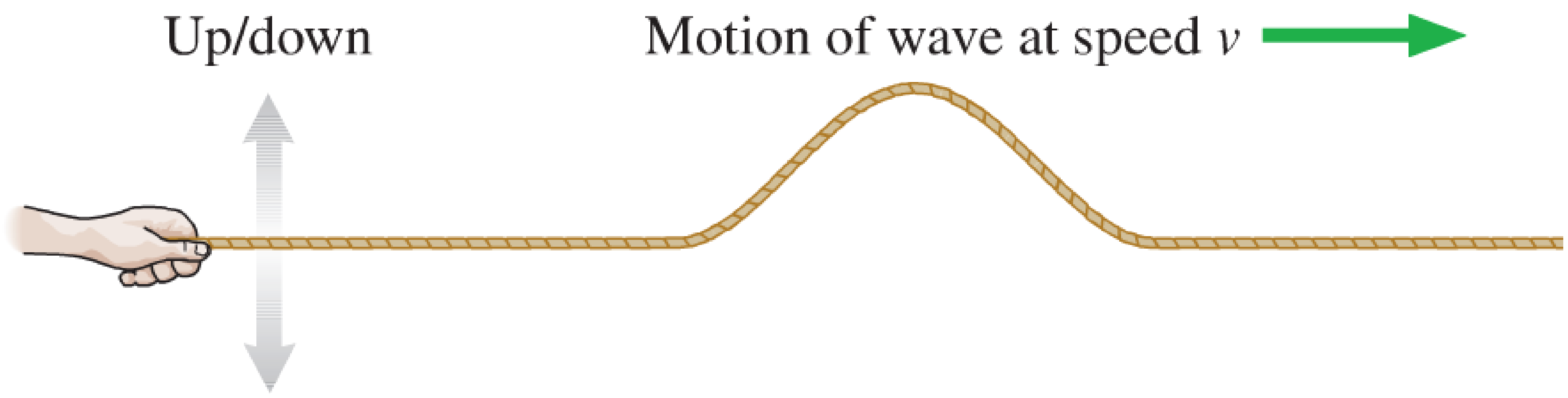
"It often happens that the wave flees the place of its creation, while the water does not; like the waves made in a field of grain by the wind, where we see the waves running across the field while the grain remains in place."

So, a traveling wave is the propagation of a disturbance in a medium, but the medium itself does not propagate.

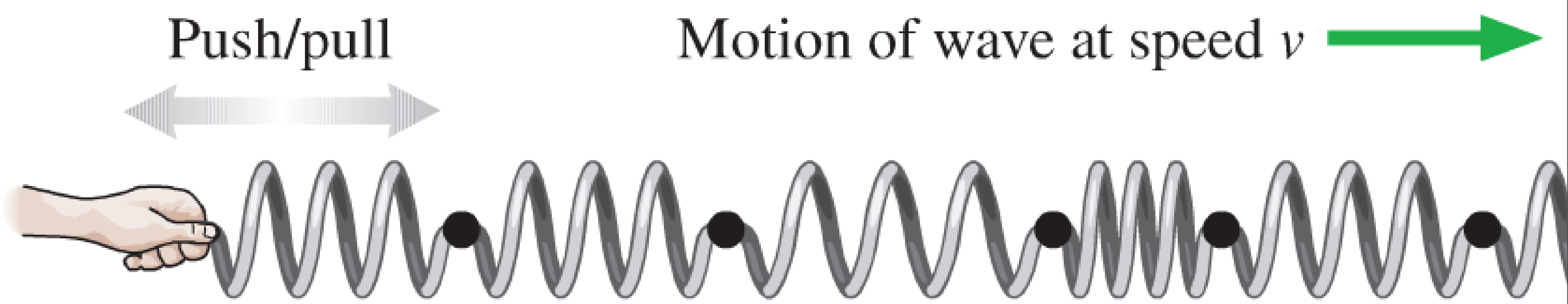
# Traveling Waves

Traveling waves come in two types

Transverse (like light)




Longitudinal (like sound)



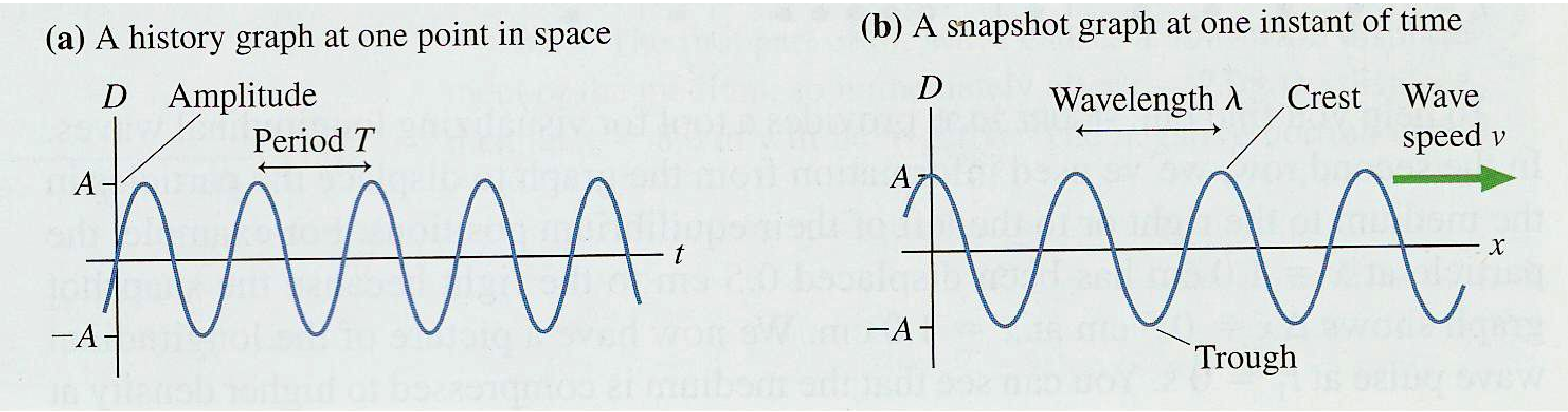
# Traveling Waves

Most of the time, we think of waves as mechanical waves - pulse on a string, waves on the ocean, etc.

However, if light is a wave, what is waving?? The puzzling thing about light is that it seems to propagate without a medium....when sunlight travels through space to us, where is the string? where is the ocean?

It turns out that EM waves are not mechanical waves and do not require a medium. They are a self-sustaining oscillation of the EM field. We'll talk more about EM fields later in the course, but you do not need do understand them in order to calculate many interesting properties of light. EM waves are describable by the same sort of mathematics as mechanical waves.

# Wave Properties Review - Periodic Waves




- Amplitude($A$): Maximum value of a disturbance
- Period ($T$): Time for one complete wave cycle to pass a fixed point
- Frequency ($f$): Number of wave cycles passing a given point per unit time $f = \frac{1}{T}$
- Wave speed ($v$): During one period a fixed observer sees one complete wavelength go by $v = \frac{\lambda}{T} = \lambda f$

# Sinusoidal Waves

It is worth noting that

$$\lambda = \frac{v}{f} = \frac{\text{property of medium}}{\text{property of source}}$$

In other words, the speed of light does not depend on whether you use a lightbulb or a laser beam. However, the frequency does depend on the source....and so does the wavelength.

# Sinusoidal Waves – Simulation

Simulation of transverse waves on a rope: http://phet.colorado.edu/

# Mathematics of Sinusoidal Waves

We can define the displacement $D$ of a particle in a medium due to the passage of a sinusoidal wave (snapshot at $t = 0$):

$$D(x, t = 0) = A \sin\left(2\pi\frac{x}{\lambda} + \phi_0\right)$$

This equation is periodic with period $\lambda$ ($D(x) = D(x + \lambda)$). We can turn this into a moving wave by making the position depend on time:
$x \rightarrow x - vt$

$$D(x, t) = A \sin\left(2\pi\frac{x - vt}{\lambda} + \phi_0\right) = A \sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) + \phi_0\right)$$

# Mathematics of Sinusoidal Waves

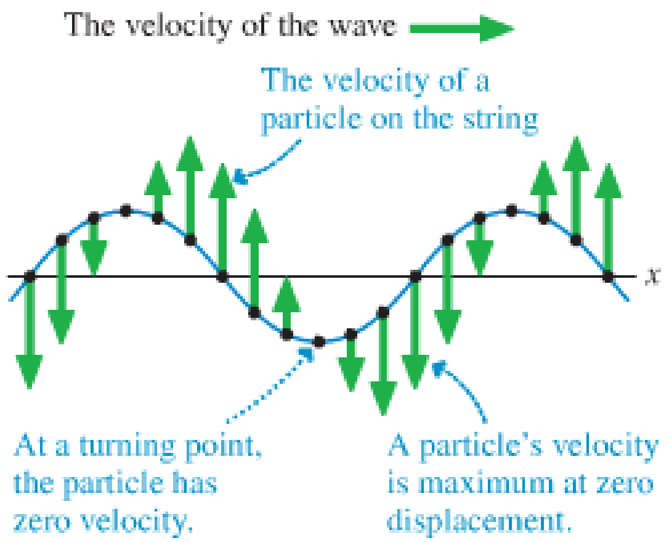
We can re-express this in different notation to clean things up a little. Let's define the angular frequency ($\omega$) and wave number ($k$) as

$$\omega = 2\pi f = \frac{2\pi}{T}, k = \frac{2\pi}{\lambda}$$

Wave number is the spatial analogue of frequency, it describes the number of radians of wave cycle per unit distance. This allows us to write the equation from the previous page as

$$D(x,t) = A\sin(kx - \omega t + \phi_0)$$

# Example: Waves on a String




In this case displacement is $y$:

$$y(x,t) = A\sin(kx - \omega t + \phi_0)$$

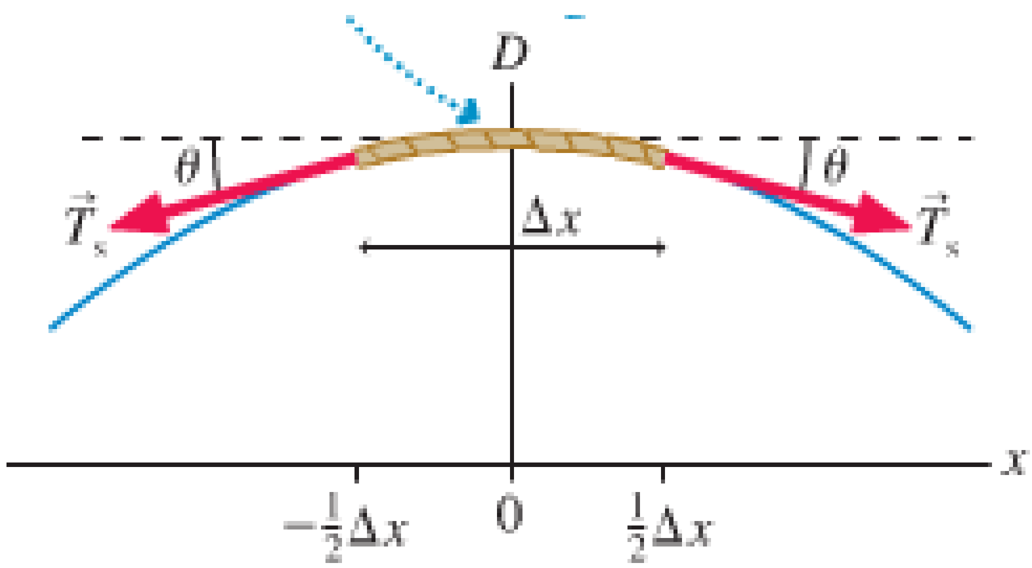
The velocity of a particle on the string is given by the first derivative of this expression, the acceleration by the second derivative

$$v_y = \frac{dy}{dt} = -\omega A\cos(kx - \omega t + \phi_0)$$

$$a_y = \frac{dv_y}{dt} = -\omega^2 A\sin(kx - \omega t + \phi_0)$$

This is referring to the particle speed.

# Example: Waves on a String

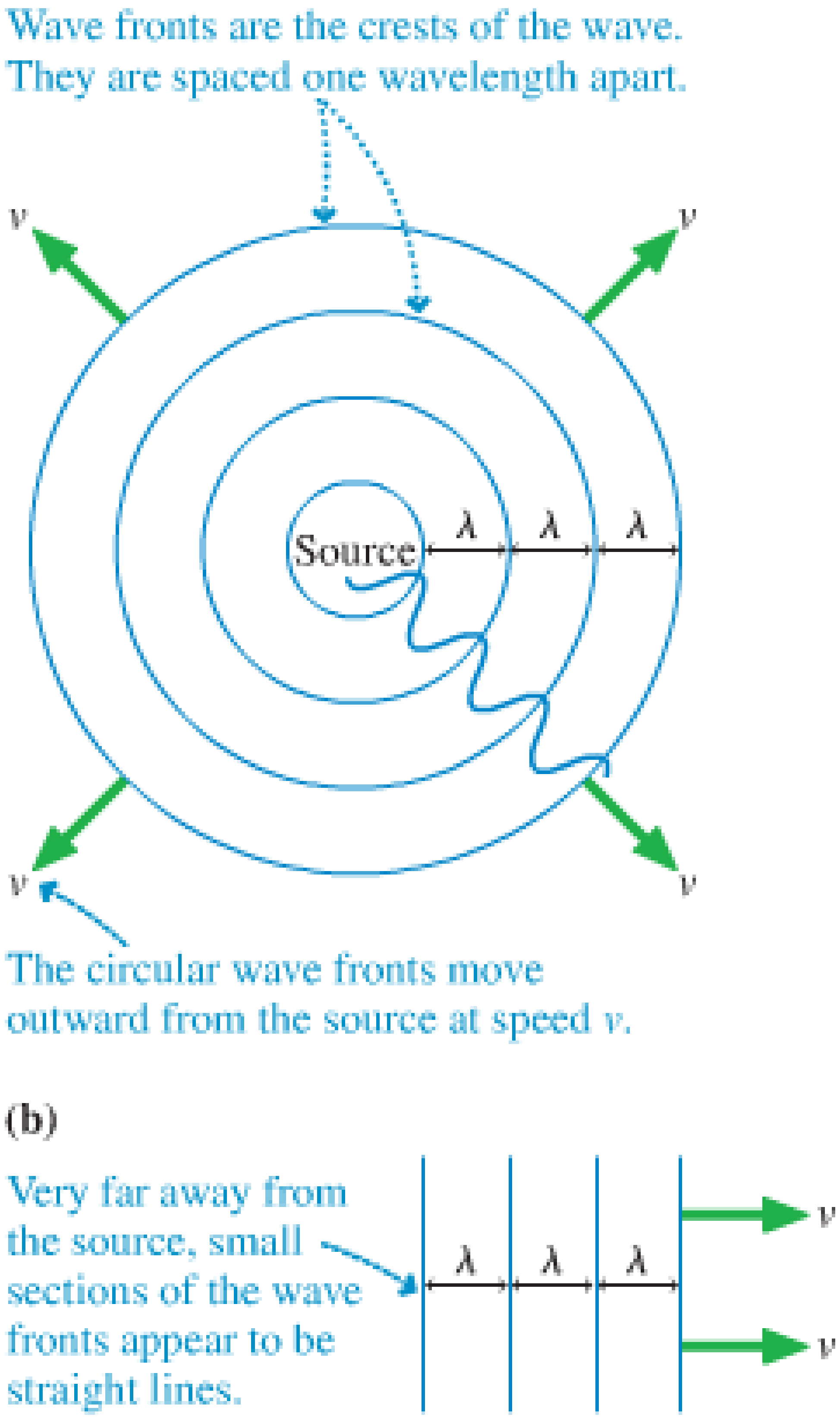



Your text (page 613) then goes through a number of small algebraic steps looking at the net force on a small segment of string. Since Knight does every step, I will not reproduce it here...but the answer is quite interesting!

$$v = \sqrt{\frac{T_s}{\mu}}$$

where $T_s$ is the tension and $\mu$ is the linear density. This defines the wave speed. Please note that it depends only on properties of the medium and what that dependence is.

# Waves in 2 or 3 Dimensions

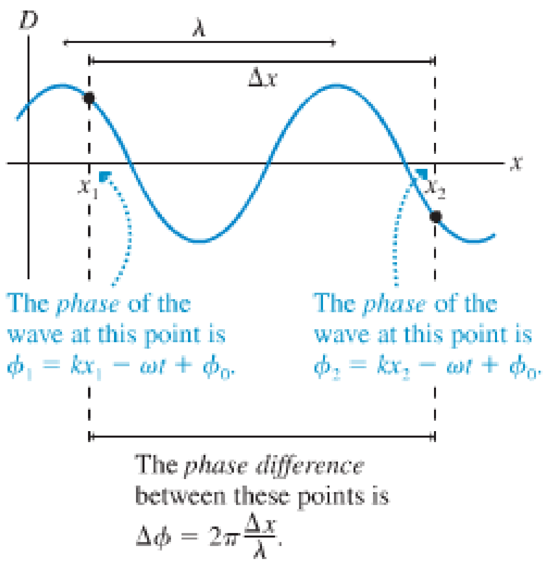



So far we have discussed waves in 1-D. What do they look like in more than one dimension? This circular wave is made of wave fronts. In 3-D you get spherical waves. Very far from the source, these waves look like plane waves. Mathematically, little changes:

$$D(r,t) = A(r)\sin(kr - \omega t + \phi_0)$$

(note amplitude change as wave spreads)

# Phase and Phase Difference




The phase of the wave is

$$(kx - \omega t + \phi_0)$$

Very soon the phase difference will be very important since we will talk about interference. The phase difference is

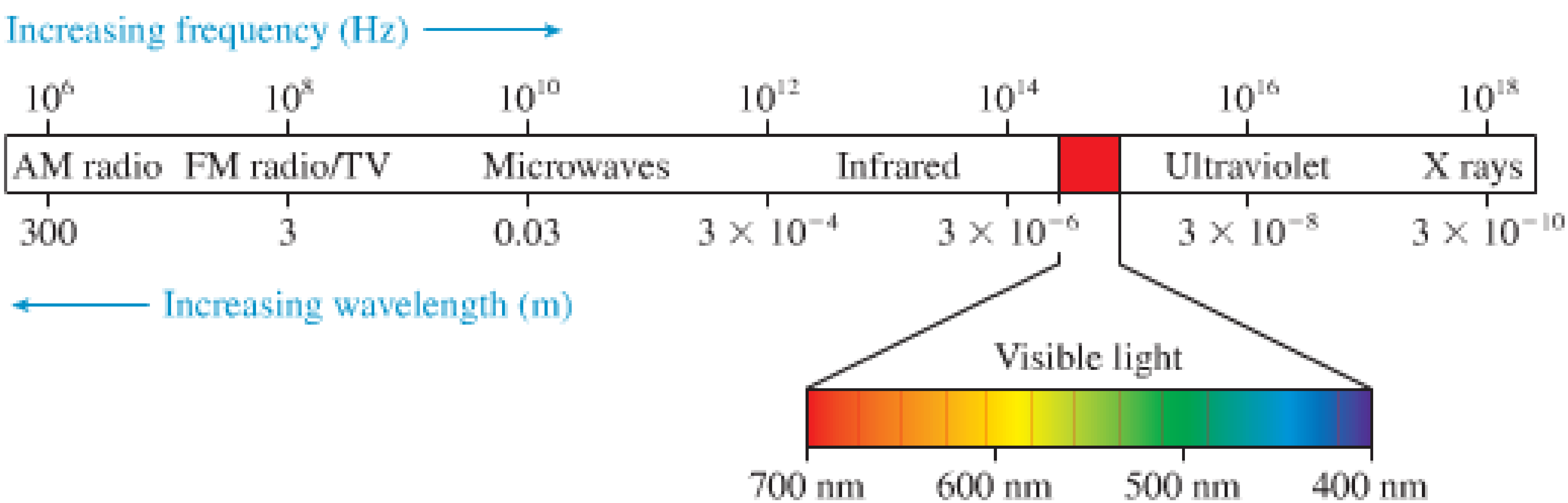
$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 = (kx_2 - \omega t + \phi_0) - (kx_1 - \omega t + \phi_0) \\ &= k(x_2 - x_1) = k\Delta x = 2\pi\frac{\Delta x}{\lambda}\end{aligned}$$

Moving from one crest to another represents a phase shift of $2\pi$

# EM Waves

As previously mentioned, light is an EM wave which travels at speed $c$ in a vacuum. While it is not a mechanical wave like the ones we have been talking about so far, a lot of the same physics applies. It is a transverse wave with very high frequency:

$$f = \frac{v}{\lambda} = \frac{3.00 \times 10^8 m/s}{600 nm} = 5.00 \times 10^{14} Hz$$



# Waves Changing Medium

What happens to an EM wave when it changes from a medium with a certain index of refraction to a different medium with a different index of refraction? As previously mentioned, we can define the index of refraction as

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in material}} = \frac{c}{v}$$

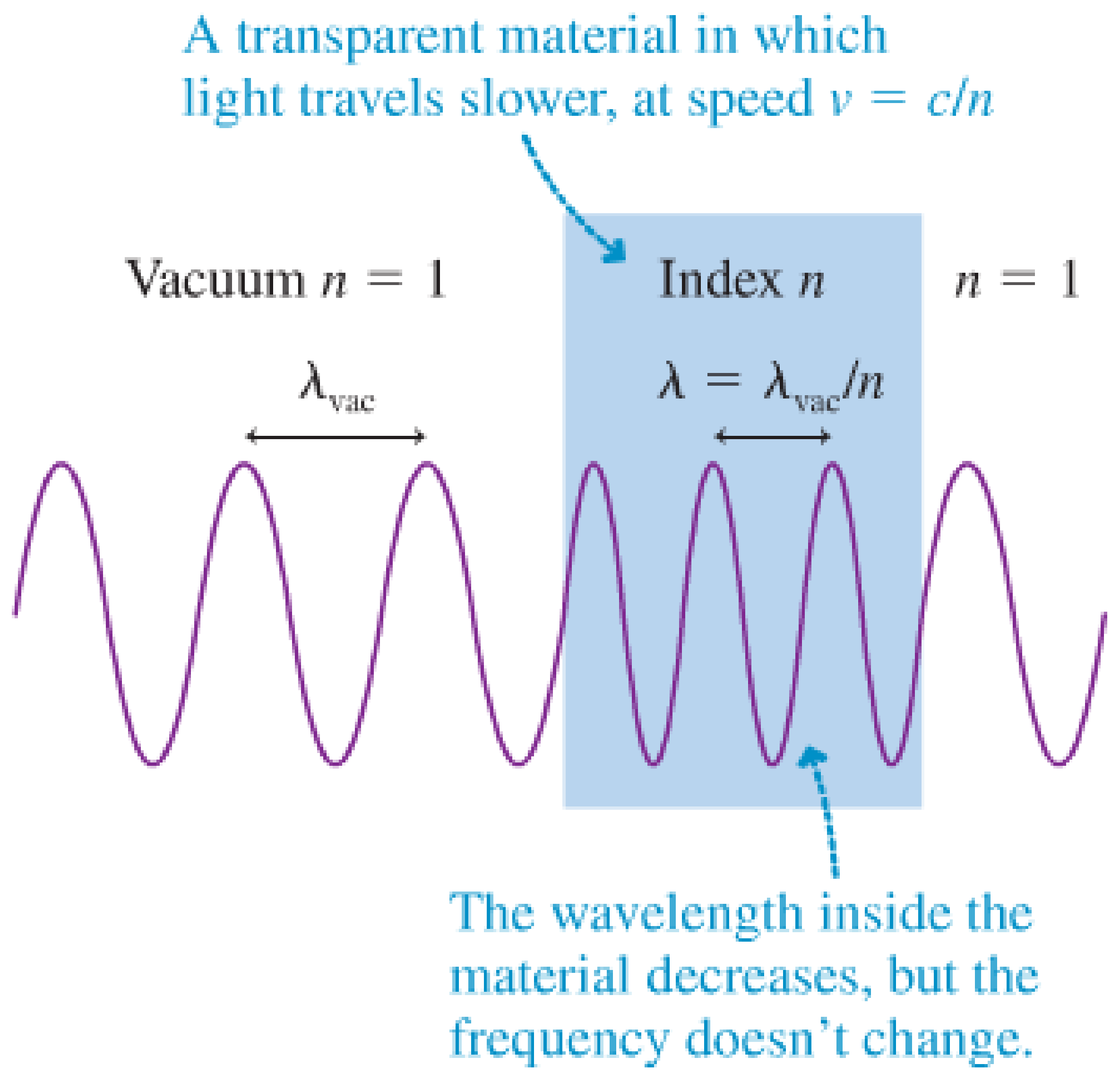
It is clear that $v < c$ means that $n > 1$ always (remember this on assignments and in P131). The speed of light changes when the

medium changes. We also know that

$$v = \lambda f$$

So, if $v$ changes, then either $\lambda$ changes or $f$ changes or both change.

# Waves Changing Medium




The frequency depends only on the source, not the medium. However, the wavelength changes. So, in $v = \lambda f$ we have $v$ and $\lambda$ changing while $f$ is fixed when the medium changes.

# Wave Power and Intensity

The power of a wave is the rate at which it transfers energy. The intensity also includes the area over which that power is spread:

$$I = \frac{P}{a}$$

measured in $W/m^2$. This is particularly interesting in the case of spherical waves

$$I = \frac{P_{source}}{4\pi r^2}$$

Now think about the Sun...

# Wave Power and Intensity

For SHM with amplitude $A$ you have seen that energy is

$$E = \frac{1}{2}kA^2$$

So, if energy is proportional to the square of the amplitide, then so is intensity:

$$I = cA^2$$

where $c$ is some constant which depends on which sort of wave we are talking about.