

Traveling Waves

Leonardo Da Vinci (15th century)

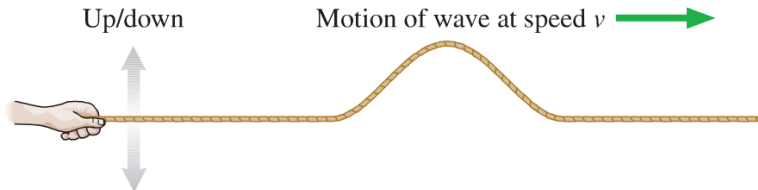
“It often happens that the wave flees the place of its creation, while the water does not; like the waves made in a field of grain by the wind, where we see the waves running across the field while the grain remains in place.”

So, a traveling wave is the propagation of a disturbance in a medium, but the medium itself does not propagate.

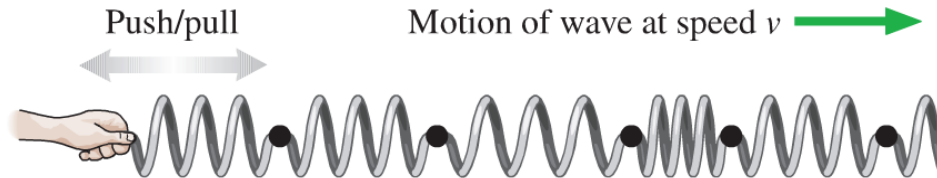
Traveling Waves

Traveling waves come in two types

Transverse (like light)



Longitudinal (like sound)



Traveling Waves

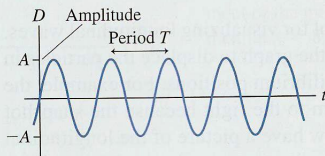
Most of the time, we think of waves as **mechanical waves** - pulse on a string, waves on the ocean, etc.

However, if light is a wave, what is waving?? The puzzling thing about light is that it seems to propagate without a medium....when sunlight travels through space to us, where is the string? where is the ocean?

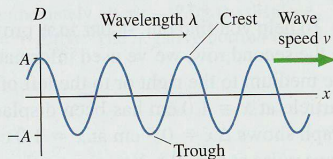
It turns out that EM waves are not mechanical waves and do not require a medium. They are a self-sustaining oscillation of the EM field. We'll talk more about EM fields later in the course, but you do not need to understand them in order to calculate many interesting properties of light. EM waves are describable by the same sort of mathematics as mechanical waves.

Wave Properties Review - Periodic Waves

(a) A history graph at one point in space



(b) A snapshot graph at one instant of time



- Amplitude(A): Maximum value of a disturbance
- Period (T): Time for one complete wave cycle to pass a fixed point
- Frequency (f): Number of wave cycles passing a given point per unit time $f = \frac{1}{T}$
- Wave speed (v): During one period a fixed observer sees one complete wavelength go by $v = \frac{\lambda}{T} = \lambda f$

It is worth noting that

$$\lambda = \frac{v}{f} = \frac{\text{property of medium}}{\text{property of source}}$$

In other words, the speed of light does not depend on whether you use a lightbulb or a laser beam. However, the frequency does depend on the source....and so does the wavelength.

Sinusoidal Waves – Simulation

Simulation of transverse waves on a rope: <http://phet.colorado.edu/>

Mathematics of Sinusoidal Waves

We can define the displacement D of a particle in a medium due to the passage of a sinusoidal wave (snapshot at $t = 0$):

$$D(x, t = 0) = A \sin\left(2\pi \frac{x}{\lambda} + \phi_0\right)$$

This equation is periodic with period λ ($D(x) = D(x + \lambda)$). We can turn this into a moving wave by making the position depend on time:

$$x \rightarrow x - vt$$

$$D(x, t) = A \sin\left(2\pi \frac{x - vt}{\lambda} + \phi_0\right) = A \sin\left(2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right) + \phi_0\right)$$

Mathematics of Sinusoidal Waves

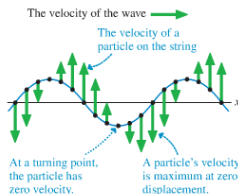
We can re-express this in different notation to clean things up a little. Let's define the **angular frequency** (ω) and **wave number** (k) as

$$\omega = 2\pi f = \frac{2\pi}{T}, k = \frac{2\pi}{\lambda}$$

Wave number is the spatial analogue of frequency, it describes the number of radians of wave cycle per unit distance. This allows us to write the equation from the previous page as

$$D(x, t) = A \sin(kx - \omega t + \phi_0)$$

Example: Waves on a String



In this case displacement is y :

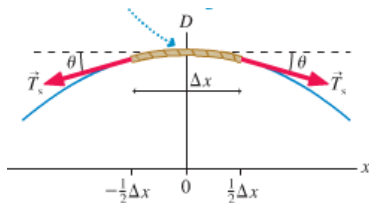
$$y(x, t) = A \sin(kx - \omega t + \phi_0)$$

The velocity of a particle on the string is given by the first derivative of this expression, the acceleration by the second derivative

$$v_y = \frac{dy}{dt} = -\omega A \cos(kx - \omega t + \phi_0)$$
$$a_y = \frac{dv_y}{dt} = -\omega^2 A \sin(kx - \omega t + \phi_0)$$

This is referring to the **particle speed**.

Example: Waves on a String



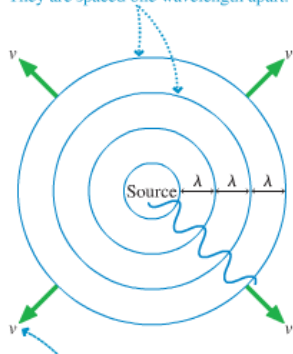
Your text (page 613) then goes through a number of small algebraic steps looking at the net force on a small segment of string. Since Knight does every step, I will not reproduce it here...but the answer is quite interesting!

$$v = \sqrt{\frac{T_s}{\mu}}$$

where T_s is the tension and μ is the linear density. This defines the **wave speed**. Please note that it depends only on properties of the medium and what that dependence is.

Waves in 2 or 3 Dimensions

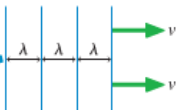
Wave fronts are the crests of the wave.
They are spaced one wavelength apart.



The circular wave fronts move outward from the source at speed v .

(b)

Very far away from the source, small sections of the wave fronts appear to be straight lines.



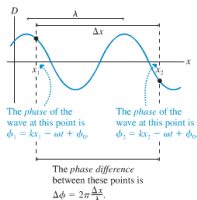
So far we have discussed waves in 1-D. What do they look like in more than one dimension? This circular wave is made of **wave fronts**. In 3-D you get **spherical waves**. Very far from the source, these waves look like **plane waves**.

Mathematically, little changes:

$$D(r, t) = A(r) \sin(kr - \omega t + \phi_0)$$

(note amplitude change as wave spreads)

Phase and Phase Difference



The **phase** of the wave is

$$(kx - \omega t + \phi_0)$$

Very soon the phase difference will be very important since we will talk about interference. The phase difference is

$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 = (kx_2 - \omega t + \phi_0) - (kx_1 - \omega t + \phi_0) \\ &= k(x_2 - x_1) = k\Delta x = 2\pi \frac{\Delta x}{\lambda}\end{aligned}$$

Moving from one crest to another represents a phase shift of 2π

EM Waves

As previously mentioned, light is an EM wave which travels at speed c in a vacuum. While it is not a mechanical wave like the ones we have been talking about so far, a lot of the same physics applies. It is a transverse wave with very high frequency:

$$f = \frac{v}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{600 \text{ nm}} = 5.00 \times 10^{14} \text{ Hz}$$

