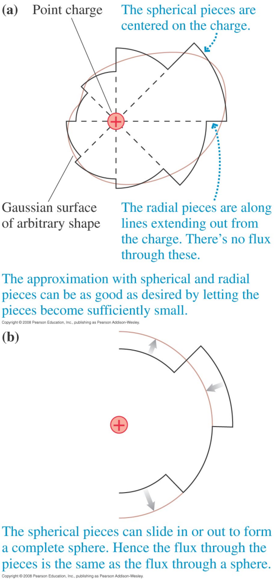
Electric Flux is Independent of Surface Shape



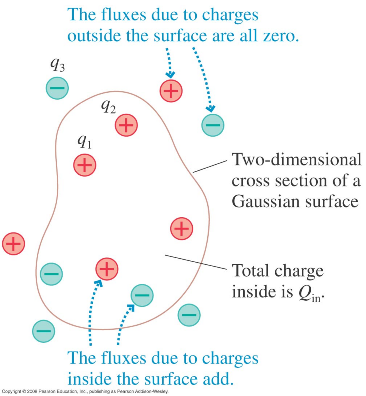
- What about the flux through some arbitrary surface shape?
- Approximate the surface by a patchwork of radial and spherical pieces. Spherical pieces are centered on the charge. Radial pieces have 0 flux through them.
- We can make the pieces arbitrarily small to get the best model of the surface.
- We can slide the pieces around and make a sphere. So, the flux through any closed surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

Charge Outside the Surface

- We have only been talking about charges enclosed by the surface. What about charges outside the surface?
- Anything outside the surface will contribute nothing to the total flux!
- Any line entering the surface in one place will exit it in another....there is no net flux unless the charge is enclosed.

Multiple Charges



- What is the flux from a set of charges through a surface?
- We know that the electric field vectors just add, and so will the fluxes

$$\Phi_e = \Phi_1 + \Phi_2 + \Phi_3 + \dots$$

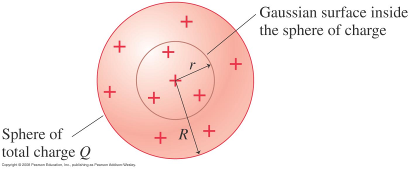
$$\Phi_e = \left(\frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} + \cdots \frac{q_i}{\epsilon_0}\right)_{inside} + (0)_{out}$$

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

where $Q_{in} = q_1 + q_2 + \cdots + q_i$

Using Gauss' Law - Example 28.4

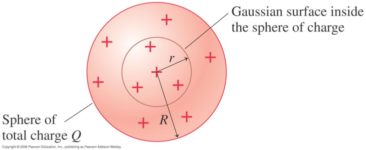
Example 28.4



What is the electric field **inside** a uniformly charged sphere?

Solving this with Coulomb's Law and superposition would not be very fun. However, there is a clear spherical symmetry in the problem. Let's try it with a spherical Gaussian surface inside the original sphere.

Using Gauss' Law - Example 28.4



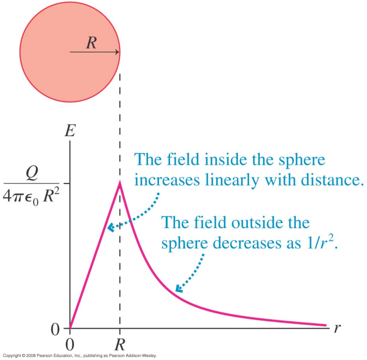
- $\vec{E}$ is perpendicular to the surface and has the same strength everywhere on the surface. The flux is therefore

$$\Phi_e = EA_{sphere} = 4\pi r^2 E = \frac{Q_{in}}{\epsilon_0}$$

- Q_{in} is the charge inside the Gaussian sphere (ie. the little one). The charge is uniform and the volume charge density is

$$\rho = \frac{Q}{V_R} = \frac{Q}{\frac{4}{3}\pi R^3}$$

Using Gauss' Law - Example 28.4



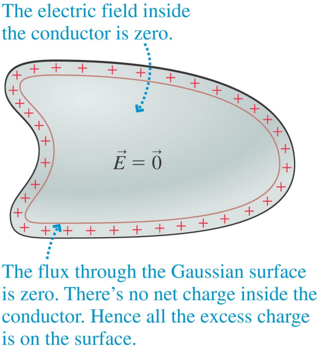
- The charge enclosed by the sphere is then

$$Q_{in} = \rho V_r = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{r^3}{R^3} Q$$

- Gauss' Law becomes

$$E = \frac{1}{4\pi r^2}\frac{Q_{in}}{\epsilon_0} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3} r$$

Conductors in Electrostatic Equilibrium (28.6)

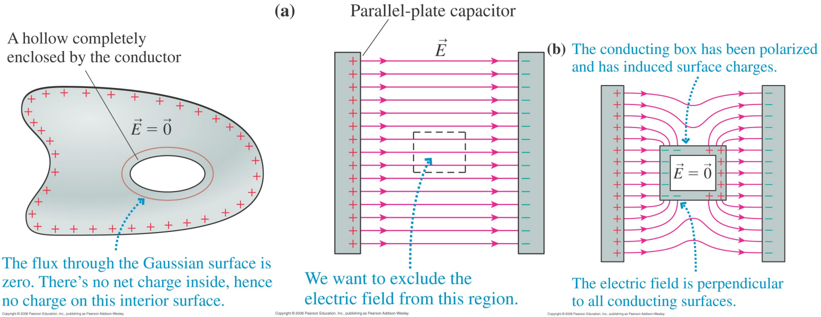


- We know that a conductor has any excess charge on its surface and $\vec{E}$ is zero anywhere on the inside (else charges would be moving).
- If you assume $E = 0$ everywhere and draw a Gaussian surface then you know that $Q_{in} = 0$ inside the surface. Any net charge must be on the outer edge...hey, we knew that already!
- The electric field at the surface is perpendicular to the surface and has magnitude (see your text):

$$\vec{E}_{surface} = \frac{\eta}{\epsilon_0}$$

where η is surface charge density.

Conductors in Electrostatic Equilibrium



- If we make a hole in the conductor we can analyze it with a Gaussian surface close to the hole. It is easy to conclude that $Q_{in} = 0$.
- Since $\vec{E}$ is also zero inside the hole, this has practical applications: we can build a Faraday cage

The Electric Potential (Chapter 29)

- We are working our way towards really practical stuff like circuit-building.
- To get there, we need to talk about energy. Chapter 29* is all about electric potential.
- We have mostly focused on static charges so far. If we eventually want to understand the motion of charges we will need to understand electric potential.

*Ch 25 for Phys141

Mechanical Energy

- The treatment of electric energy is developed by analogy to mechanical energy (in particular using comparisons to gravitational interactions). So, let's review.
- Remember that mechanical energy is conserved by particles which interact via conservative forces (eg. gravity, electric).

$$\Delta E_{mech} = \Delta K + \Delta U = 0$$

- Kinetic energy of a system is the sum of the kinetic energies of all of the particles in the system.
- Potential energy is the interaction energy of the system.

Mechanical Energy

- We define the change in potential energy in terms of the work done by forces:

$$\Delta U = U_f - U_i = -W_{interaction\ forces}$$

- The component of force along the direction of motion does work on an object. Non-constant forces need the “chop-up and integrate” trick:

$$W = \sum_j (F_s)_j \Delta s_j \rightarrow \int_{s_i}^{s_f} F_s ds = \int_i^f \vec{F} \cdot d\vec{s}$$