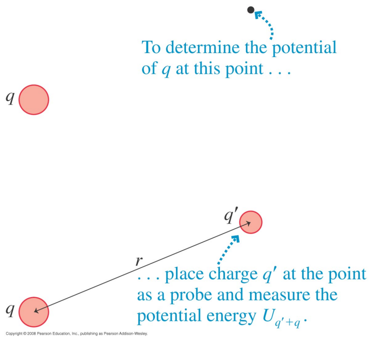
The Electric Potential of a Point Charge (29.6)



- We have only looked at a capacitor...let's not forget our old friend the point charge!
- Use a second charge (q') to probe the electric potential at a point in space. The potential energy will be

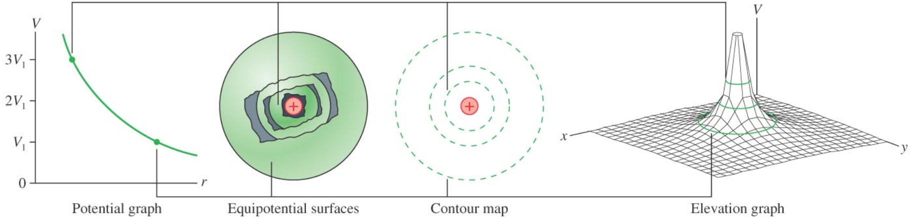
$$U_{q'+q} = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r}$$

- The potential is then

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

- This extends through all of space with $V = 0$ at infinity.

Visualizing the Potential of a Point Charge



The Electric Potential of a Charged Sphere

- The electric potential of a charged sphere (from outside) is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

- We call the potential right at the surface V_0

$$V_0 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

- A sphere of radius R charged to V_0 has total charge $Q = 4\pi\epsilon_0 R V_0$. Substituting this in gives

$$V = \frac{R}{r}V_0$$

The Electric Potential of Many Charges (29.7)

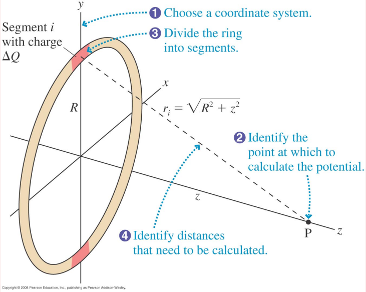
- The electric potential, like the electric field, obeys the principle of superposition:

$$V = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i}$$

  where r_i is the distance from the charge q_i to the point in space at which the potential is being calculated.
- As we did before, we will extend our calculation to a continuous charge distribution. Often this involves an integration.

The Potential of a Ring of Charge

The Potential of a Ring of Charge (Example 29.11)



A thin, uniformly charged ring of radius R has total charge Q. Find the potential at distance z on the axis of the ring.

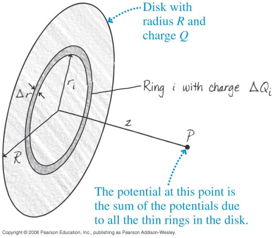
- The distance r_i is

$$r_i = \sqrt{R^2 + z^2}$$

- The potential is then the sum:

$$V = \sum_{i=1}^{N} \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{r_i} = \frac{1}{4\pi\epsilon_0} \frac{1}{\sqrt{R^2 + z^2}} \sum_{i=1}^{N} \Delta Q = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{R^2 + z^2}}$$

The Potential of a Disk of Charge (Example 29.12)



A thin, uniformly charged disk of radius R has total charge Q. Find the potential at distance z on the axis of the disk.

- The disk has uniform charge density

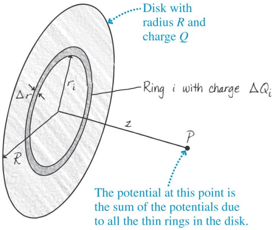
$$\eta = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

- We know the potential due to any ring

$$V_i = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q_i}{\sqrt{r_i^2 + z^2}}$$

Phys141 text uses instead σ of η.

The Potential of a Disk of Charge



- Now sum over rings

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{\Delta Q_i}{\sqrt{r_i^2 + z^2}}$$

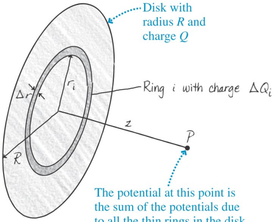
- To turn this into an integral we need to substitute for ΔQ_i

$$\Delta Q_i = \eta \Delta A_i = \frac{Q}{\pi R^2} 2\pi r \Delta r = \frac{2Q}{R^2} r_i \Delta r$$

- The potential at P is then

$$V = \frac{Q}{2\pi\epsilon_0 R^2} \int_0^R \frac{rdr}{\sqrt{r^2 + z^2}}$$

The Potential of a Disk of Charge



- You can do this integral by substitution

$$
\begin{aligned}
u &= r^2 + z^2 \\
rdr &= \frac{1}{2}du \\
V &= \frac{Q}{2\pi\epsilon_0 R^2}\int_{z^2}^{R^2+z^2} \frac{\frac{1}{2}du}{u^{1/2}} \\
&= \frac{Q}{2\pi\epsilon_0 R^2} u^{1/2}\Big|_{z^2}^{R^2+z^2} \\
&= \frac{Q}{2\pi\epsilon_0 R^2}\left(\sqrt{R^2+z^2} - |z|\right)
\end{aligned}
$$

Potential and Field (Chapter 30)

- Now we start Chapter 30* of your text.
- We will learn the connection between potential and electric field. You will be able to calculate the potential from the field and vice versa.
- We will learn sources of potential - batteries.
- We will learn Kirchhoff's Law.
- We will learn how to use capacitors.

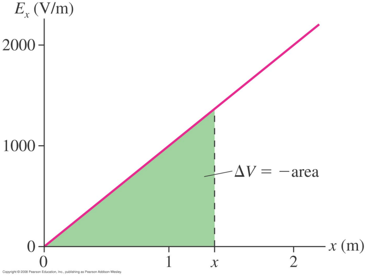
*Ch 26 in Phys141 textbook.

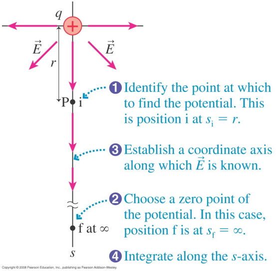
Finding Potential from Electric Field

- Given the electric field in some region of space, how do we calculate the potential?
- The potential difference is

$$\Delta V = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

- We know that an integral is the area under a curve, so





- Let’s do a practical example with a zero of potential chosen at infinity
- Integrating along a straight line from P to $f = \infty$ gives

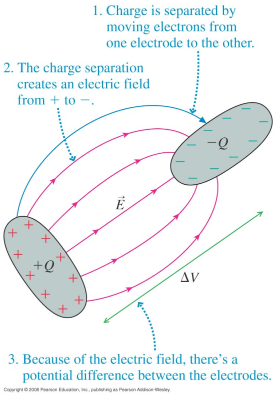
$$\Delta V = V(\infty) - V(r) = -\int_r^\infty E_s ds$$

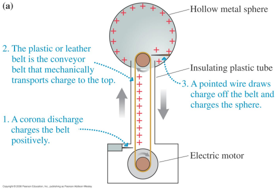
- We know the electric field is

$$E_s = \frac{1}{4\pi\epsilon_0}\frac{q}{s^2}$$

- Thus the potential at r is

$$V(r) = V(\infty) + \frac{q}{4\pi\epsilon_0}\int_r^\infty \frac{ds}{s^2} = \frac{q}{4\pi\epsilon_0}\left.\frac{-1}{s}\right|_r^\infty = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$



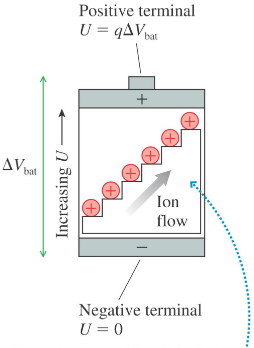


- Any charge separation causes a potential difference.
- There is a potential difference between electrodes (1st picture) of

$$\Delta V = V_{pos} - V_{neg} = -\int_{neg}^{pos} E_s ds$$

- In class we have seen a mechanical charge separator - a Van de Graaff generator.

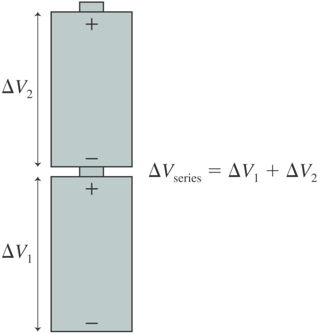
Batteries and emf



The charge escalator "lifts" charge from the negative side to the positive side. Charge q gains energy $\Delta U = q\Delta V_{bat}$.

- We are more familiar with a chemical charge separator - a battery
- We will ignore the chemistry and discuss the battery as a charge escalator which lifts positive charges from the negative terminal to the positive terminal.
- Doing this requires work...energy from chemical reactions. In an ideal battery $\Delta U = W_{chem}$
- The work done per charge in a battery is known as its emf ($\mathcal{E}$). For example $\mathcal{E} = 1.5V, 9V, etc$

Batteries and emf

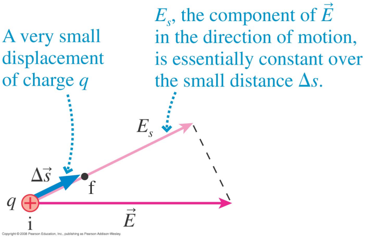


- The voltage between the terminals is called the terminal voltage and is just slightly less than $\mathcal{E}$.
- If you put multiple batteries in series (see left) the combined voltage is simply the sum of the two individual voltages.

$$\Delta V_{series} = \Delta V_1 + \Delta V_2 + \cdots$$

Finding the Electric Field from the Potential (30.3)

Sec. 25.10 in Phys141 textbook



- The work done by the electric field as q moves through a small distance Δs is

$$W = F_s \Delta s = q E_s \Delta s$$

- The potential difference is

$$\Delta V = \frac{\Delta U_{q+sources}}{q} = \frac{-W}{q} = -E_s \Delta s$$

- Rearranging and making Δs infinitely small gives

$$E_s = -\frac{dV}{ds}$$

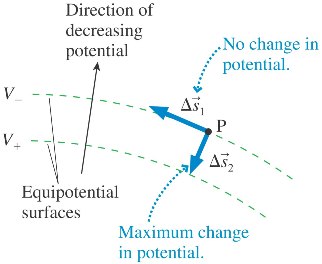
Finding the Electric Field from the Potential

- Let's use a point charge as an example
- Choose the s axis to be in the radial direction, parallel to $\vec{E}$ and we see

$$E_r = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{q}{4\pi\epsilon_0 r}\right) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

- Similarly, we could have started with potential and derived the $\vec{E}$ of a charged ring or disk with a simple derivative!
- The geometric interpretation is that the $\vec{E}$ is the slope of the V vs. s graph.

The Geometry of Potential and Field

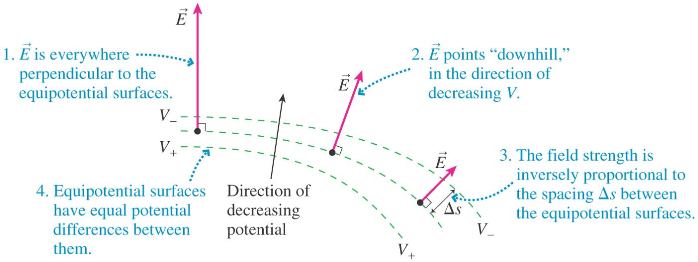


- The figure shows 2 equipotential surfaces, one at positive potential with respect to the other.
- Consider 2 displacements $\Delta\vec{s}_1$ and $\Delta\vec{s}_2$.
- For $\Delta\vec{s}_1$ the change in potential is zero...so the component of $\vec{E}$ along that direction must be zero ($E = -dV/ds$).
- $\Delta\vec{s}_2$ is perpendicular to the surface. There is definitely a potential difference there and

$$E_\perp = -\frac{dV}{ds} \approx -\frac{V_+ - V_-}{\Delta s}$$

- $\vec{E}$ is perpendicular to equipotential surfaces and points "downhill".

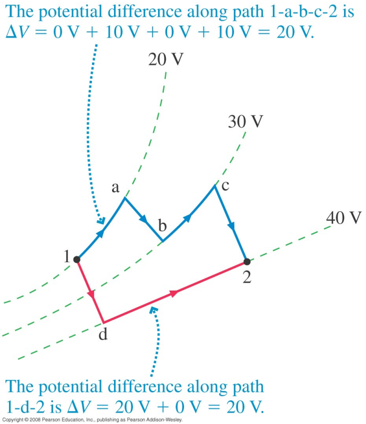
The Geometry of Potential and Field



$$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$

$$\vec{E} = -\nabla V$$

The Geometry of Potential and Field



- The work done moving a charge from point 1 to point 2 is independent of the path.
- Moving from point 1 to 2 on the blue path leads to a potential increase of 20V. Moving from 2 to 1 on the red path leads to a decrease of 20V.
- Kirchhoff's Loop Law says that the potential differences encountered while moving around a loop is zero

$$\Delta V_{loop} = \sum_i (\Delta V)_i = 0$$