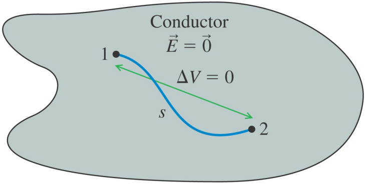
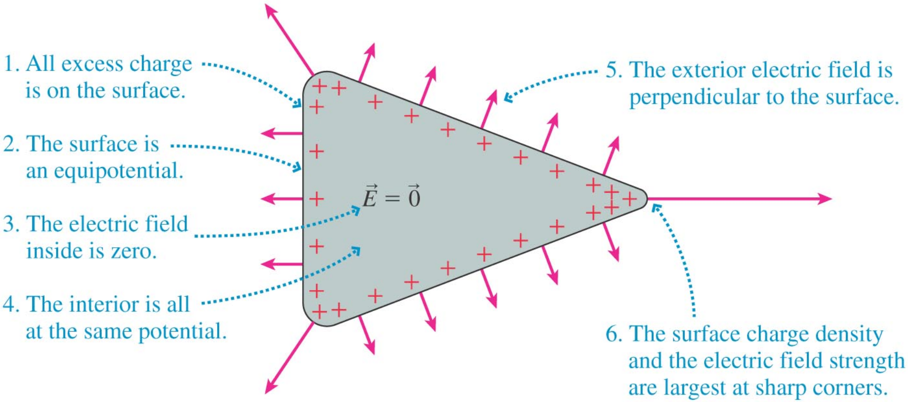
A Conductor in Electrostatic Equilibrium (30.4)

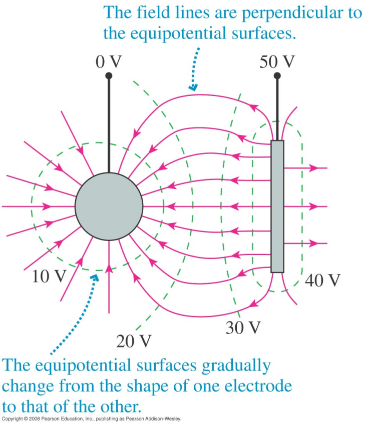


- We have already learned that electric field is zero everywhere inside a conductor.
- That also means that the potential difference between any two points in a conductor is zero.
- In electrostatic equilibrium the entire conductor is at one potential.
- So, there is no $\vec{E}$ inside but there is $\vec{E}$ outside, what happens at the surface??
- The surface is an equipotential surface - $\vec{E}$ must be perpendicular to it!

A Conductor in Electrostatic Equilibrium

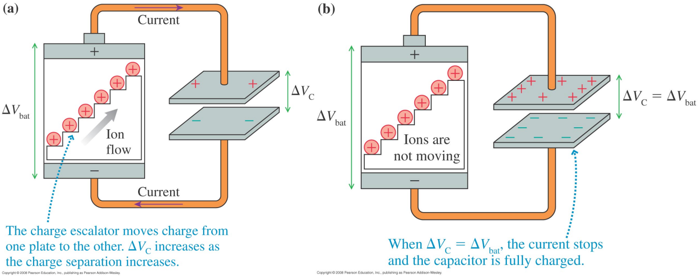


A Conductor in Electrostatic Equilibrium



- The field and potential between two conductors then needs to have a funny shape.
- The field must be perpendicular to each conductor surface, no matter what shapes those conductors have.
- An equipotential surface close to an electrode must roughly match the shape of the electrode.

Capacitance and Capacitors (30.5)



- We have been using capacitors a lot without defining capacitance or describing how to charge-up these devices.
- A battery will create a potential difference across a capacitor which is equal to the potential difference in the battery.

Capacitance and Capacitors

- We know that the potential difference in a capacitor is related to its electric field by $\Delta V_C = Ed$ and the electric field is

$$E = \frac{Q}{\epsilon_0 A}$$

- Combining these gives

$$Q = \frac{\epsilon_0 A}{d} \Delta V_C$$

So, the charge is directly proportional to the potential difference.

- The ratio of charge to potential difference is called capacitance:

$$C \equiv \frac{Q}{\Delta V_C} = \frac{\epsilon_0 A}{d}$$

- The unit of capacitance is the farad (F).

$$1\text{farad} = 1\frac{\text{coulomb}}{\text{volt}} = 1\frac{\text{C}}{\text{V}}$$

Charging a Capacitor (Example 30.6)

Example 30.6 - Charging a Capacitor

The spacing between the plates of a $1\mu F$ capacitor is 0.050mm. (a) What is the surface area of the plates? (b) How much charge is on the plates if this capacitor is attached to a 1.5V battery?

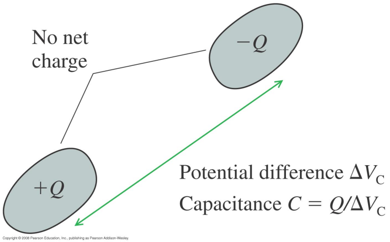
- The area is

$$A = \frac{dC}{\epsilon_0} = 5.65 \text{ m}^2$$

- The charge is

$$Q = C\Delta V_C = 1.5 \times 10^{-6} \text{ C} = 1.5 \ \mu\text{C}$$

Forming a Capacitor (30-22)

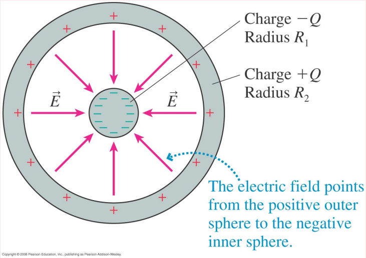


- We have been drawing our capacitors as parallel plates since those are the most useful ones. However, any two electrodes will form a capacitor.
- The capacitance is

$$C = \frac{Q}{\Delta V_C}$$

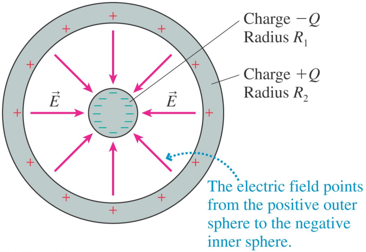
A Spherical Capacitor (Example 30.7)

Example 30.7



A metal sphere of radius R_1 is inside and concentric with a hollow metal sphere of radius R_2. What is the capacitance of this spherical capacitor?

A Spherical Capacitor (Example 30.7)



- The potential difference between the two spheres is

$$\Delta V = V_f - V_i = -\int_{s_i}^{s_f} E_s ds$$

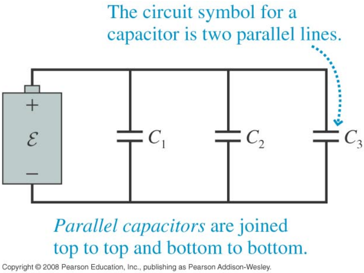
- The electric field contribution from the outer sphere is zero.
- Integrate on a radial line from $s_i = R_1$ to $s_f = R_2$. The field component points inward, so is negative.

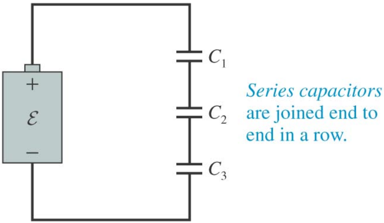
$$\Delta V_C = -\int_{R_1}^{R_2} \left(\frac{-Q}{4\pi\epsilon_0 s^2} \right) ds = \frac{Q}{4\pi\epsilon_0} \int_{R_1}^{R_2} \frac{ds}{s^2} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

- Capacitance is then

$$C = 4\pi\epsilon_0 \left(\frac{1}{R_1} - \frac{1}{R_2} \right)^{-1}$$

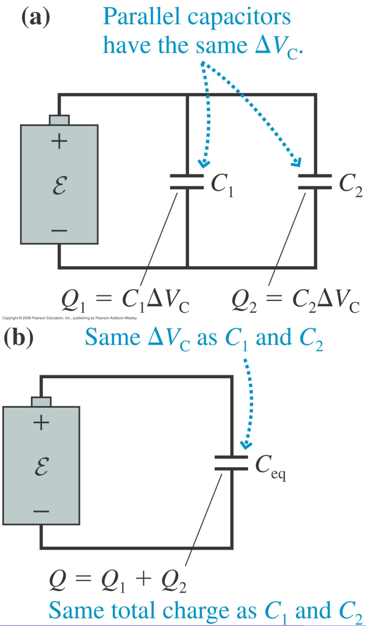
Combinations of Capacitors





- We can join capacitors together in parallel or in series.
- In either case we will learn to replace a system of capacitors with a single equivalent capacitor.

Parallel Capacitors

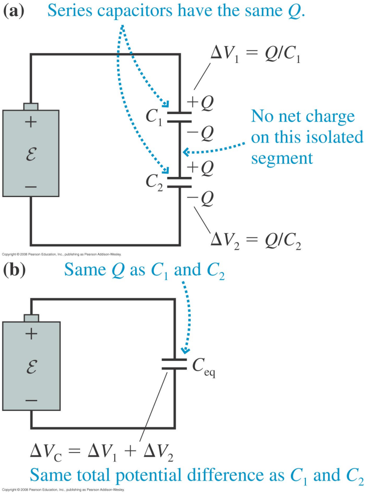


- The two top electrodes are connected by a conducting wire, so form a single conductor in equilibrium.
- The two top electrodes are therefore at the same potential. Two or more capacitors connected in parallel all have the same potential difference between electrodes.
- The battery has to do the work to move $Q = Q_1 + Q_2$ to the top plates. So, the equivalent capacitance is

$$C_{eq} = \frac{Q_1 + Q_2}{\Delta V_C} = C_1 + C_2$$

Just sum the capacitances!

Series Capacitors

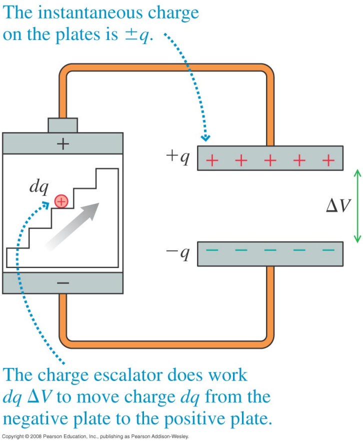


- The battery sees only the top plate of the first capacitor and the bottom plate of the second one. It cannot add or remove charge from the others.
- It will remove the same amount of charge from the bottom of the second capacitor as it adds to the top of the first.
- The potential difference across both capacitors is $\Delta V_C = \Delta V_1 + \Delta V_2$.
- The capacitance is then

$$C = \frac{Q}{\Delta V_C} = \frac{Q}{\Delta V_1 + \Delta V_2}$$

$$\frac{1}{C} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

The Energy Stored in a Capacitor (30.6)



- Charging a capacitor uses energy from the battery. Energy is conserved, therefore it "goes" somewhere.
- As the battery uses energy, the potential energy stored in the capacitor increases

$$dU = dq\Delta V = \frac{qdq}{C}$$

- Integrating over all of the charging time gives

$$U_C = \frac{1}{C}\int_0^Q qdq = \frac{Q^2}{2C} = \frac{C(\Delta V)^2}{2}$$

The Energy Stored in a Capacitor

- The energy stored is proportional to the square of the potential difference - reminds me of a spring $U = \frac{1}{2}k\Delta x^2$
- An important feature of a capacitor is that it can be discharged very quickly (after an arbitrarily long charge). It is a device to store energy in a circuit. (eg. defibrillator, flashbulb)
- What is the energy stored in a 2.0 μF capacitor charged to 5000 V?

$$U_C = \frac{C(\Delta V_C)^2}{2} = 25 \text{ J}$$

- What is the power dissipated if that energy is released in $10\mu s$?

$$P = \frac{\Delta E}{\Delta t} = 2.5 \text{ MW}$$

that's “megawatts”.