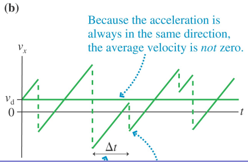
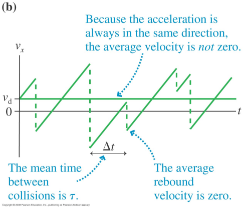
A Model of Conduction

- Suppose an electron just had a collision with an ion and has rebounded with velocity $\vec{v}_0$. The acceleration of the electron between collisions is
$$a_x = \frac{F}{m} = \frac{eE}{m}$$
- This causes the x-component of the electron velocity to increase linearly in time
$$v_x = v_{0x} + a_x \Delta t = v_{0x} + \frac{eE}{m}\Delta t$$
- Each collision transfers kinetic energy, heating the metal.
- This accelerating and colliding leads to the non-zero average velocity v_d



A Model of Conduction



- The average speed at which electrons are pushed along by the electric field is

$$v_d = \frac{e\tau}{m} E$$

  where τ is the mean time between collisions. It depends on the metal's temperature but not on E.
- Substituting into our expression for electron current gives

$$i_e = n_e A v_d = \frac{n_e e \tau A}{m} E$$

Current and Current Density (31.3)

- Our understanding of current at the macroscopic level predates our understanding at the microscopic level.
- So far we have been discussing electron current instead of current. Time to relate the two.
- The traditional definition of current is as the rate of charge flow

$$I \equiv \frac{dQ}{dt}$$

- For a steady current this gives $Q = I\Delta t$
- The unit for current is the ampère: 1 A = 1 C/s
- The current is related to the electron current by

$$I = \frac{Q}{\Delta t} = \frac{eN_e}{\Delta t} = ei_e$$

Current and Current Density

- So, the current and electron current are only different by a scale factor (e)
- However, there is an historical accident which still plagues us. The direction of current is defined to be the direction in which positive charges seem to move.
- In other words, we now know that electrons are carrying the charges and are negative, but all practical circuits are discussed in terms of the flow of positive charge!
- The microscopic understanding does not mesh well with the traditional convention. Nonetheless, we still use it. It makes no difference to practical calculations.

The Current Density in a Wire

- We know the current is

$$I = ei_e = n_e e v_d A$$

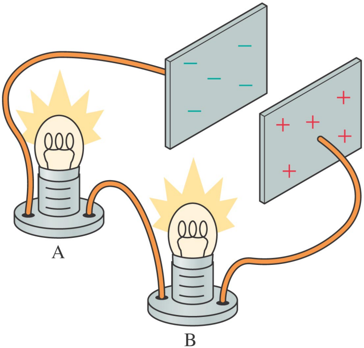
- Some of these factors depend on the wire, others on the electric field. We separate them by introducing current density

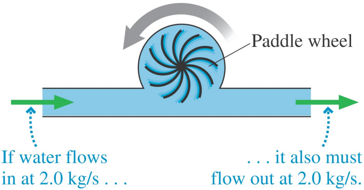
$$J \equiv \frac{I}{A} = n_e e v_d$$

- It can be useful to rearrange this and say that a specific piece of metal shaped into a wire with cross-sectional area A has current

$$I = JA$$

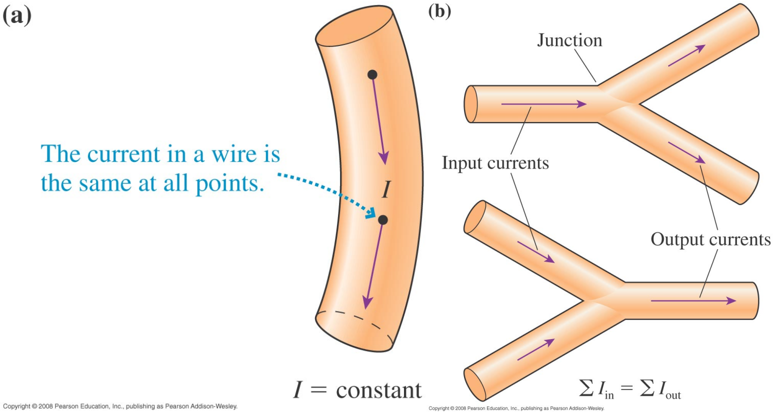
Conservation of Current





- Two identical lightbulbs are placed in series in a circuit, Which one glows brighter?
- Neither, they are equally bright! The first lightbulb does not destory or “use-up” any of the electrons passing through it.
- Further, the electrons do not slow down (see water example).
- The law of current reads: The current is the same at all points on a current-carrying wire.

Conservation of Current



The sum of the currents entering a junction must equal the sum of those leaving is Kirchhoff's junction law.

Conductivity and Resistivity (31.4)

- The current density can be expressed as

$$J = n_e e v_d = n_e e \left(\frac{e\tau E}{m} \right) = \frac{n_e e^2 \tau}{m} E$$

- The coefficent in front of the electric field depends only on the conducting material and is referred to as the conductivity

$$\sigma = \frac{n_e e^2 \tau}{m}$$

  All pieces of copper have the same conductivity (at same T).
- So, the current density becomes simply

$$J = \sigma E$$

  Current density depends only on the material and the electric field.
- It is also useful to define resistivity

$$\rho = \frac{1}{\sigma} = \frac{m}{n_e e^2 \tau}$$

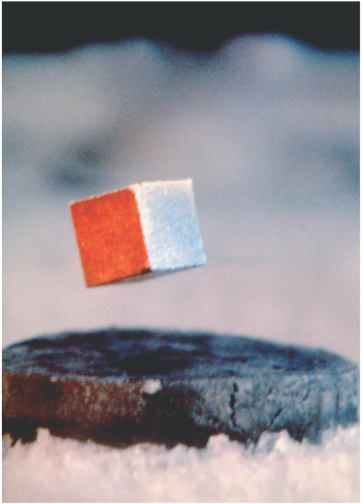
Conductivity and Resistivity

TABLE 31.2 Resistivity and conductivity of conducting materials

Material	Resistivity (Ω m)	Conductivity (Ω^{-1} m^{-1})
Aluminum	2.8×10^{-8}	3.5×10^{7}
Copper	1.7×10^{-8}	6.0×10^{7}
Gold	2.4×10^{-8}	4.1×10^{7}
Iron	9.7×10^{-8}	1.0×10^{7}
Silver	1.6×10^{-8}	6.2×10^{7}
Tungsten	5.6×10^{-8}	1.8×10^{7}
Nichrome*	1.5×10^{-6}	6.7×10^{5}
Carbon	3.5×10^{-5}	2.9×10^{4}

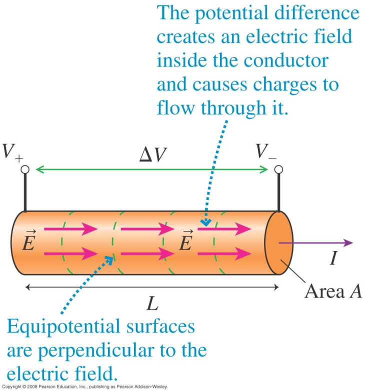
*Nickel-chromium alloy used for heating wires.

Superconductivity



- A very interesting feature of metals is that their resistivity gradually drops with temperature.
- However, at very low temperatures there is a sudden transition to **zero** resistivity! This is known as superconductivity.
- Electrons in a superconductor move with friction. Superconducting wires can sustain enormous currents.
- The discovery of “high temperature” superconductors has had a big impact on their usefulness.

Resistance and Ohm's Law (31.5)



- We know that current is related to electric field and that electric field is related to potential.
- So, it is natural that current and potential are also related.
- The field strength inside a wire is

$$E = -\frac{dV}{ds} = \frac{\Delta V}{\Delta s} = \frac{\Delta V}{L}$$

- The current in terms of field is

$$I = JA = A\sigma E = \frac{A}{\rho}E$$

Resistance and Ohm's Law

- In terms of potential difference:

$$I = \frac{A}{\rho L}\Delta V$$

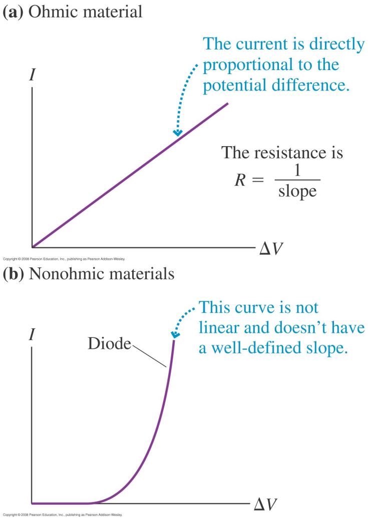
  tells us that current depends on wire properties and the potential difference.
- We encapsulate the properties of the wire in the resistance

$$R = \frac{\rho L}{A}$$

- The resistivity is about the material, the resistance is about the specific piece of conductor (ie. including its length).
- The unit of resistance is the Ohm $1\Omega \equiv 1\frac{V}{A}$
- We can now write Ohm's Law as

$$I = \frac{\Delta V}{R}$$

More on Ohm’s Law



- Ohm’s Law only applies to Ohmic materials.
- Wires are metals with small resistivity. Ideally they have $R = 0\Omega$ and $\Delta V = 0$
- Resistors are poor conductors with R ranging from 10 Ω to 1M Ω. They are used to control the current in a circuit.
- Insulators have very high resistance $R = \infty$. There is no current, even if there is a potential difference across it (eg. glass, plastic, air).