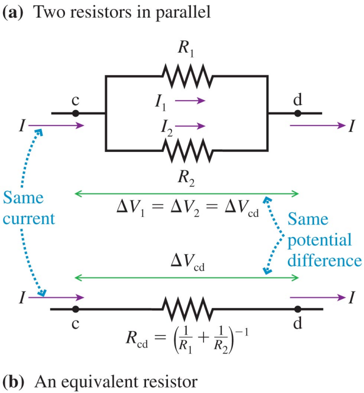
Parallel Resistors (32.6)



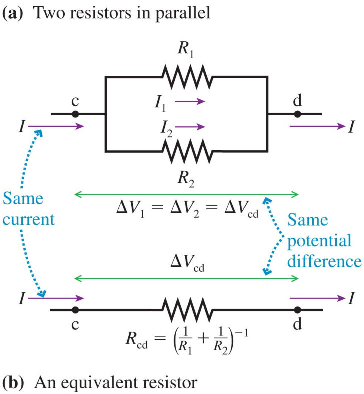
- Resistors connected at both ends are called parallel resistors
- The important thing to note is that: the two left ends of the resistors are at the same potential. Also, the two right ends are at the same potential. Therefore, the ΔV for each resistor is the same!
- Kirchhoff's Junction Law means that

$$I = I_1 + I_2$$

- Using Ohm's Law

$$I = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} = \frac{\Delta V}{\Delta R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Parallel Resistors (32.6)



- Starting with

$$I = \Delta V \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

- and using Ohm's Law again:

$$R = \frac{\Delta V}{I} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1}$$

- So, for many parallel resistors

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \right)^{-1}$$

Example 32.9

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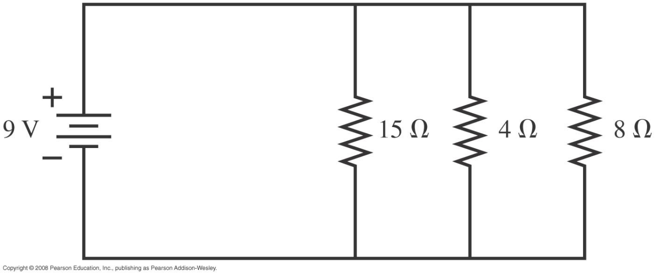
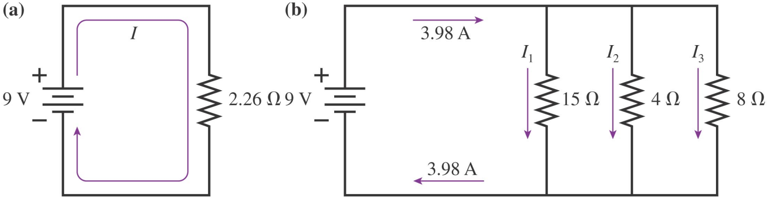


Three resistors are connected in parallel to a 9V battery. Find the potential difference across and the current through each resistor. (Note: assume an ideal battery and ideal wires)

Example 32.9



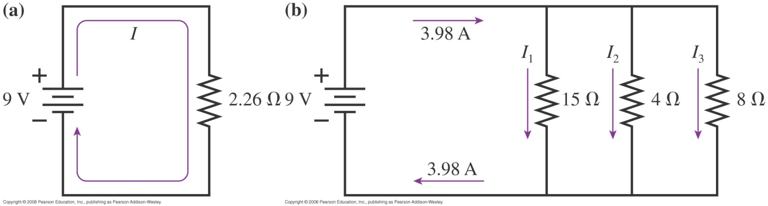
- We replace the 3 parallel resistors with a single equivalent resistor:

$$R_{eq} = \left(\frac{1}{15\Omega} + \frac{1}{4\Omega} + \frac{1}{8\Omega}\right)^{-1} = (0.4417\Omega^{-1})^{-1} = 2.26\Omega$$

- The equivalent resistance is LESS than any one resistor!
- We can then determine the current across the equivalent circuit

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{9V}{2.26\Omega} = 3.98A$$

Example 32.9

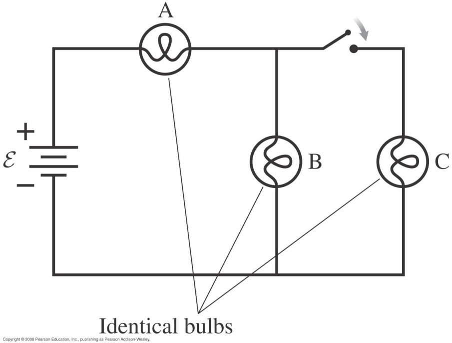


- Now, we know that the current divides at each junction (not equally!!) and that the potential is the same across each resistor

$$I_1 = \frac{9V}{15\Omega} = 0.6A, \quad I_2 = \frac{9V}{4\Omega} = 2.25A, \quad I_3 = \frac{9V}{8\Omega} = 1.13A$$

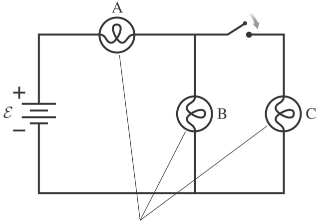
- The sum of the currents is 3.98A as expected.

Lightbulb Puzzle



What happens to the brightness of A and B when the switch is closed? How does the brightness of C compare to that of A and B?

Lightbulb Puzzle



- When the switch is open, A and B are in series with $R_{eq} = 2R$ and current is

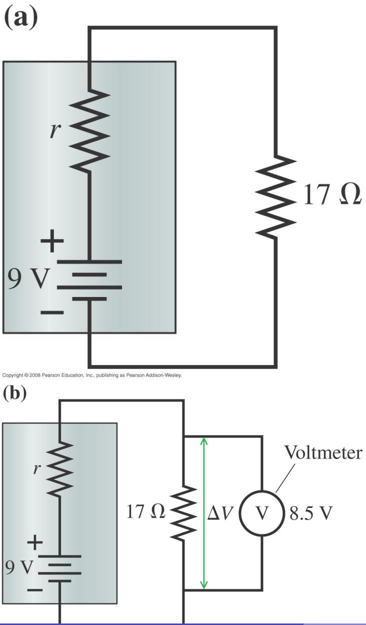
$$I_{before} = \frac{\mathcal{E}}{2R}$$

- Closing the switch puts B and C in parallel with $R_{eq} = R/2$. They are in series with A giving total resistance of $\frac{3}{2}R$

$$I_{after} = \frac{\mathcal{E}}{3R/2} = \frac{2}{3}\frac{\mathcal{E}}{R} > I_{before}$$

- So, A gets brighter, B gets dimmer ($\mathcal{E}/3R$) and C is equal to B.

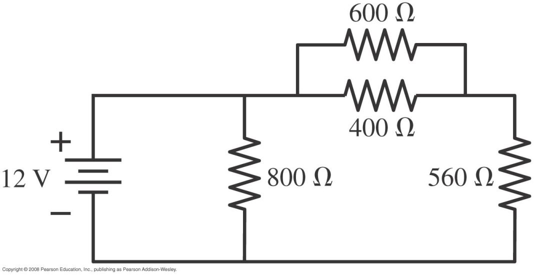
Voltmeters



- A voltmeter measures the potential difference across an element of a circuit.
- It must be connected in parallel so that it has the same ΔV as the element.
- It should have nearly infinite resistance.

Resistor Circuits (32.7)

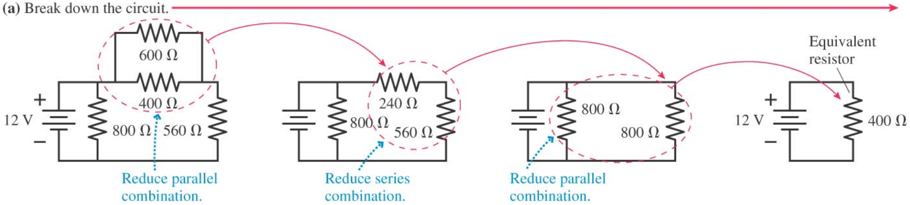
Example 32.11



Find the current through and the potential difference across each of the four resistors in the circuit above.

- We need to develop a strategy for complex circuits.
- We will break the circuit down into pieces and make an equivalent circuit, then build it back up piece by piece.

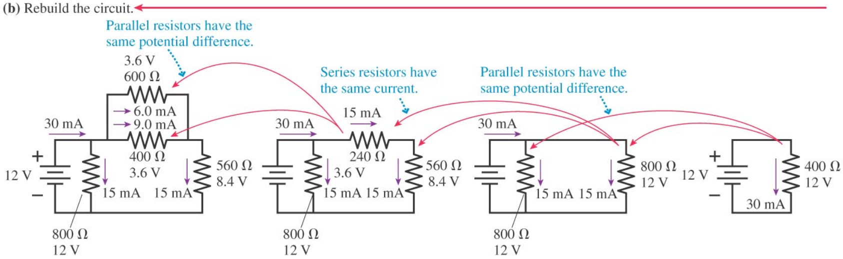
Example 32.11 - Break it Down



- Add the 600 and 400 Ohm resistors in parallel = 240 Ohm
- Add 240 and 560 in series = 800 Ohm
- Add 800 and 800 Ohm in parallel = 400 Ohm
- The current is

$$I = \frac{\mathcal{E}}{R} = \frac{12V}{400\Omega} = 30mA$$

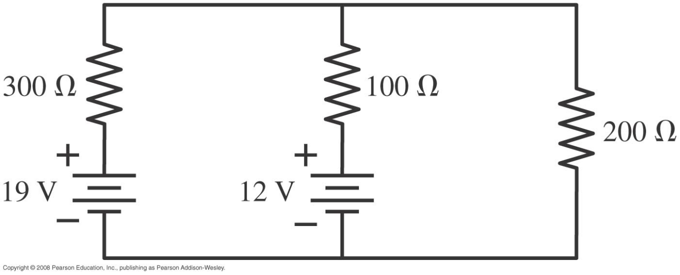
Example 32.11 - Build it Up



- Two parallel 800Ω resistors split the current equally.
- Splitting the 800Ω resistor leaves the same current in two series resistors (15mA) and known resistances, so voltages can be calculated.
- Parallel resistors have the same potential difference and known resistances, so current can be calculated.

Example 32.12 - A 2-Loop Circuit

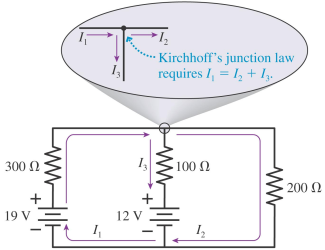
Example 32.12



Find the current and the potential difference across the 100Ω resistor.

None of these resistors are in series or parallel, you cannot simplify it.
It is a 2-loop circuit.

Example 32.12 - A 2-Loop Circuit



- Apply the loop law twice. Choose a direction for the center wire.
- For the left loop

$$19V - (300\Omega)I_1 - (100\Omega)I_3 - 12V = 0$$

- For the right loop

$$12V + (100\Omega)I_3 - (200\Omega)I_2 = 0$$

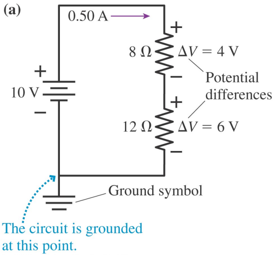
- Using $I_3 = I_1 - I_2$ gives 2 equations and 2 unknowns

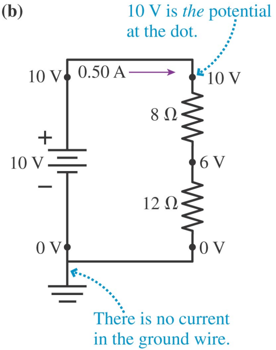
$$\begin{aligned} 400I_1 - 100I_2 &= 7 \\ -100I_1 + 300I_2 &= 12 \end{aligned}$$

and we can solve for I_1, I_2 and I_3

I1=0.03, I2=0.05, I3 = 0.02 A

Grounding Circuits (32.8)





- We often hear about circuits being grounded. Why?
- Within a single circuit you only care about relative potential differences. However, if two circuits are connected it suddenly becomes necessary to worry about a common reference point.
- A circuit connected to the earth by a single wire (not in a loop) is said to be grounded.
- The ground wire carries no current, so does not effect the circuit. However, it is now possible to specify absolute voltages, not just differences.
- Grounding is an important saftey feature, keep the case surrounding a circuit at $V = 0V$ at all times.