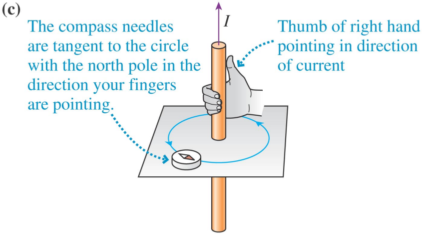
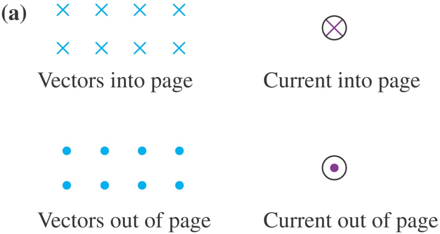
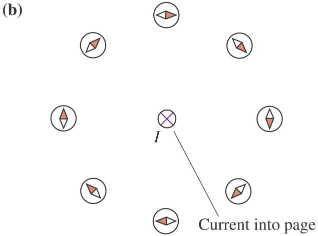
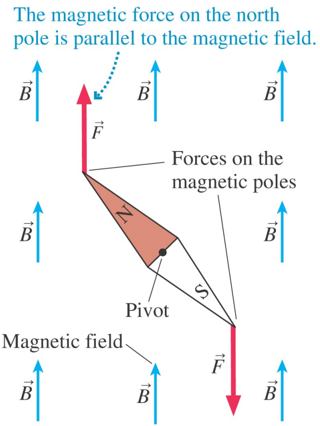
The Direction of Magnetic Field





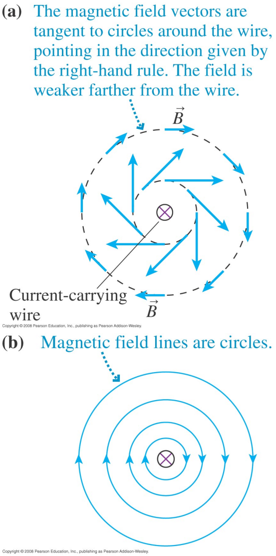


The Magnetic Field



- We introduced electric field to explain-away long-range electric forces. Charges create a field throughout space with which other charges interact.
- Properties of a magnetic field:
  1. A magnetic field is created at all points in space around a current-carrying wire.
  2. Like $\vec{E}$, $\vec{B}$ is a vector field
  3. $\vec{B}$ exerts forces on magnetic poles. North poles point along $\vec{B}$.
- So, a compass needle experiences a torque in a magnetic field until it is aligned with that field.

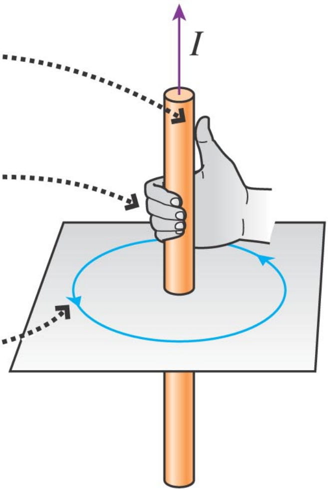
The Magnetic Field



- We can represent the field by drawing field vectors. These show the direction a magnet would point at each spot. The length is the strength (see how it drops with distance).
- Another representation is with magnetic field lines. The field direction is tangent to a field line. The more close-packed the field lines, the stronger the field.
- Given a current in a wire, use the right-hand rule to get the direction.

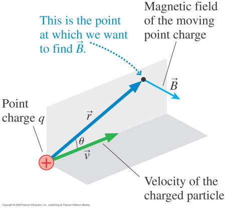
The Right-Hand Rule

1. Point your *right* thumb in the direction of the current.
2. Curl your fingers around the wire to indicate a circle.
3. Your fingers point in the direction of the magnetic field lines around the wire.



The Source of Magnetic Field: Moving Charges (33.3)

- Since current seems to lead to magnetic field. Let's assume that moving charges are the source of magnetic field.
- We need the equivalent of Coulomb's Law. When a charge is moving, how "big" is the magnetic field at some distance r away?



- The Biot-Savart Law is

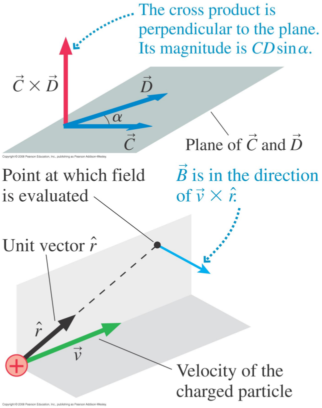
$$|\vec{B}_{point\ \ charge}| = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$$

  The direction of the vector is given by the right-hand rule. μ_0 is the permeability constant.
- The unit of magnetic field strength is the Tesla.

Superposition

- Like electric fields, magnetic fields obey the principle of superposition. If there are n moving point charges the net field is given by the **vector** sum:

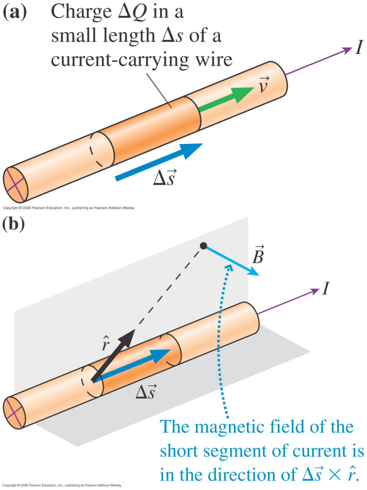
$$\vec{B}_{tot} = \vec{B}_1 + \vec{B}_2 + \cdots + \vec{B}_n$$



- If we want our Biot-Savart Law to have direction as well as magnitude we need again to introduce unit vector $\hat{r}$.
- We also need a cross product:

$$\vec{B}_{point\ charge} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

- This agrees completely with our previous Biot-Savart definition but now has the direction built-in!



- Rather than a single point charge, let's look at the magnetic field from a current.
- Divide a current-carrying wire into segments of length $\Delta\vec{s}$ containing charge ΔQ moving at velocity $\vec{v}$.
- The magnetic field created by this charge is proportional to $(\Delta Q)\vec{v}$:

$$(\Delta Q)\vec{v} = \Delta Q \frac{\Delta\vec{s}}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta\vec{s} = I\Delta\vec{s}$$

- The Biot-Savart Law for a short segment is:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{s}\times\hat{r}}{r^2}$$

Example 33.3: $\vec{B}$ of a Long Wire

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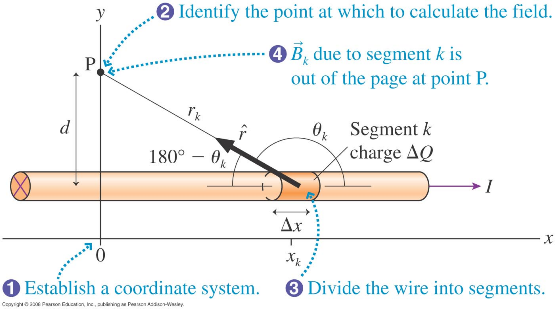
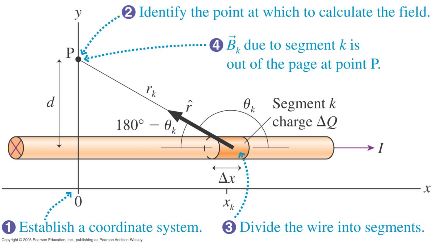


A long straight wire carries current I in the positive x direction. Find the magnetic field at a point which is a distance d from the wire.

We know the direction of the field already by the right-hand rule. The field points along the z axis only.

Example 33.3: $\vec{B}$ of a Long Wire



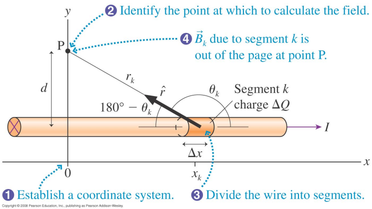
- We can use Biot-Savart to find the $(B_k)_z$, noting that the cross product $\Delta\vec{s} \times \hat{r} = (\Delta\vec{s})(1)(\sin\theta_k)$:

$$(B_k)_z = \frac{\mu_0}{4\pi}\frac{I\Delta x \sin\theta_k}{r_k^2} = \frac{\mu_0}{4\pi}\frac{I\sin\theta_k}{x_k^2 + d^2}\Delta x$$

- Also note that $\sin\theta_k$ is:

$$\sin(\theta_k) = \sin(180 - \theta_k) = \frac{d}{r_k} = \frac{d}{\sqrt{x_k^2 + d^2}}$$

Example 33.3: $\vec{B}$ of a Long Wire



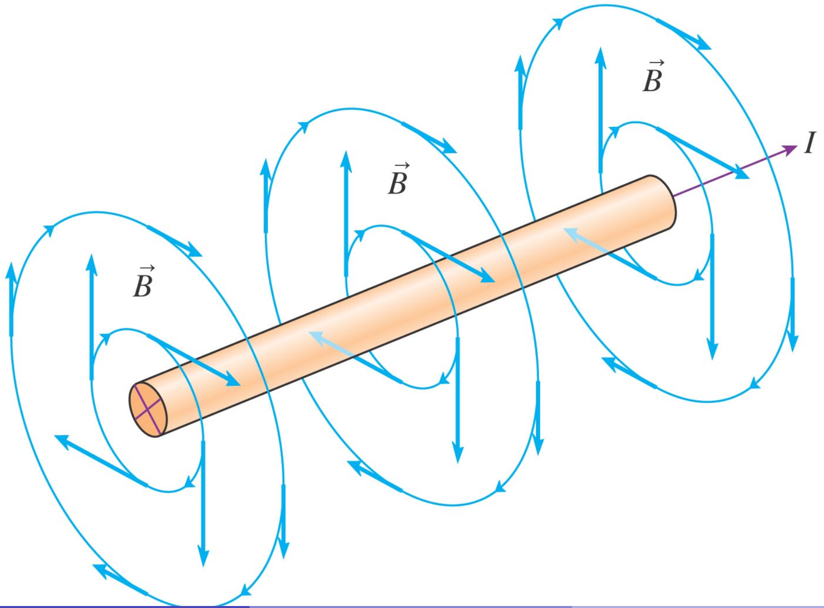
- Substituting these back into Biot-Savart

$$B = \frac{\mu_0 Id}{4\pi} \sum_k \frac{\Delta x}{(x_k^2 + d^2)^{3/2}}$$

$$B = \frac{\mu_0 Id}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{(x^2 + d^2)^{3/2}}$$

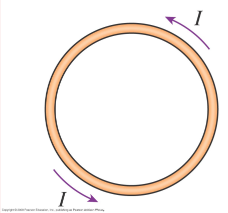
$$B = \frac{\mu_0 Id}{4\pi} \frac{x}{(x^2 + d^2)^{1/2}} \Bigg|_{-\infty}^{\infty} = \frac{\mu_0}{2\pi} \frac{I}{d}$$

Example 33.3: $\vec{B}$ of a Long Wire



Example 33.5: $\vec{B}$ of a Current Loop

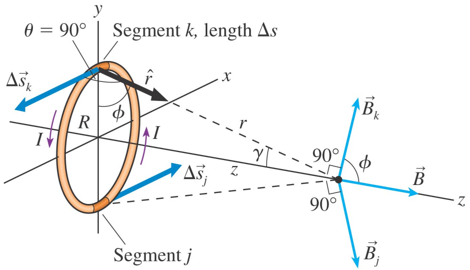
(b) An ideal current loop



A circular loop of wire of radius R carries a current I. Find the magnitude of the field of the current loop at distance z on the axis of the loop

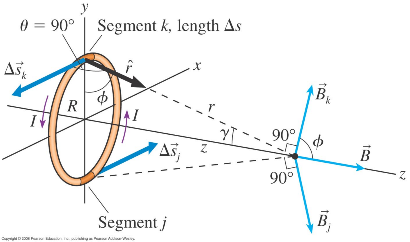
Hey, back to our favourite type of example - a ring!

Example 33.5: $\vec{B}$ of a Current Loop



- Assume CCW current and the loop in the $x - y$ plane. Look at the field from one small segment of loop .
- Note that the segment at the top (k) has opposite current flow from the segment at the bottom (j). The direction of the field is given by $\Delta\vec{s} \times \hat{r}$.
- The y components of k and j cancel.
- For every segment on the ring we can find a partner on the opposite side to cancel the y and x components.

Example 33.5: $\vec{B}$ of a Current Loop



- We will use the Biot-Savart Law to get the z component. Note that $\Delta\vec{s}_k \times \hat{r} = \Delta s(1)\sin 90 = \Delta s$

$$(B_k)_z = \frac{\mu_0}{4\pi}\frac{I\Delta s}{r^2}\cos\phi$$

- From triangles we know that

$$\cos\phi = \frac{R}{r}, \quad r = (z^2 + R^2)^{1/2}$$

Example 33.5: $\vec{B}$ of a Current Loop

- This gives the magnetic field for one segment as:

$$(B_k)_z = \frac{\mu_0}{4\pi}\frac{IR}{(Z^2+R^2)^{3/2}}\Delta s$$

- Rings don't need integrals!

$$\begin{aligned}
B_{loop} &= \frac{\mu_0 IR}{4\pi(z^2+R^2)^{3/2}}\sum_k \Delta s \\
B_{loop} &= \frac{\mu_0 IR}{4\pi(z^2+R^2)^{3/2}} 2\pi R \\
B_{loop} &= \frac{\mu_0}{2}\frac{IR^2}{(z^2+R^2)^{3/2}}
\end{aligned}$$

- We have many devices containing a coil of N loops. For $z = 0$:

$$B_{\text{centre of coil}} = \frac{\mu_0}{2}\frac{NI}{R}$$