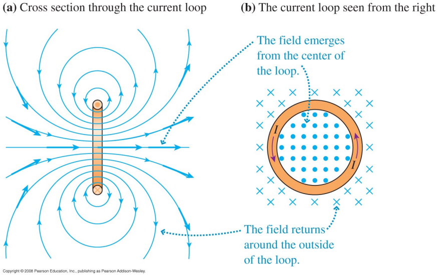
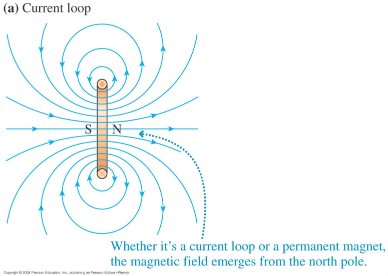
Magnetic Dipoles (33.5)

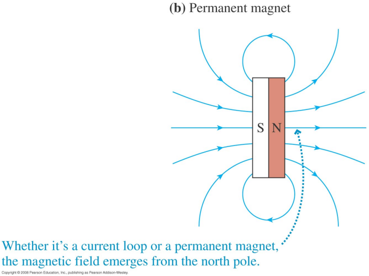


- To determine the magnetic field direction in a loop either
  1. Point your right thumb in the direction of the current and curl your fingers around the loop. Your fingers point in the direction the field leaves the loop.
  2. Curl the fingers of your right hand around the loop in the direction of the current. Your thumb is then pointing in the direction in which the field leaves the loop.

Magnetic Dipoles

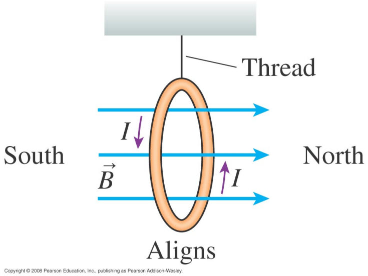
- A current loop has two distinct sides, rather like a magnetic dipole.
- In fact, a current loop **is** a magnetic dipole! It is an electromagnet

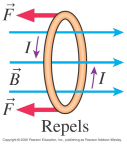


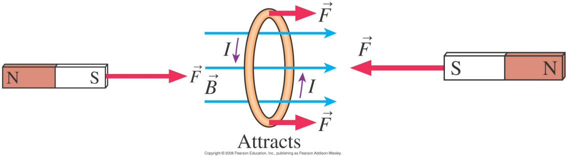


- The magnetic field emerges from the north face of each.

Magnetic Dipoles







Magnetic Dipole Moment

- The on-axis magnetic field of a current loop is:

$$B_{loop} = \frac{\mu_0}{2} \frac{IR^2}{(z^2 + R^2)^{3/2}}$$

- Seen from far away ($z \gg R$) we have:

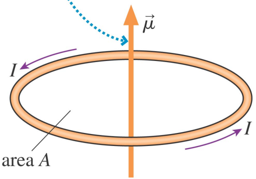
$$B_{loop} \approx \frac{\mu_0}{2} \frac{IR^2}{z^3} = \frac{\mu_0}{4\pi} \frac{2(\pi R^2)I}{z^3} = \frac{\mu_0}{4\pi} \frac{2AI}{z^3}$$

  where $A = \pi R^2$ is the area of the loop.
- This actually works even if the loop is not circular (of course A changes).
- We define the magnetic dipole moment of a current loop enclosing area A to be

$$\vec{\mu} = AI, (direction\ from\ right\ hand\ rule)$$

Magnetic Dipole Moment

The magnetic dipole moment is perpendicular to the loop, in the direction of the right-hand rule. The magnitude of $\vec{\mu}$ is AI.



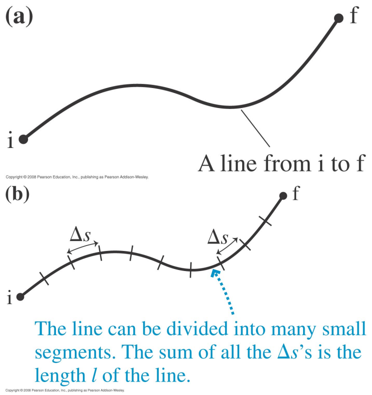
On axis field is:

$$\vec{B}_{dipole} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}$$

Ampere's Law and Solenoids (33.6)

- We could use the Biot-Savart Law to calculate any magnetic field we want just as you can use Coulomb's Law to calculate any electric field through superimposing a large number of charges.
- But that would be hard.
- In the case of electric field we were able to solve problems with complicated shapes by noticing the symmetries of the problem and applying Gauss' Law.
- We can do something similar for magnetic fields using Ampere's Law

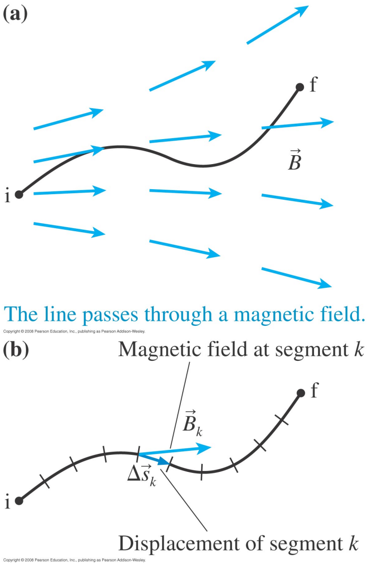
Line Integrals



- To apply Ampere's Law we will need to do some line integrals
- Imagine that, instead of integrating along the x-axis, you want to integrate along some arbitrary line. Maybe the line isn't even straight!
- Take the line on the left and divide it into little segments, then sum over the segments

$$\ell = \sum_k \Delta s_k \to \int_i^f ds$$

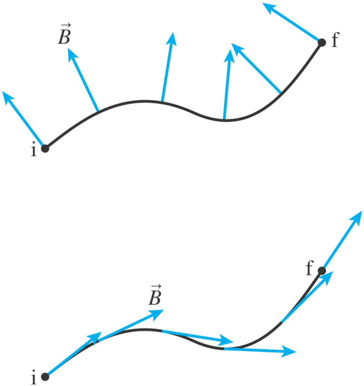
Line Integrals



- Imagine we have a magnetic field passing through the line.
- We could take each segment of the line and calculate the dot product of $\vec{B}$ with a vector along the segment, then sum them up:

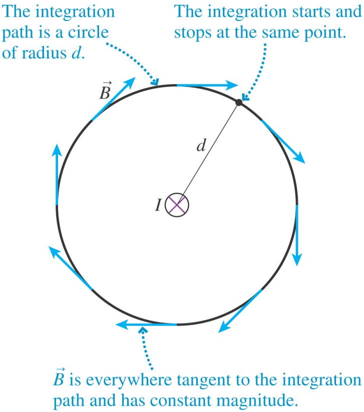
$$\sum_k B_k \cdot \Delta s_k \rightarrow \int_i^f \vec{B} \cdot d\vec{s}$$

Line Integrals



- As in Gauss' Law, we confine outselves to 2 simple cases:
  1. $\vec{B}$ is everywhere perpendicular to the line
  $$\int_i^f \vec{B} \cdot d\vec{s} = 0$$
  2. $\vec{B}$ is everywhere tangent to the line
  $$\int_i^f \vec{B} \cdot d\vec{s} = B\ell$$

Ampere's Law

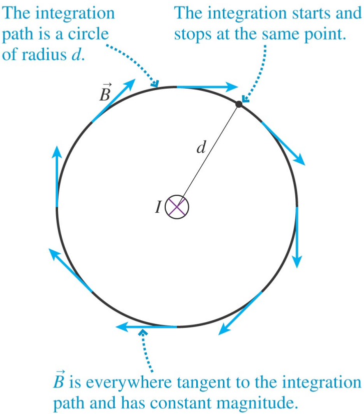


- If we draw a circle of radius d around a wire carrying current I, we know the magnetic field will be tangent to the circle at every point.
- We also know that the magnetic field will have the same magnitude everywhere around the circle.
- So, how about we integrate around that circle?

$$\oint \vec{B} \cdot d\vec{s}$$

(note it is a closed curve)

Ampere's Law



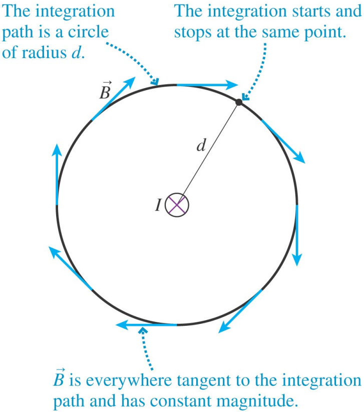
- This is one of those easy cases

$$\oint \vec{B} \cdot d\vec{s} = B\ell = B(2\pi d)$$

- We know B, it is

$$B = \frac{\mu_0 I}{2\pi d}$$

Ampere's Law

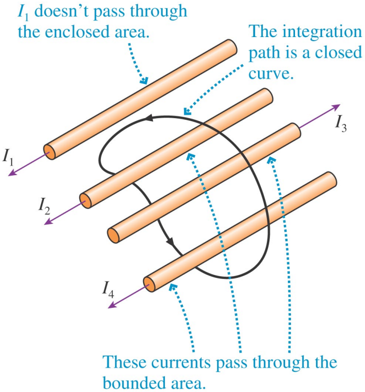


- Substituting $\vec{B}$ into the expression gives Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

- Notice it does not depend on the radius of the circle! (just like Gauss' law and flux)
- We could also show that
  - it does not depend on the shape of the curve
  - it does not depend on where the current is within the curve
  - it depends only on the amount of current flowing inside the curve

Ampere's Law

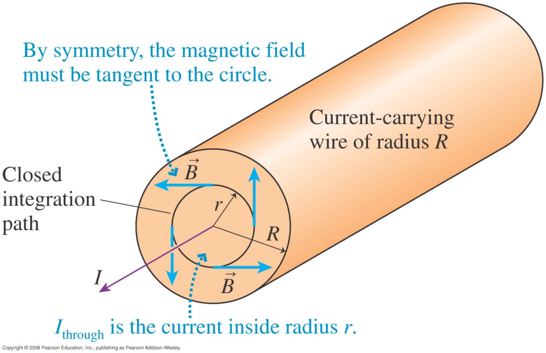


- We do need to understand if the current is positive or negative.
- Place the fingers of your right hand along the direction of the integral. The direction in which your thumb is pointing is positive, the opposite is negative.
- So, in the figure on the left, the total current through the curve is

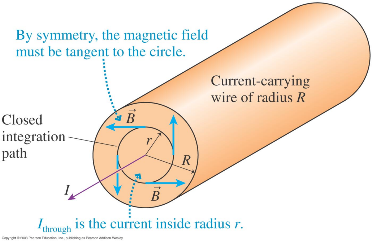
$$I = I_2 - I_3 + I_4$$

Example 33.8

A wire of radius R carries current I. Find the magnetic field inside the wire at distance $r < R$ from the axis.



Example 33.8



- This wire has cylindrical symmetry and the magnetic field is tangent to circles concentric with the wire.
- Assuming the current density is uniform across the wire, the current through the circle is

$$I_{through} = JA_{circle} = \pi r^2 J$$

where J is current density and is given by

$$J = \frac{I}{A} = \frac{I}{\pi R^2}$$

Example 33.8

- The current through a circle of radius r is then

$$I_{through} = \frac{r^2}{R^2} I$$

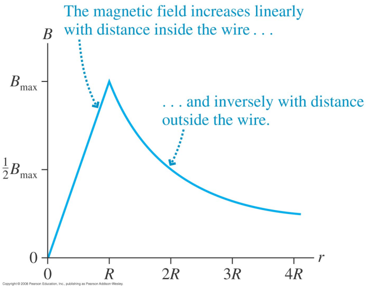
- Integrating around the circle gives

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{through} = \frac{\mu_0 r^2}{R^2} I$$

- $\vec{B}$ is tangent to the circle so

$$\oint \vec{B} \cdot d\vec{s} = B\ell = 2\pi r B$$

Example 33.8

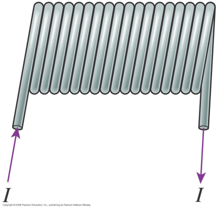


- Substituting into Ampere's Law gives

$$2\pi r B = \frac{\mu_0 r^2}{R^2} I$$
$$B = \frac{\mu_0 I}{2\pi R^2} r$$

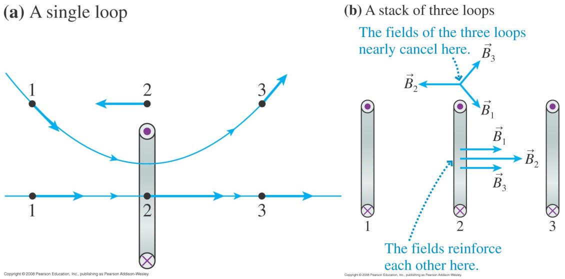
- The magnetic field strength increases linearly with distance until the outer edge of the wire, then follows the Biot-Savart Law.

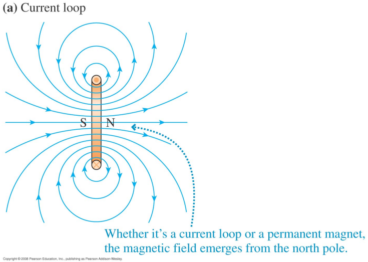
The Magnetic Field of a Solenoid



- When studying electric fields we talked a lot about parallel plate capacitors because they created a nice uniform field.
- The equivalent for magnetic fields is a solenoid.
- A solenoid is a helical coil of wire with the same current I passing through each of the loops. Each loop is often referred to as a turn.
- Of course, we have already studied a single loop of wire and know that superposition works for $\vec{B}$, so...

The Magnetic Field of a Solenoid





- The superposition of fields inside the loop is reinforcing and gives a field roughly parrallel to the axis.
- The superposition outside the loop is cancelling.
- An ideal solenoid has a strong uniform field inside and no field outside!