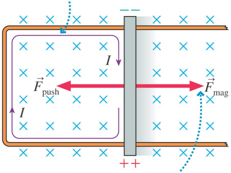
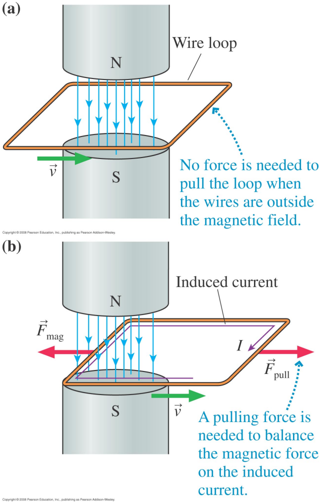
# A Generator!





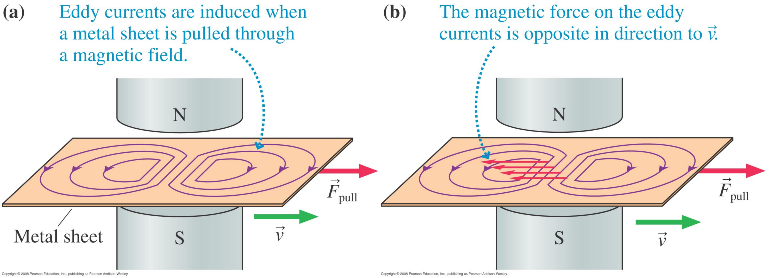
- Pulling or pushing a wire through a magnetic field creates a motional EMF in the wire and a current $I = \mathcal{E}/R$ in the circuit.
- To keep the wire moving you must supply a force to overcome the newly creating magnetic field, this does work on the circuit.
- The work done by pulling or pushing is exactly balanced by the energy dissipated by the current produced.
- We have a generator!

# Eddy Currents



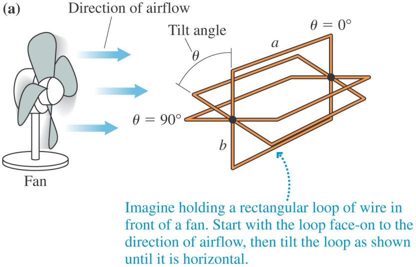

- In the top left figure there is no magnetic force on the wire as no magnetic field goes through the copper.
- In the bottom left figure an induced current is created by dragging the copper with velocity $\vec{v}$ through the field $\vec{B}$.
- The induced current then generates a magnetic force which opposes the motion. So, to get the wire out of the field, you may have to pull pretty hard.

# Eddy Currents




- If you drag a sheet of metal through a magnet then a current is induced but there is no wire to define their path.
- Whirlpools of current form called eddy currents.
- Again, a magnetic force is produced which opposes the motion. So, it is difficult to push a sheet of metal through a magnetic field.
- This is undesirable if you wish to pull some metal quickly through a field but very desirable if you want to use magnetic braking.

# Magnetic Flux (34.3)

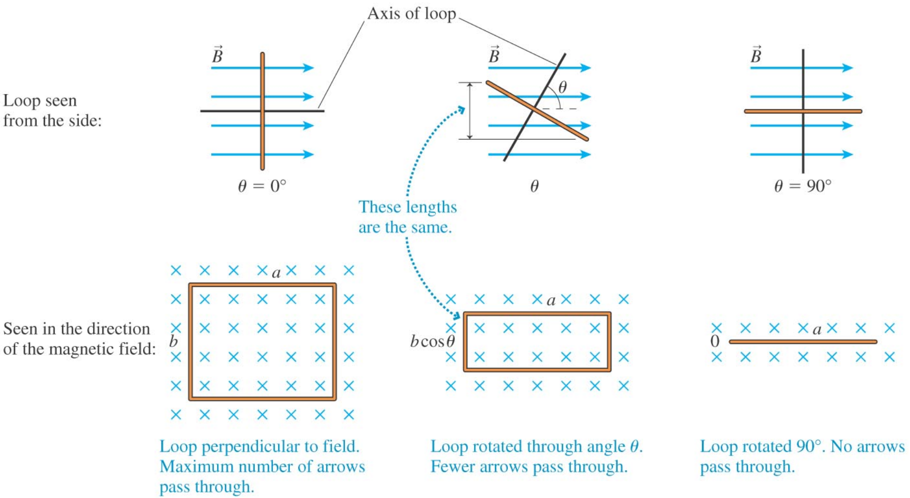





- We have already seen the concept of flux with electric fields and Gauss' Law.
- It is also possible to define flux of a magnetic field in the same way.
- Again we picture magnetic field lines like airflow through the loop of current. The effective area of the loop is

$$A_{eff} = ab\cos\theta = A\cos\theta$$

So, no air flows through at 90° and maximum flow is at 0°.


Axis of loop
$\vec{B}$
Loop seen from the side:
$\theta = 0°$
$\theta$
$\theta = 90°$
These lengths are the same.
Seen in the direction of the magnetic field:
a
b
$b\cos\theta$
0
Loop perpendicular to field. Maximum number of arrows pass through.
Loop rotated through angle $\theta$. Fewer arrows pass through.
Loop rotated 90°. No arrows pass through.

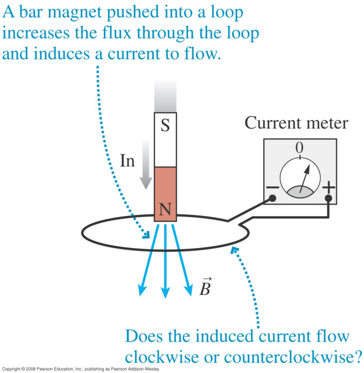
# Magnetic Flux (34.3)

- So, the magnetic flux can be defined mathematically in a very similar way to electric flux

$$\Phi_m = AB\cos\theta$$

- Now, if we define an area vector $\vec{A}$ perpendicular to the loop and of magnitude $A$, we can write this as

$$\Phi_m = \vec{A} \cdot \vec{B}$$



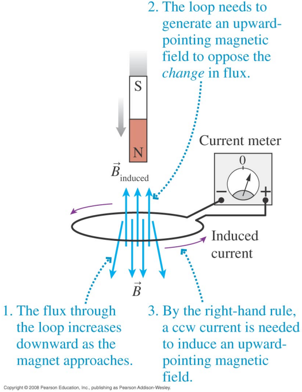



- Pushing a bar magnet into a loop induces a current in the loop.
- Pulling it back out induces a current in the opposite direction.

**Lenz's Law**

There is an induced current in a closed conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the **change** in flux.

# Lenz's Law


2. The loop needs to generate an upward-pointing magnetic field to oppose the *change* in flux.
S
N
$\vec{B}_{\text{induced}}$
Current meter
0
−
+
Induced current
$\vec{B}$
1. The flux through the loop increases downward as the magnet approaches.
3. By the right-hand rule, a ccw current is needed to induce an upward-pointing magnetic field.
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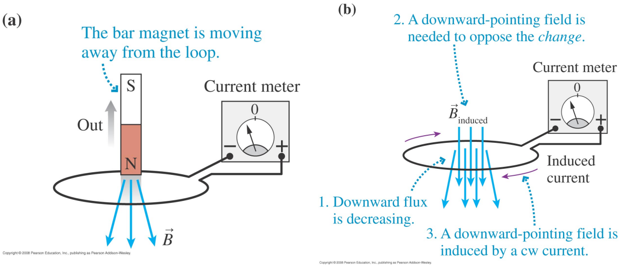
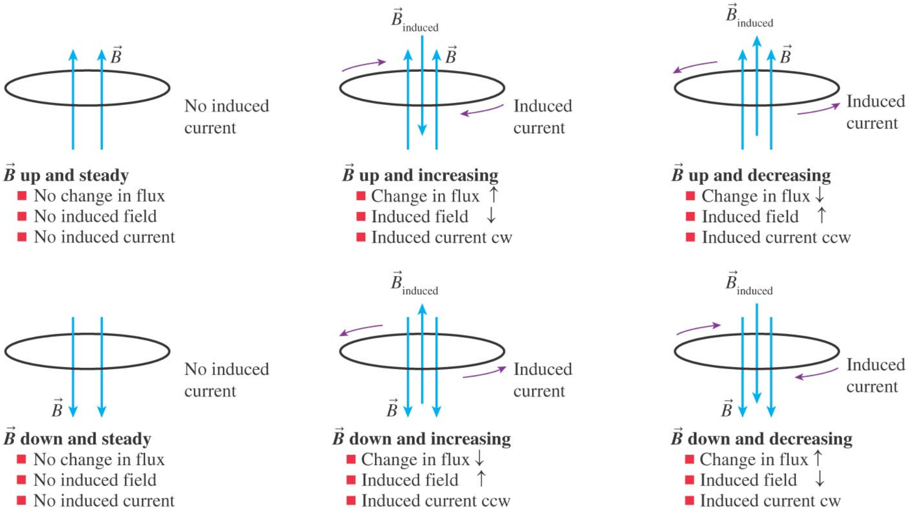

(a)
The bar magnet is moving away from the loop.
S
N
Out
Current meter
0
−
+
$\vec{B}$
(b)
2. A downward-pointing field is needed to oppose the *change*.
Current meter
0
−
+
$\vec{B}_{\text{induced}}$
Induced current
1. Downward flux is decreasing.
3. A downward-pointing field is induced by a cw current.

# Lenz's Law



$\vec{B}$
No induced current
$\vec{B}$ up and steady
No change in flux
No induced field
No induced current
$\vec{B}_{induced}$
$\vec{B}$
Induced current
$\vec{B}$ up and increasing
Change in flux ↑
Induced field ↓
Induced current cw
$\vec{B}_{induced}$
$\vec{B}$
Induced current
$\vec{B}$ up and decreasing
Change in flux ↓
Induced field ↑
Induced current ccw
$\vec{B}$
No induced current
$\vec{B}$ down and steady
No change in flux
No induced field
No induced current
$\vec{B}_{induced}$
$\vec{B}$
Induced current
$\vec{B}$ down and increasing
Change in flux ↓
Induced field ↑
Induced current ccw
$\vec{B}_{induced}$
$\vec{B}$
Induced current
$\vec{B}$ down and decreasing
Change in flux ↑
Induced field ↓
Induced current cw

# Faraday's Law (34.5)

- We have already talked about Faraday's discovery and we have described Faraday's Law qualitatively. Time to put some math behind it.
- We have understood that a current is induced in a wire dragged through a magnetic field through understanding the forces on the charges on the wire. We have also seen that a changing magnetic field induces a current (but have not yet seen why).
- In either case, there must be an induced emf $\mathcal{E}$. The current induced is (Ohm's Law):

$$I_{induced} = \frac{\mathcal{E}}{R}$$

(in the direction given by Lenz's Law)

# Faraday's Law

- So, it is the induced emf which determines the current.

**Faraday's Law**

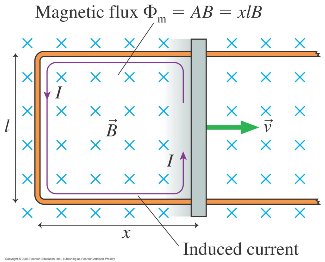
An emf $\mathcal{E}$ is induced around a closed loop if the magnetic flux through the loop changes. The magnitude of the emf is

$$\mathcal{E} = \left| \frac{d\Phi_m}{dt} \right|$$

and the direction of the emf is given by Lenz's Law.

This is a new law of physics!

# Faraday's Law

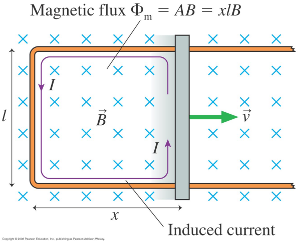



- So, it seems the current is all about the rate of change of magnetic flux.
- Consider the example on the left: a wire moving on a U-shaped rail.

- The field is perpendicular to the loop, so the flux is $\Phi = AB$ where $A$ is the area of the loop.
- As the wire slides the enclosed area changes. If $x$ is the distance from the end of the loop to the wire then $A = xL$ and the flux is

$$\Phi_m = AB = xLB$$

# Faraday's Law




- The flux through the loop increases as the wire moves, so

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| = \frac{d}{dt}(xLB) = \frac{dx}{dt}lB = vLB$$

- Now we can calculate the induced current as

$$I = \frac{\mathcal{E}}{R} = \frac{vLB}{R}$$

The same expression we got before!

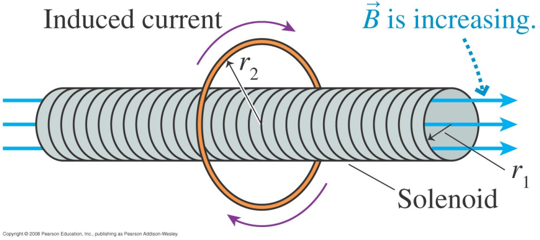
# What Does Faraday's Law Tell Us?

- If we write Faraday's Law as

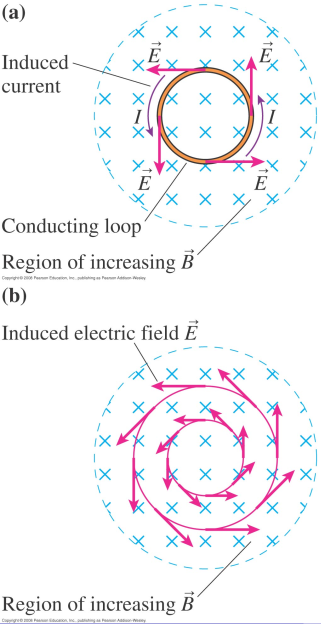
$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| = \left|\vec{B} \cdot \frac{d\vec{A}}{dt} + \vec{A} \cdot \frac{d\vec{B}}{dt}\right|$$

- Each term represents a fundamentally different way to change the flux and induce and emf
  1. The loop can expand or rotate
  2. The magnetic field can change
- We need a physical model to understand that second part.

# A Funny Example



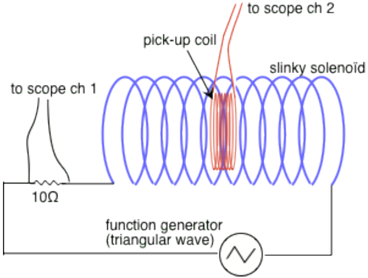

- We know that an ideal solenoid has magnetic field inside but no field outside.
- So, if we change the magnetic field in this solenoid does a current get induced in the loop of wire?
- Yes! But what is the physical mechanism which can explain this? How does the loop of wire "know" that the magnetic flux has changed???




- In the figure there is a changing magnetic field. Somehow it must be causing an electric field in the wire in order to induce the current.
- The induced electric field must be tangent to the wire and is the physical mechanism that makes the current flow.
- Even if the loop of wire is not there, the electric field is still induced.
- This strange induced field does not come from charges. It is a non-Coulomb electric field. Yet another sign that an electric field is not just a mathematical concept, it is a real, physical "thing".

# Induction in a Solenoid, an experiment

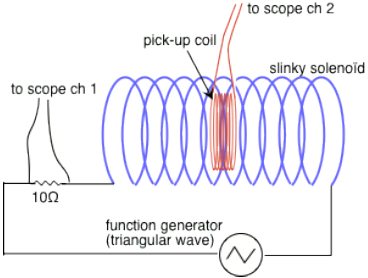
- Passing a current through a solenoid creates a magnetic field.
- If the current is varying in time then the changing magnetic flux can induce an EMF in a loop of wire inside the solenoid.



# Induction in a Solenoid, an experiment

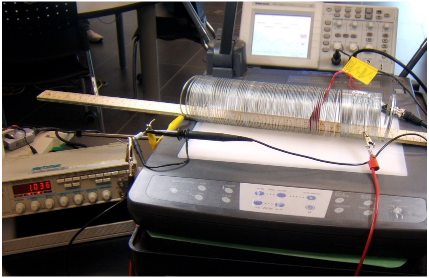
In this experiment we will use the following setup:

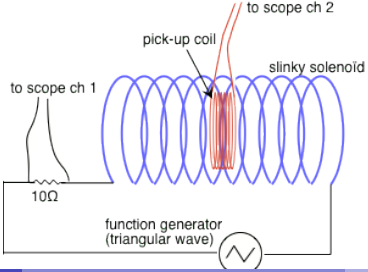
- A slinky is the solenoid.
- A 10-Ω resistor in series allows current to be measured.
- The pick-up coil is made of 10 loops of ordinary hook-up wire.
- The oscilloscope monitors the current in the slinky on Channel 1 and the induced EMF in the pick-up coil on Channel 2.
- A function generator provides a triangular-shaped voltage-vs-time waveform to the slinky-resistor combo.



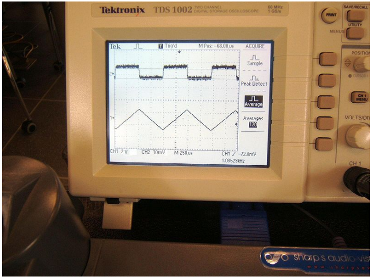
# Induction in a Solenoid, setup

Here's the experiment set up on a Document Presenter:



# Induction in a Solenoid, data




We want to predict the induced EMF, $\mathcal{E}$. Here are the data:

- $n = 440$ turns/m : Number of turns/metre of slinky
- $A_p = 1.08 \times 10^{-3}\ \mathrm{m}^2 \pm 8\%$ : Area inside pick-up coil
- $N_p = 10$ : Number of turns in pickup coil
- $\Delta V = 3.28 \pm 0.05$ V : P-P voltage change across 10-$\Omega$ resistor
- $\Delta t = 483\mu s$ : Time for $\Delta V$ is 1/2 period, ($f = 1.035$ kHz)

Find $\mathcal{E}$ during the rising half cycle.

# Induction in a Solenoid, calculations & conclusion

Find $\mathcal{E}$ during the rising half cycle.
The peak-to-peak change in $B$ in the solenoid is $\Delta B$:

$$\Delta B = \mu_0 n \Delta I = \mu_0 n \Delta V / R$$

$$= (4\pi \times 10^{-7}\ \text{ T m/A})(440\ \text{m}^{-1})(3.28\ \text{V}/10\ \Omega) = 1.81 \times 10^{-4}\ \text{T}\ (\pm 10\%)$$

$$\mathcal{E} = N_p \frac{d\Phi_B}{dt} = N_p \frac{A_p \Delta B}{\Delta t}$$

$$= (10) \frac{(1.08 \times 10^{-3}\ \text{m}^2)(1.81 \times 10^{-4}\ \text{T})}{483\ \mu\text{s}} = 4.05\ \text{m}V\ (\pm 20\%)$$

On the scope we see that the voltage across the pick-up coil changes from about –4 mV to $+4$ mv, which is within experimental error of our calculation.