Calculating the Induced Field

- The net work pushing the charge around the loop is not zero.
- The induced electric field is non-conservative. So, we cannot associate a potential with a non-Coulomb electric field. It is a different sort of electric field.
- However, we also defined the emf as the work per unit charge

$$\mathcal{E} = \frac{W}{q}$$

- If the charge moves through a small displacement $d\vec{s}$, then the work done by the field is

$$dW = \vec{F} \cdot d\vec{s} = q\vec{E} \cdot d\vec{s}$$

- The work done by the electric field as the charge moves around a closed loop is

$$W_{closed\ curve} = q \oint \vec{E} \cdot d\vec{s}$$

Calculating the Induced Field

- The emf around a closed loop is then

$$\mathcal{E} = \frac{W_{closed\ curve}}{q} = \oint \vec{E} \cdot d\vec{s}$$

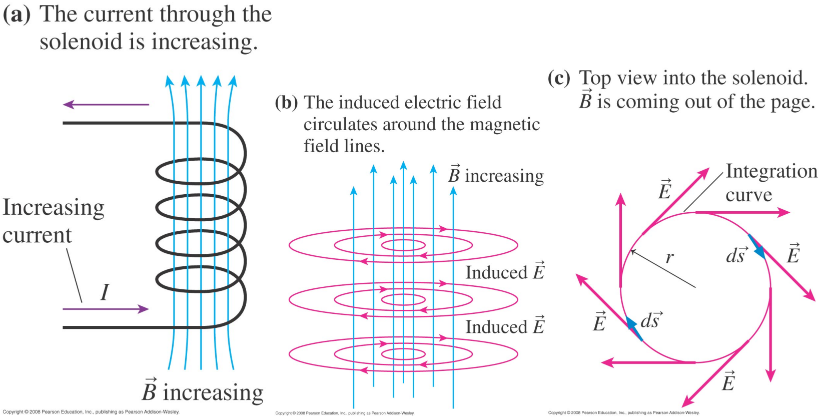
- If we restrict ourselves to non-changing loop perpendicular to the field we can write Faraday's Law as

$$\mathcal{E} = \frac{d\Phi_m}{dt} = A\left|\frac{dB}{dt}\right|$$

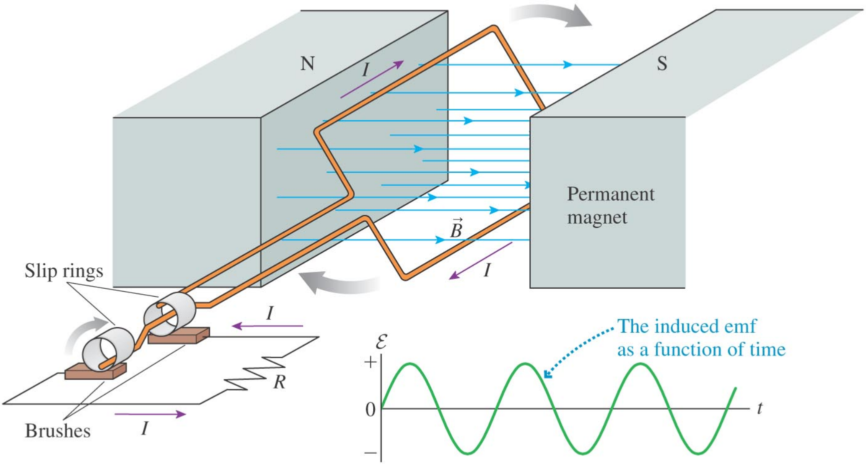
- Which leads us to an alternate expression of Faraday's Law

$$\oint \vec{E} \cdot d\vec{s} = A\left|\frac{dB}{dt}\right|$$

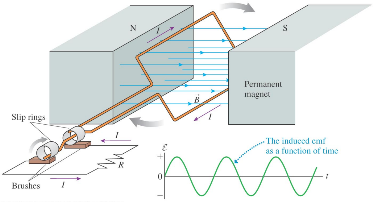
Calculating the Induced Field



Induced Currents: Generators (34.7)



Induced Currents: Generators (34.7)



- The magnetic field and area are constant, but the rotation changes the effective area, so the flux is always changing. The flux is

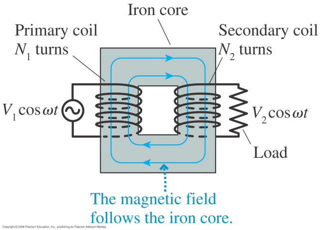
$$\Phi_m = \vec{A} \cdot \vec{B} = AB\cos\theta = AB\cos\omega t$$

where $\omega = 2\pi f$
- The induced emf is

$$\mathcal{E} = -N\frac{d\Phi_m}{dt} = -ABN\frac{d}{dt}(\cos\omega t) = \omega ABN\sin\omega t$$

where N is the number of turns in the coil.
- The generator creates and alternating current (AC) voltage.

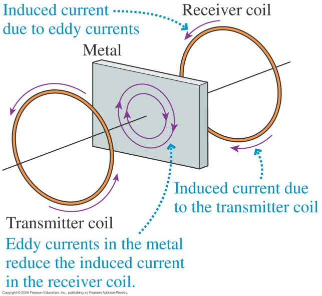
Transformers



- A primary coil with N_1 turns and a secondary coil with N_2 turns are each wrapped around an iron core.
- An alternating current through the primary creates an oscillating magnetic field through the secondary and induces a current.
- The changing B in the core is proportional to N_1: $B \propto 1/N_1$
- The emf in the secondary coil is $\mathcal{E} \propto N_2$
- Combining these gives

$$V_2 = \frac{N_2}{N_1} V_1$$

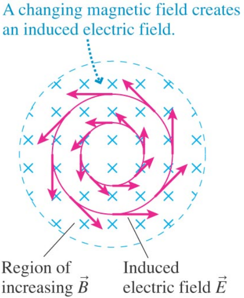
Metal Detectors

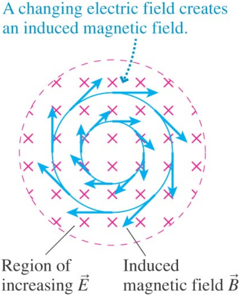


- At the airport you are scanned by a metal detector. These are very simple devices which rely on inductance.
- They have a transmitter coil and a receiver coil and you place objects between the two.

- A high-frequency alternating current is provided in the transmitter coil.
- This creates a changing magnetic flux through the receiver.
- If a metal is placed between then eddy-currents are created in a plane parallel to the coils.
- The receiver then sees a combination of the original oscillating field and the field generated by the eddy currents. The net field at the receiver decreases.
- An insulator does not allow eddy currents.

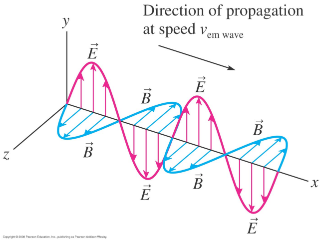
Maxwell's Theory of EM Waves





- Maxwell created a mathematical theory of electricity and magnetism which incorporated Faraday's ideas about fields.
- We know that a changing magnetic field induces a current. Maxwell thought that a changing electric field should induce a magnetic field as well (despite a lack of experimental evidence). So, he hypothesized the induced magnetic field.
- The top picture shows a changing magnetic field inducing an electric field. The bottom shows a changing electric field inducing a magnetic field.
- The pictures are almost identical, except the direction of the electric fields are reversed.

Maxwell's Theory of EM Waves

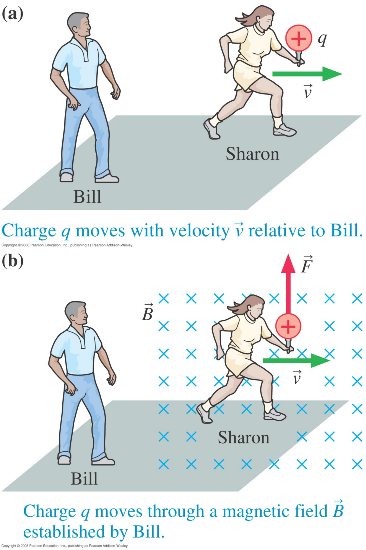


- Maxwell realized that if it were true that changing electric fields induced magnetic fields and changing magnetic fields induced electric fields it might be possible to construct a self-sustaining wave.
- A changing $\vec{E}$ creates a $\vec{B}$ which changes in just the right way to induce a $\vec{E}$ which changes to make a $\vec{B}$, etc.
- This requires no charges or currents!!!
- He calculated the speed of such a wave to be

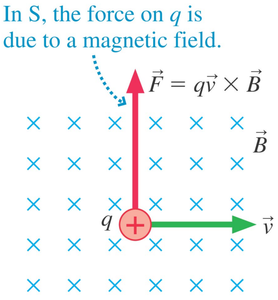
$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s}$$

Light is an EM wave!!!!!

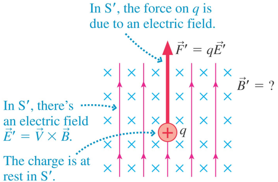
EM Waves Tidbits from Chapter 35



- Sharon is carrying a positive charge. Bill sees a moving charge and an induced magnetic field.
- However, Sharon sees the charge sitting still. So, sees no magnetic field.
- So, is there a magnetic field or not??
- It gets worse. If the runner moves through an external magnetic field the observer sees a force on her charge ($\vec{F} = q\vec{v} \times \vec{B}$).
- However, Sharon says her charge is at rest, so there are no magnetic forces!!
- So, is there a force or not??



The situation in frame S

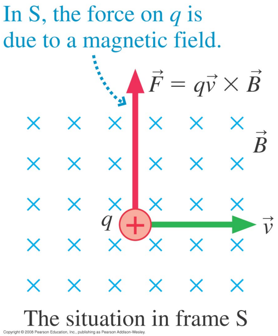


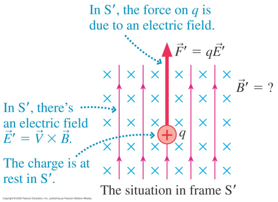
The situation in frame S′

- We know something for sure: Experimenters in all inertial reference frames agree about a force acting on a particle.
- Imagine "Bill" makes a region of uniform $\vec{B}$ but $\vec{E} = 0$ and shoots a particle of charge q through that region at velocity $\vec{v}$.
- The total force on the particle from electric and magnetic fields will be

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = q\vec{v} \times \vec{B}$$

Transformation of Electric and Magnetic Fields

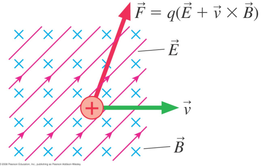




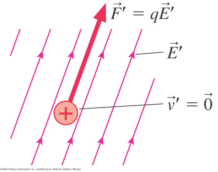
- Sharon is in a frame such that this particle is at rest. However, we agree that she **must** see an upwards force on the particle. How??
- Well, we defined and electric field as the thing that creates a force on a stationary charge.
- Therefore, she must measure an electric field in her reference frame.
- Changing reference frames converts electric to magnetic fields or vice-versa!

Transformation of Electric and Magnetic Fields

(a) The electric and magnetic fields in frame S



(b) The electric field in frame S′, where the charged particle is at rest



- In general, the total EM force on a charged particle is called the Lorentz Force

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

- The charge is at rest in frame S' moving at speed $\vec{V} = \vec{v}$, so in that frame the force is:

$$\vec{F}' = q\vec{E}'$$

- Equating $\vec{F}$ with $\vec{F}'$ gives us the transformed electric field

$$\vec{E}' = \vec{E} + \vec{v} \times \vec{B}$$

Transformation of Electric and Magnetic Fields

- **There is a single EM field that changes its appearance from $\vec{E}$ to $\vec{B}$ for different observers.**

Correspondence between $\vec{E}$ and $\vec{B}$

The Biot-Savart law for the magnetic field of a moving point charge is nothing other than the Coulomb electric field of a stationary point charge transformed into a moving reference frame!