Thomas Young (1773-1829)

...whenever two portions of the same light arrive at the eye by different routes, either exactly or very nearly in the same direction, the light becomes most intense when the difference in their routes is any multiple of a certain length, and least intense in the intermediate state of the interfering portions; and this length is different for light of different colours.

Thomas Young was a pretty brilliant experimentalist...

Superposition

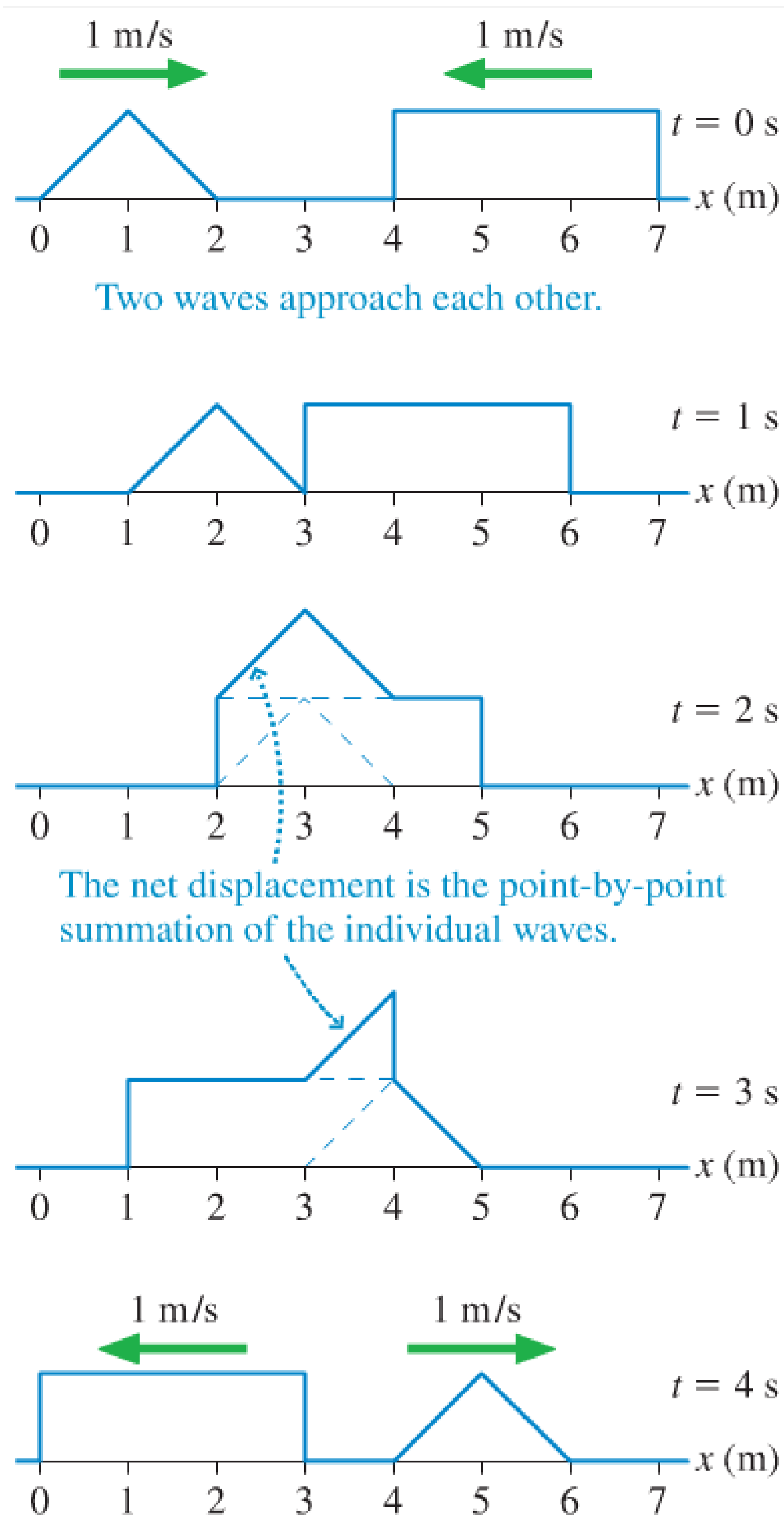
What happens when two waves of the same type meet? They interfere. That interference can be constructive or destructive or, if the frequecies are different, can create beats. To simplify things, we will study interfering waves of equal frequency and amplitude.

Principle of Superposition

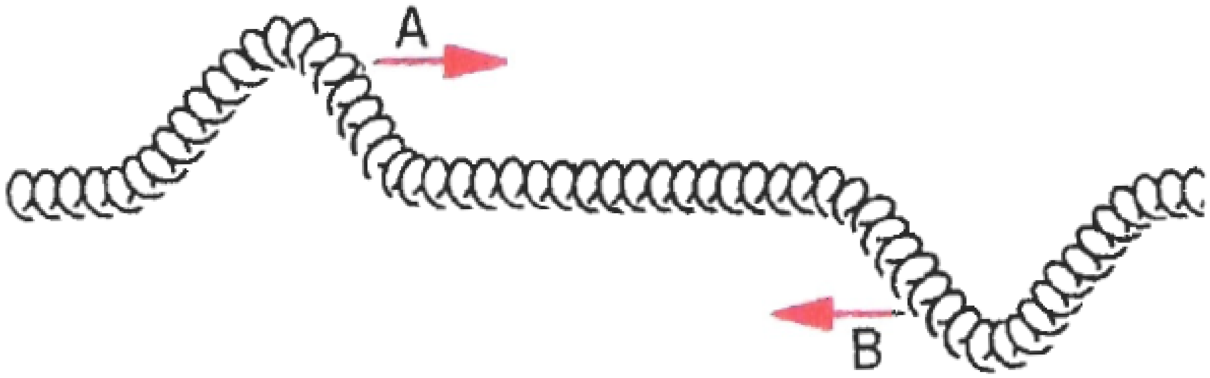
When two or more waves are simultaneously present at a single point in space, the displacement of the medium at that point is the sum of the displacements due to each individual wave.

$$D_{net} = D_1 + D_2 + \cdots = \sum_i D_i$$

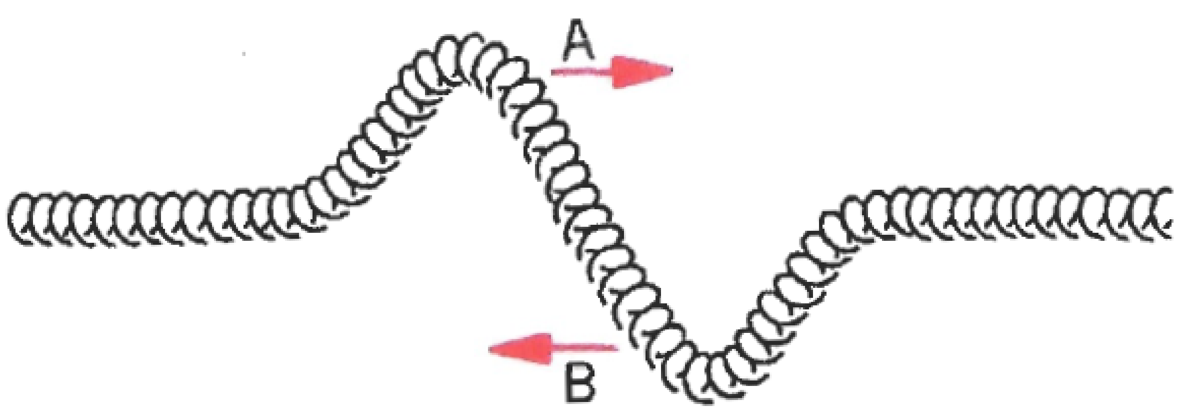
Illustrated Principle of Superposition



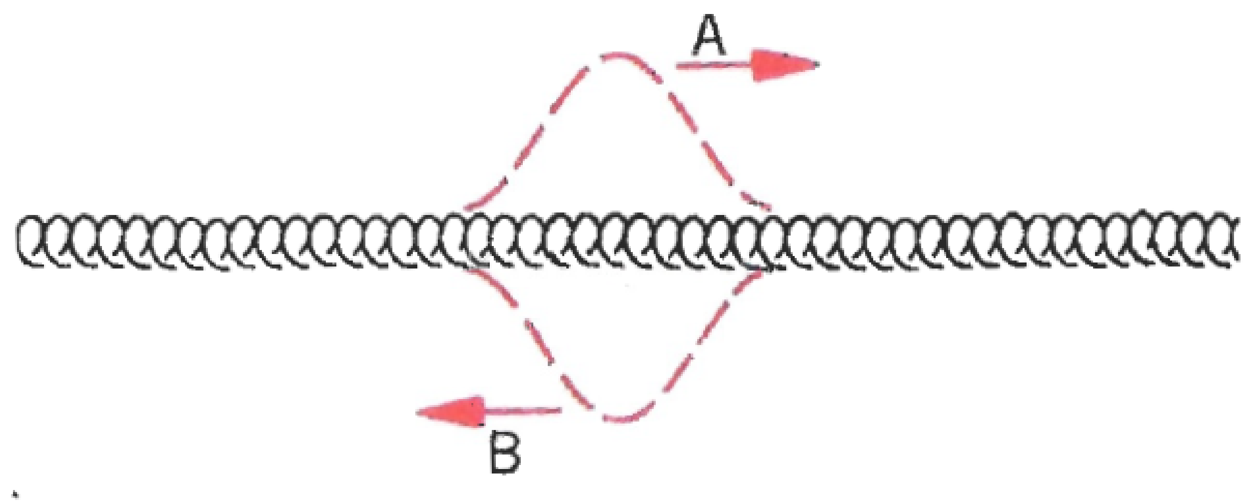
Illustrated Principle of Superposition



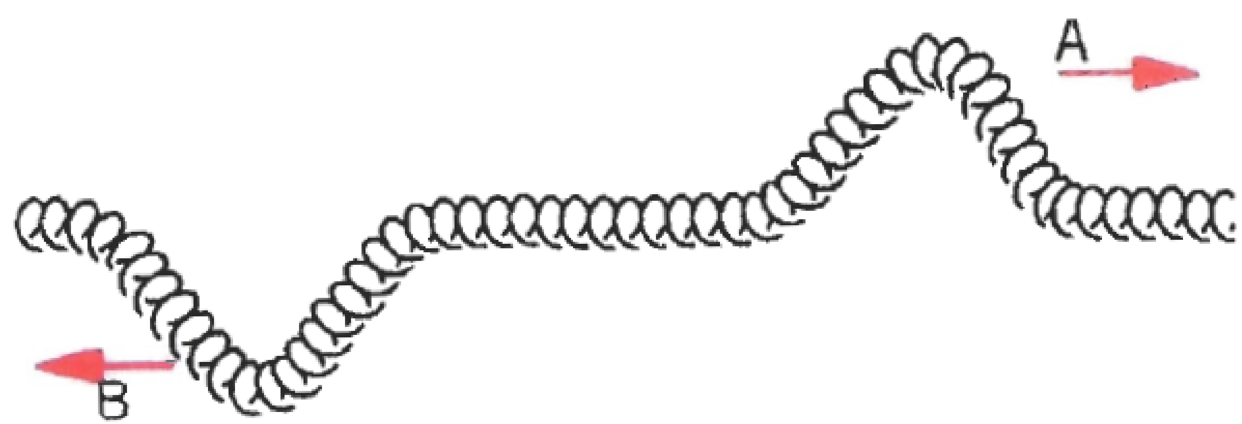
(a) The approaching pulses



(b) Overlap begins

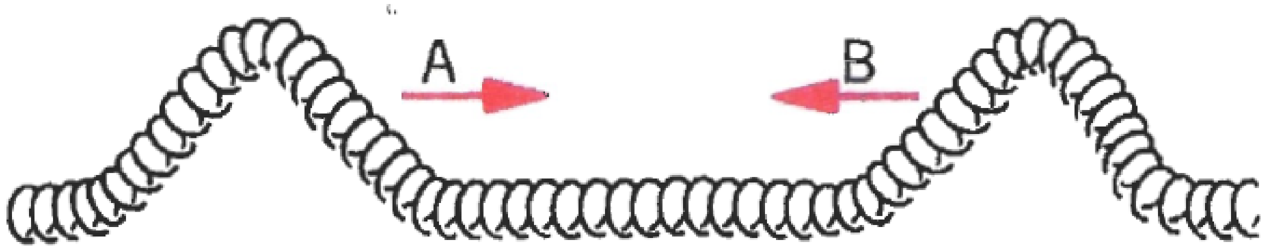


(c) Total overlap

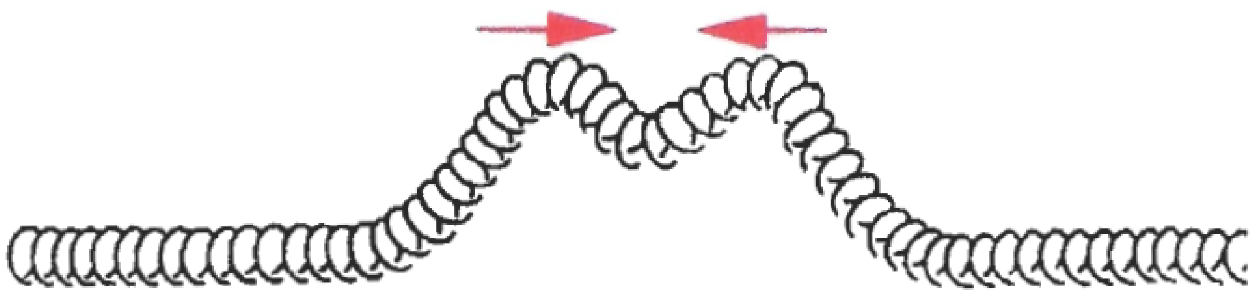


(d) The receding pulses

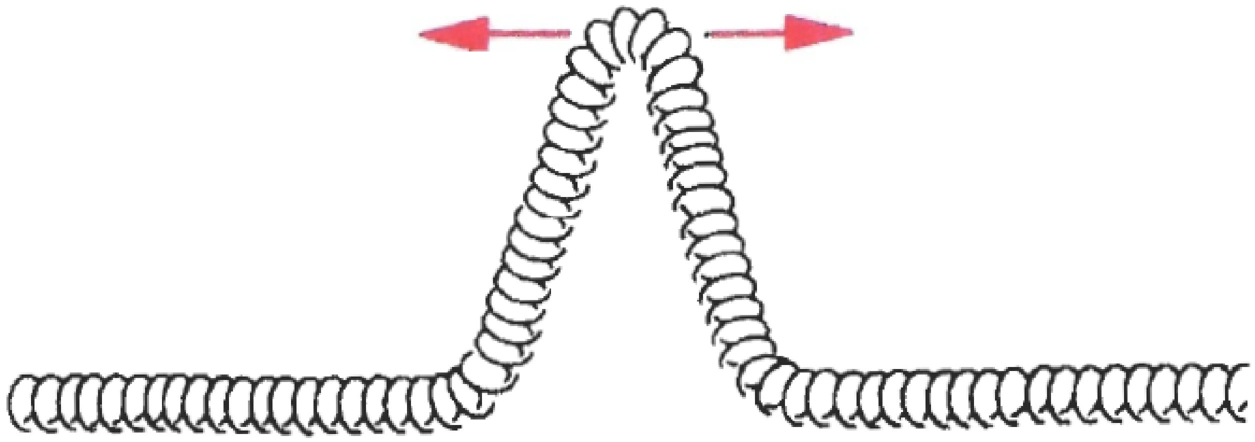
Illustrated Principle of Superposition



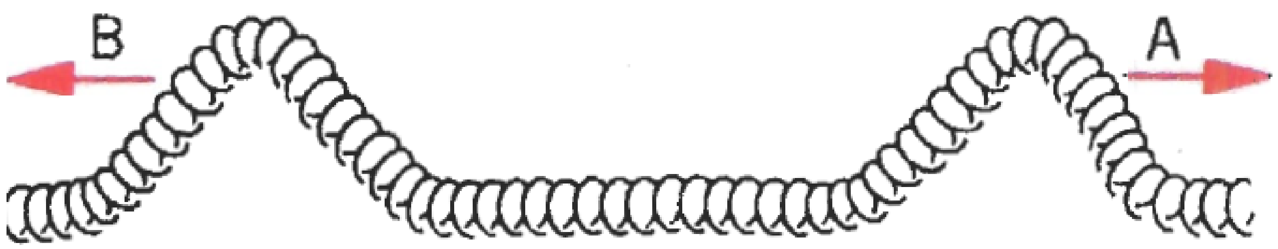
(*a*) The approaching pulses



(*b*) Overlap begins

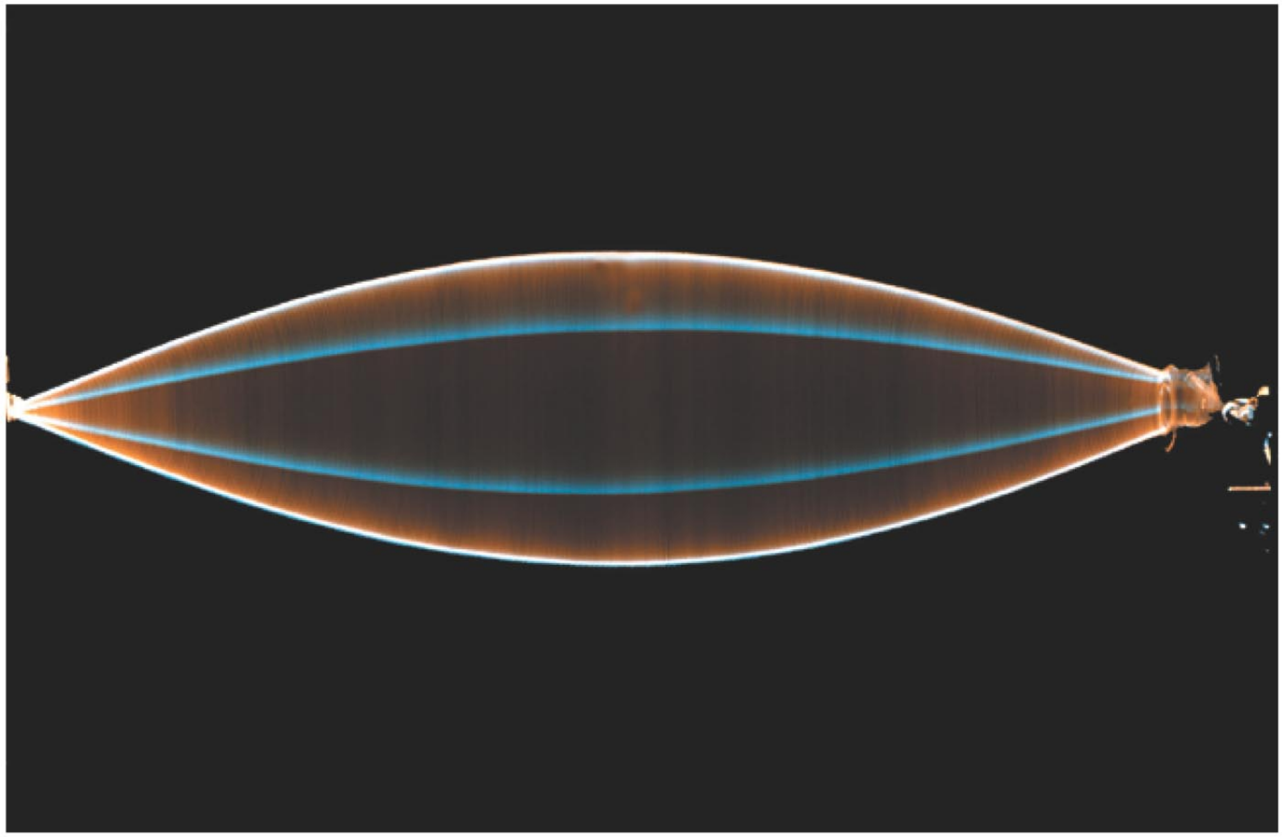


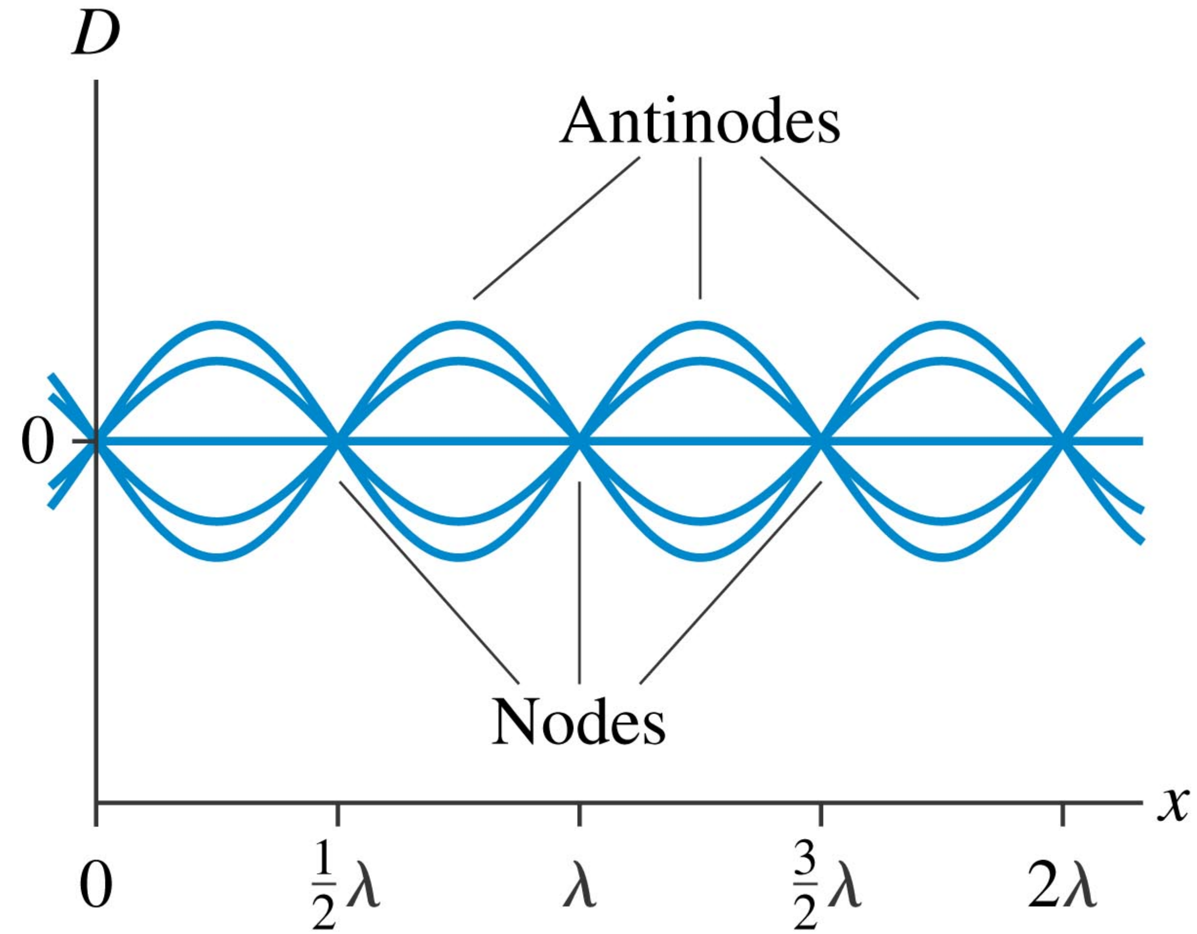
(*c*) Total overlap; the slinky has twice the height of either pulse



(*d*) The receding pulses

Standing Waves





The nodes and antinodes are spaced $\lambda/2$ apart.

- A standing wave is a superposition of two waves travelling in opposite directions.
- Constructive interference creates antinodes, destructive interference creates nodes.
- Nodes on a standing wave are spaced $\lambda/2$ apart and never move
- Antinodes are halfway between nodes.

The Mathematics of Standing Waves

We can write two equal waves travelling in opposite directions like:

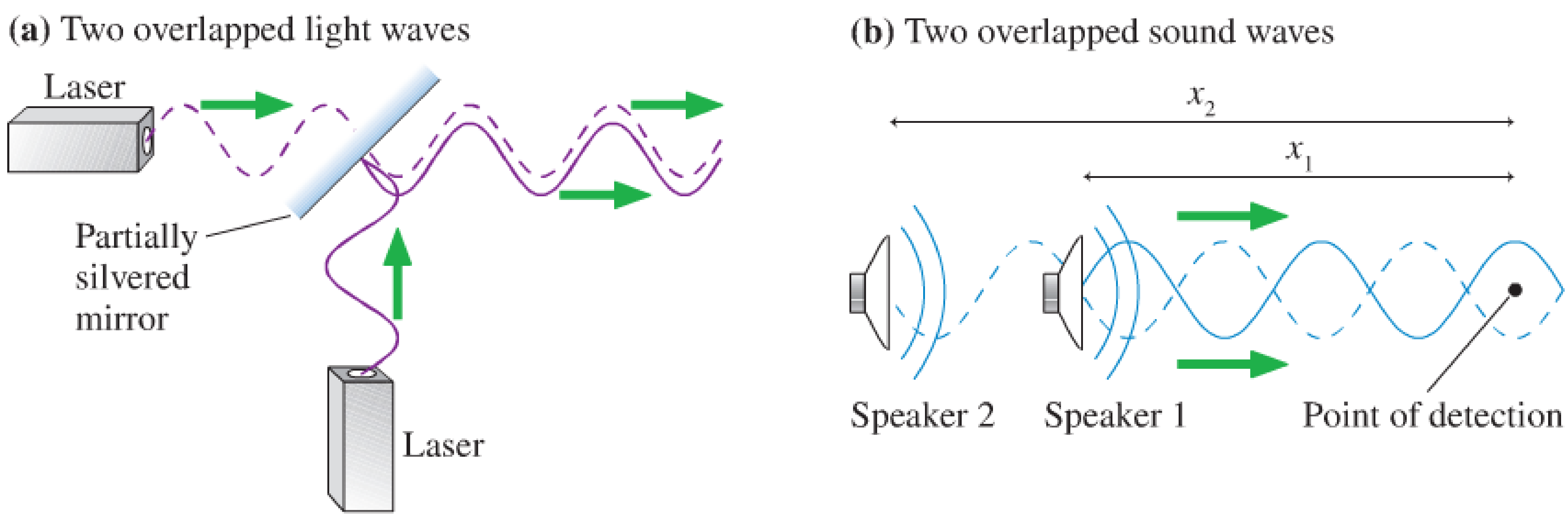
$$D(x,t) = D_R + D_L = a\sin(kx - \omega t) + a\sin(kx + \omega t)$$

This can be simplified to (see your text)

$$D(x,t) = (2a\sin kx)\cos\omega t = A(x)\cos\omega t$$

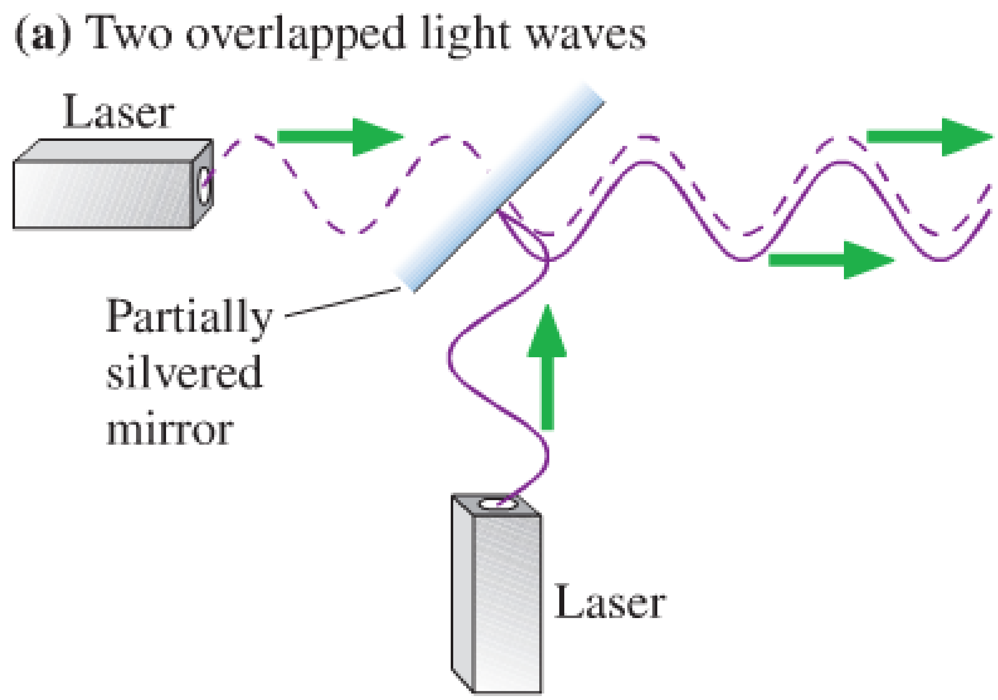
Notice the form!! This is not a travelling wave. This is the equation of a medium in which each point is executing SHM (with varying amplitude).

Interference in 1-D

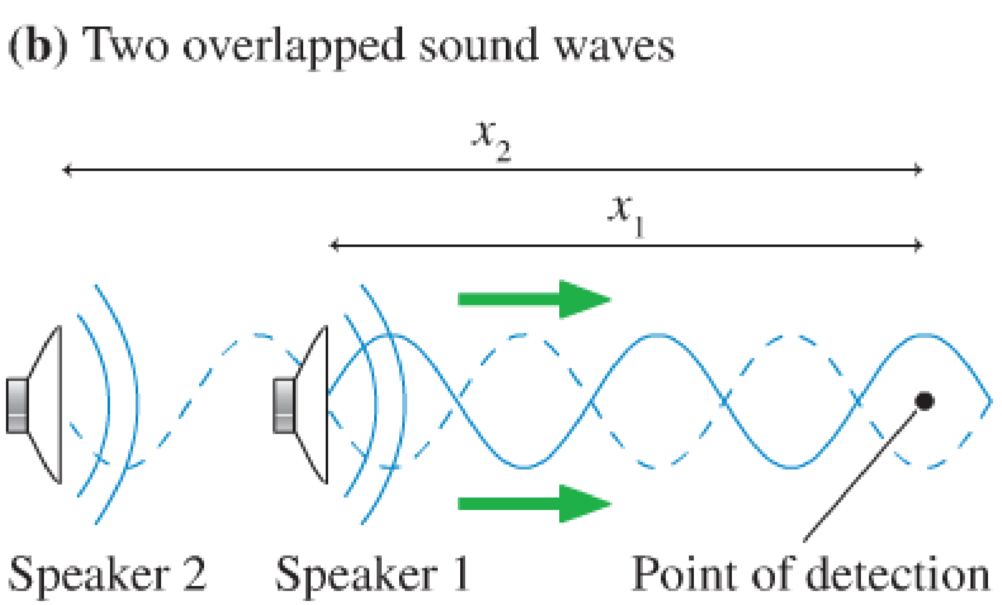


We assume sinusoidal waves of the same frequency and amplitude traveling to the right along the x-axis. The displacements are

$$\begin{aligned} D_1(x_1,t) &= a\sin(kx_1-\omega t+\phi_{10}) = a\sin\phi_1 \\ D_2(x_2,t) &= a\sin(kx_2-\omega t+\phi_{20}) = a\sin\phi_2 \end{aligned}$$



These two waves are in phase and will give constructive interference. If they are perfectly in phase and of equal amplitude a, this will lead to a combined amplitude $A = 2a$.



These two waves are out of phase and will give destructive interference. If they are 180° out of phase and of equal amplitude a, this will lead to a combined amplitude $A = a - a = 0$.

Interference in 1-D - Phase Differences

Remember our mathematical description of the two waves:

$$\begin{aligned} D_1(x_1, t) &= a\sin(kx_1 - \omega t + \phi_{10}) = a\sin\phi_1 \\ D_2(x_2, t) &= a\sin(kx_2 - \omega t + \phi_{20}) = a\sin\phi_2 \end{aligned}$$

Let's now concentrate on the phases (arguments of the sin)

$$\begin{aligned} \phi_1 &= kx_1 - \omega t + \phi_{10} \\ \phi_2 &= kx_2 - \omega t + \phi_{20} \end{aligned}$$

The phase difference is then

$$\Delta\phi = \phi_2 - \phi_1$$

Interference in 1-D - Phase Differences

Let's express the phase difference another way

$$\begin{aligned}\Delta\phi &= (kx_2 - \omega t + \phi_{20}) - (kx_1 - \omega t + \phi_{10}) \\ &= k(x_2 - x_1) + (\phi_{20} - \phi_{10}) \\ &= 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0\end{aligned}$$

There are two distinct contributions: the path length difference (Δx term) and the inherent phase difference ($\Delta\phi_0$ term).

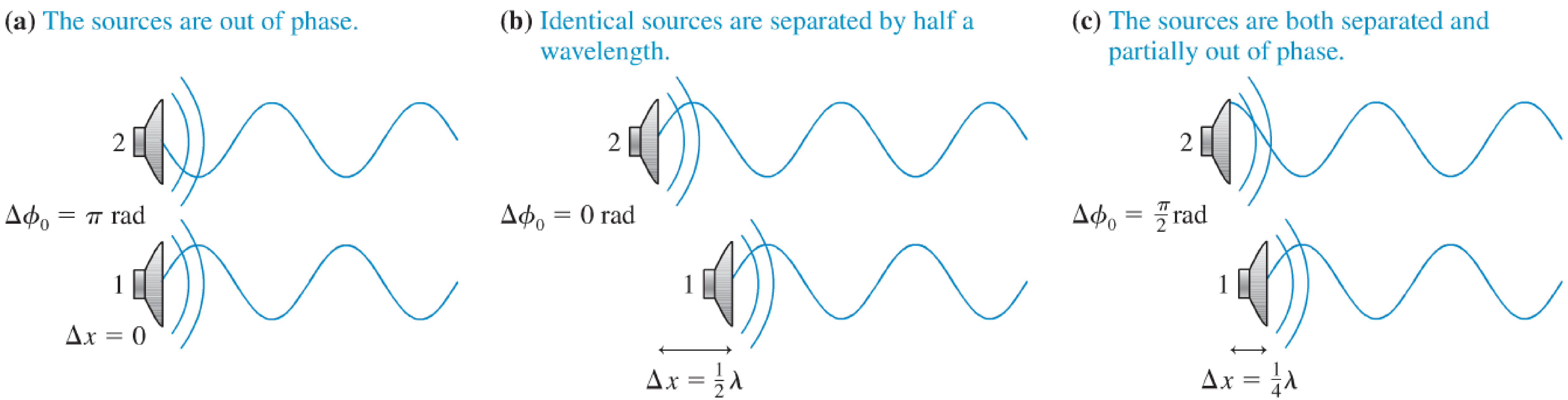
Maximum Constructive Interference

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \phi_0 = m \cdot 2\pi \text{ rad}, m = 0, 1, 2, 3, \ldots$$

Maximum Destructive Interference

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \phi_0 = \left(m + \frac{1}{2}\right) \cdot 2\pi \text{ rad}, m = 0, 1, 2, 3, \ldots$$

Interference in 1-D - Phase Differences



Mathematics of Interference in 1-D

- The displacement resulting from two waves is

$$\begin{aligned} D &= D_1 + D_2 = a \sin(kx_1 - \omega t + \phi_{10}) + a \sin(kx_2 - \omega t + \phi_{20}) \\ &= a \sin\phi_1 + a \sin\phi_2 \end{aligned}$$

- We can use the trig identity

$$\sin\phi_1 + \sin\phi_2 = 2\cos\left[\frac{1}{2}(\phi_1 - \phi_2)\right]\sin\left[\frac{1}{2}(\phi_1 + \phi_2)\right]$$

- To write:

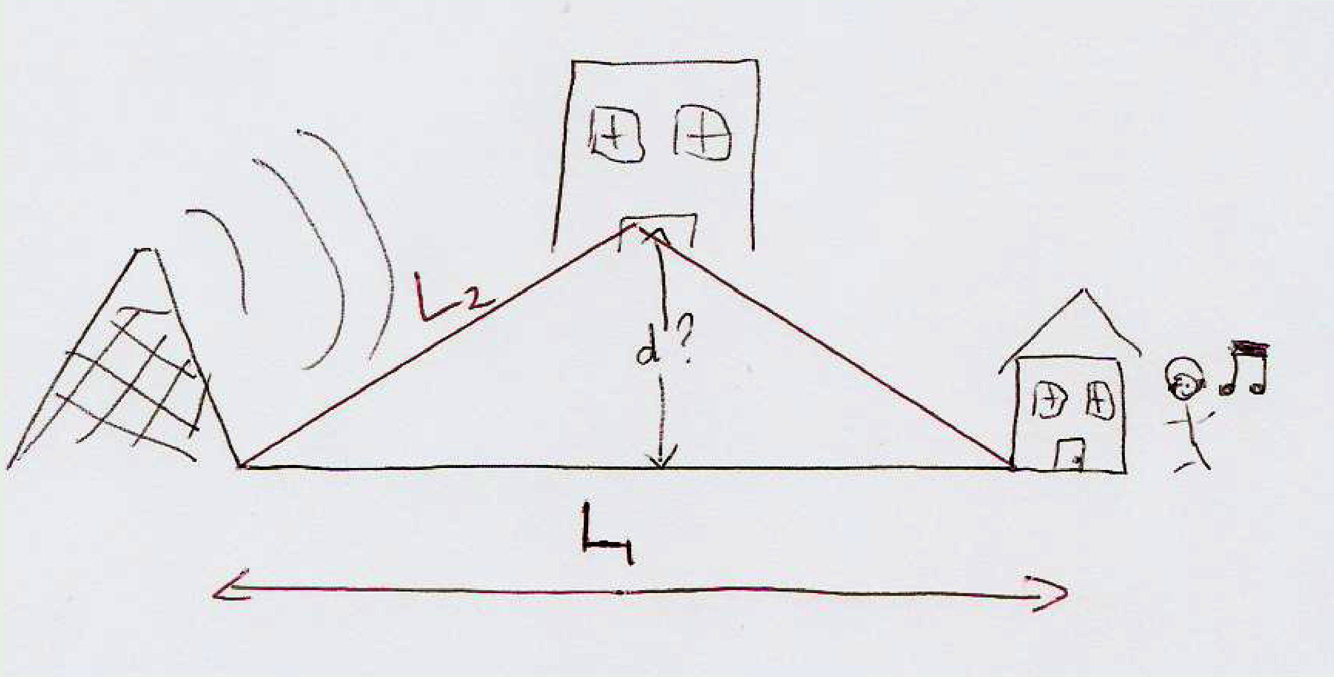
$$D = \underbrace{\left[2a\cos\frac{\Delta\phi}{2}\right]}_{\text{amplitude of new wave}} \sin(kx_{ave} - \omega t + (\phi_0)_{ave})$$

(gives us back constructive and destructive phase-differences found earlier)

Example of EM Interference

Listening to AM Radio

Suppose you are listening to AM650 (650kHz) and you live 23km from the radio tower. There is another building halfway between you and the tower and radio waves are bouncing off of that building. How far off to the side is the building if destructive interference occurs between the direct and reflected waves? (assume equal amplitudes and no phase shift on reflection)



(i.e., What is *d*??)

Example of EM Interference

- Direct path: $L_1 = 23$ km
- Indirect path:

$$L_2 = 2\sqrt{\left(\frac{L_1}{2}\right)^2 + d^2}, Pythagoras$$
$$= \sqrt{L_1^2 + 4d^2}$$

- Path difference: $L_2 - L_1 = \sqrt{L_1^2 + 4d^2} - L_1$
- We want destructive interference:

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \phi_0 = \left(m + \frac{1}{2}\right) \cdot 2\pi \text{ rad}, m = 0, 1, 2, 3, \ldots$$

Example of EM Interference

- Destructive interference:

$$\begin{aligned}\frac{\Delta x}{\lambda}2\pi &= (2m+1)\pi \\ \Delta x &= \lambda(m+\frac{1}{2}) \\ L_2 - L_1 &= \lambda(m+\frac{1}{2}) = \sqrt{L_1^2+4d^2} - L_1\end{aligned}$$

- Take the minimum d as $m = 0$

$$\begin{aligned}\frac{\lambda}{2} &= \sqrt{L_1^2+4d^2} - L_1 \\ L_1 + \frac{\lambda}{2} &= \sqrt{L_1^2+4d^2} \\ L_1^2 + L_1\lambda + \frac{\lambda^2}{4} &= L_1^2 + 4d^2\end{aligned}$$

Example of EM Interference

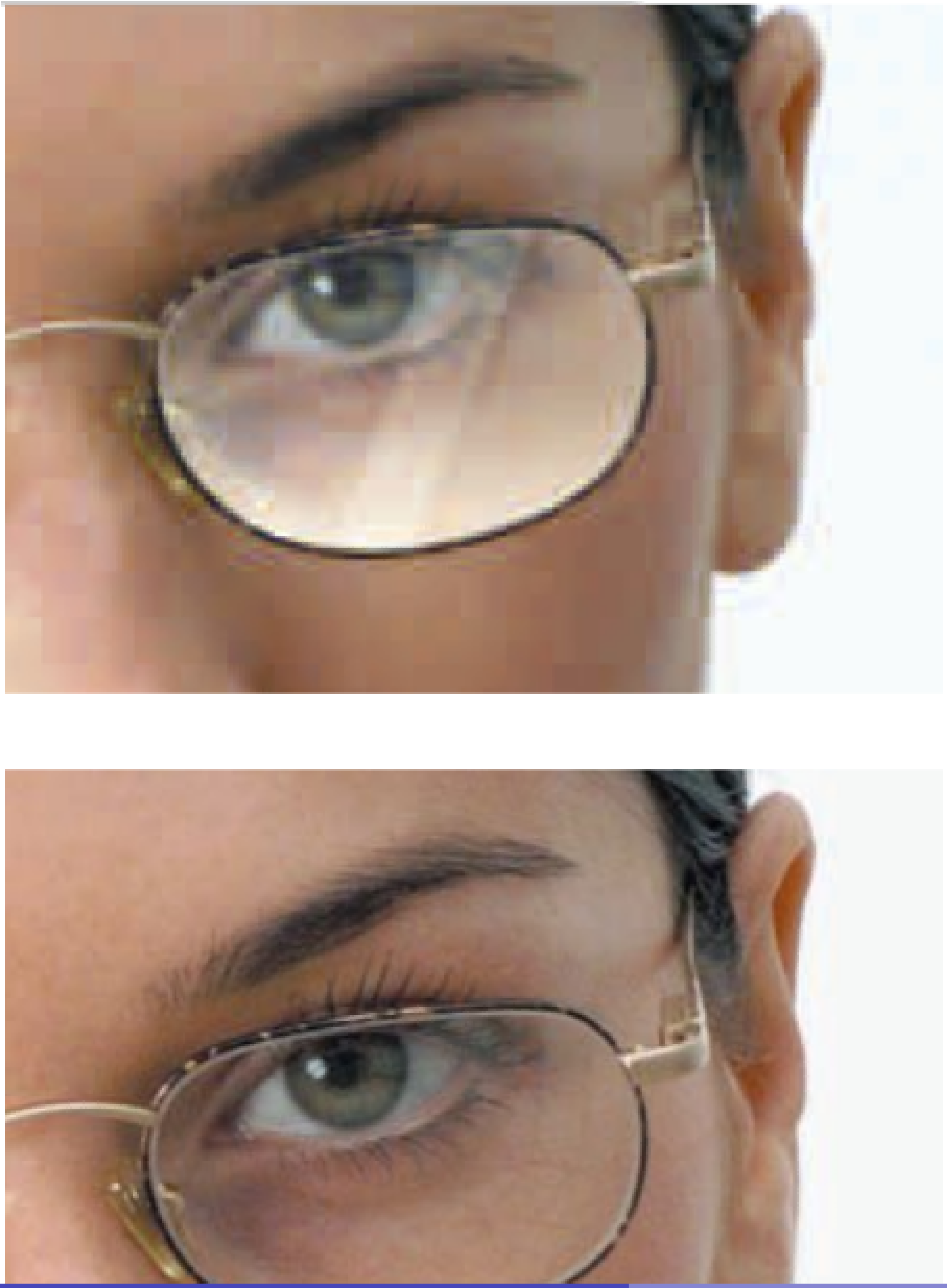
- Now solve for d:

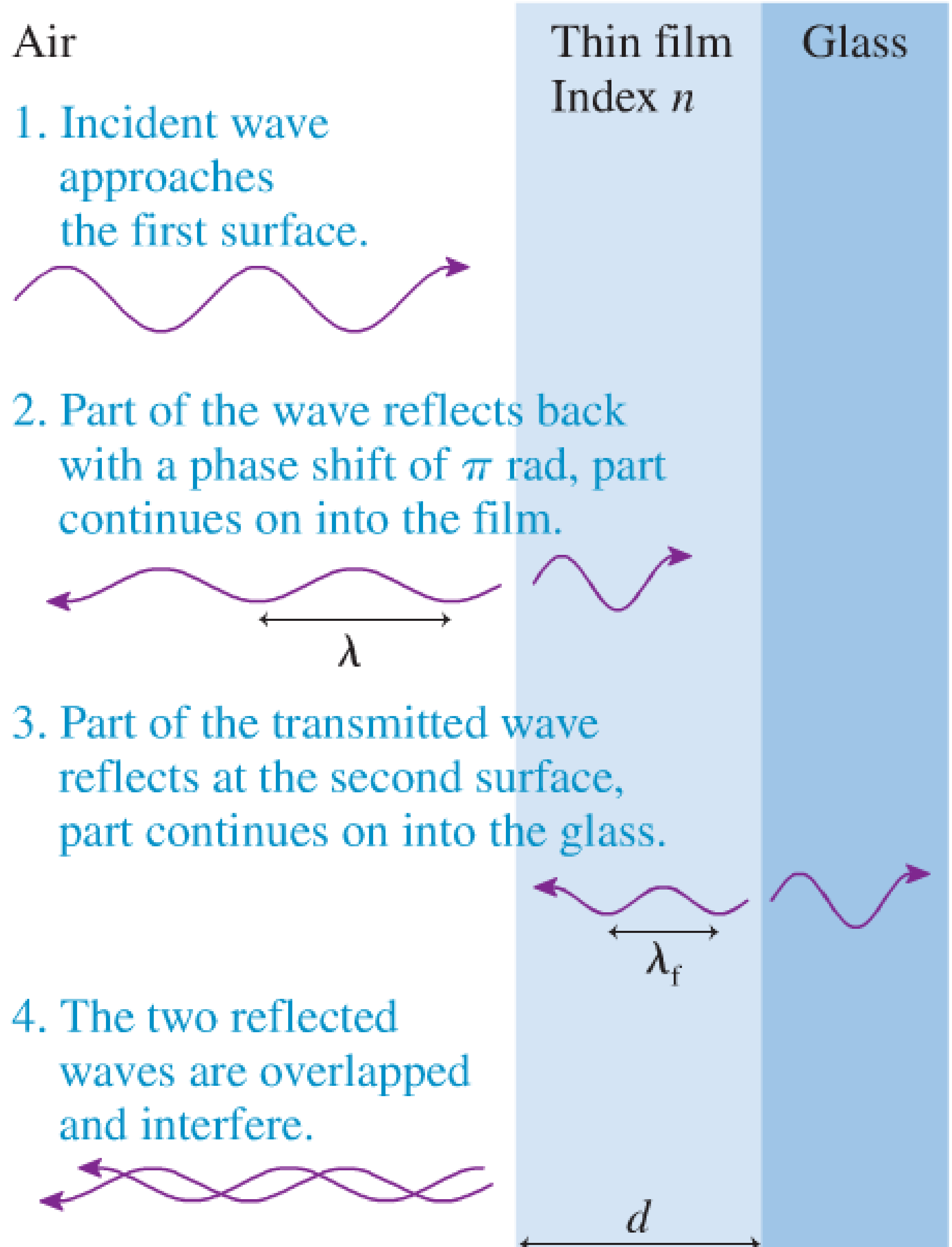
$$
\begin{aligned}
L_1^2 + L_1\lambda + \frac{\lambda^2}{4} &= L_1^2 + 4d^2 \\
\cancel{L_1^2} + L_1\lambda + \frac{\lambda^2}{4} &= \cancel{L_1^2} + 4d^2 \\
d &= \frac{1}{2}\sqrt{L_1\lambda + \frac{\lambda^2}{4}}
\end{aligned}
$$

- Plugging in the numbers:

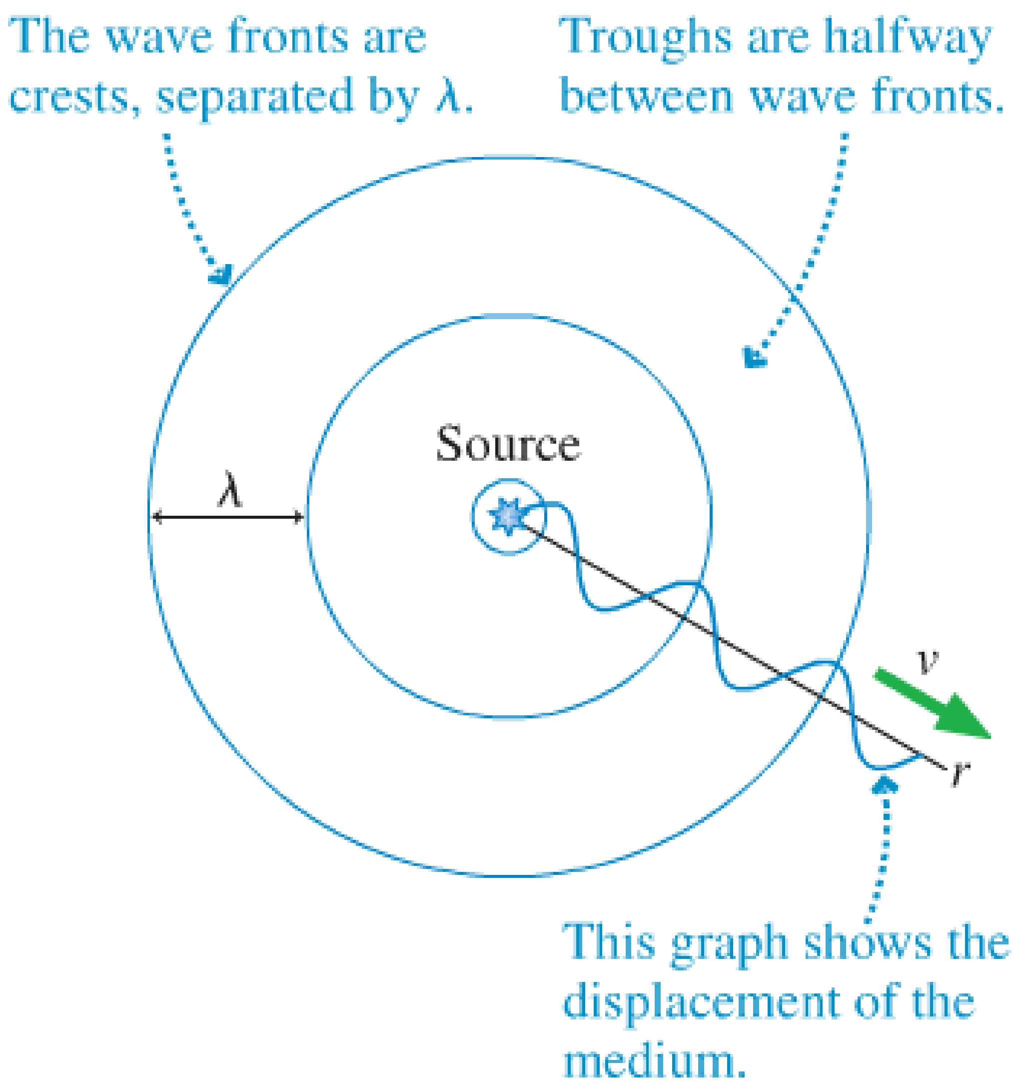
$$
\begin{aligned}
\lambda &= \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{650000 \text{ Hz}} = 462 \text{ m} \\
d &= 1.6 \text{ km}
\end{aligned}
$$

Application: anti-reflective coating





Interference in 2 or 3 Dimensions

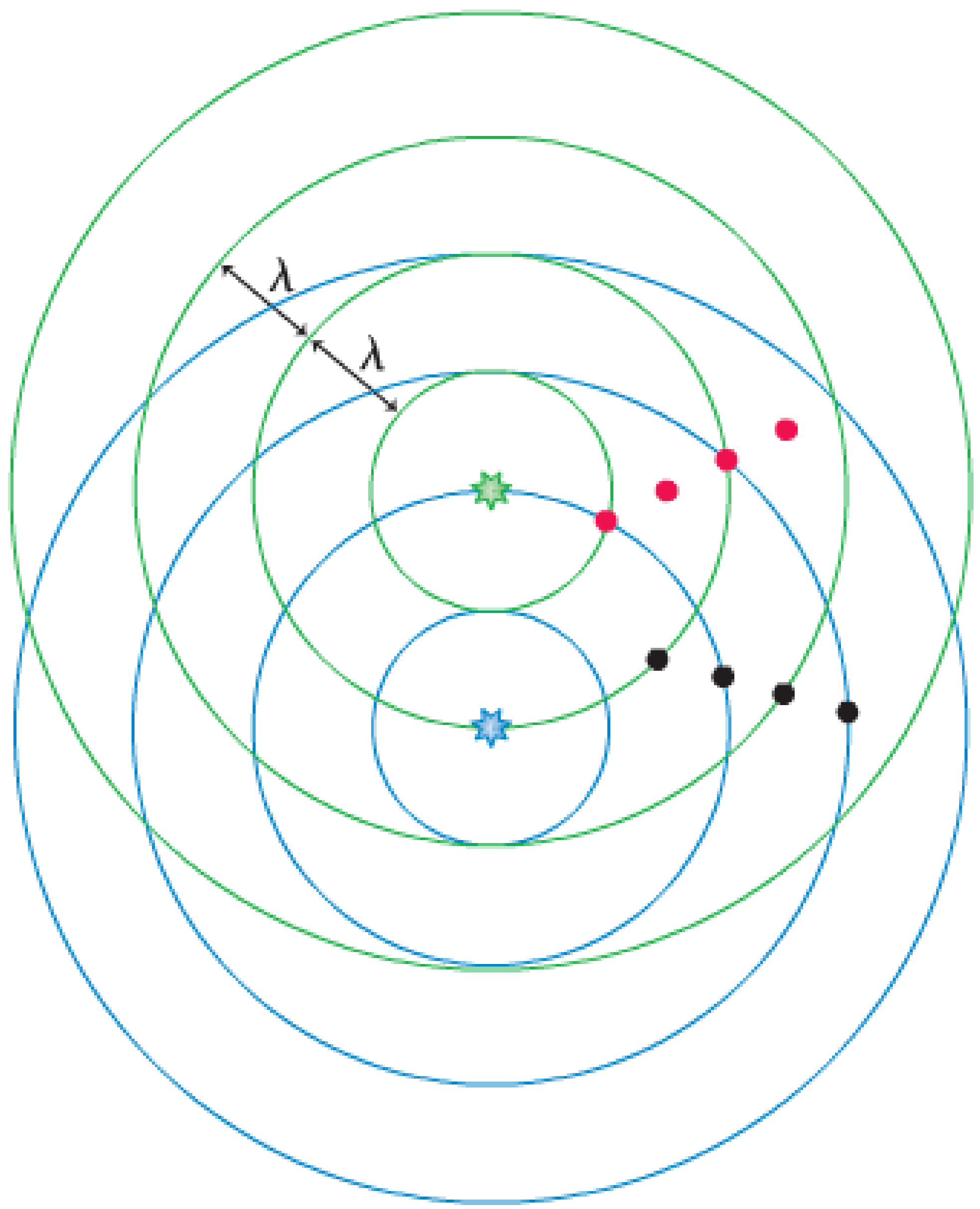


Working in 2 or 3 dimensions is not very different from working in 1-D:

$$D(r,t) = a\sin(kr - \omega t + \phi_0)$$

where r is the distance measured outwards from the source. Essentially we just replace x everywhere with a radial coordinate r...

Interference in 2 or 3 Dimensions

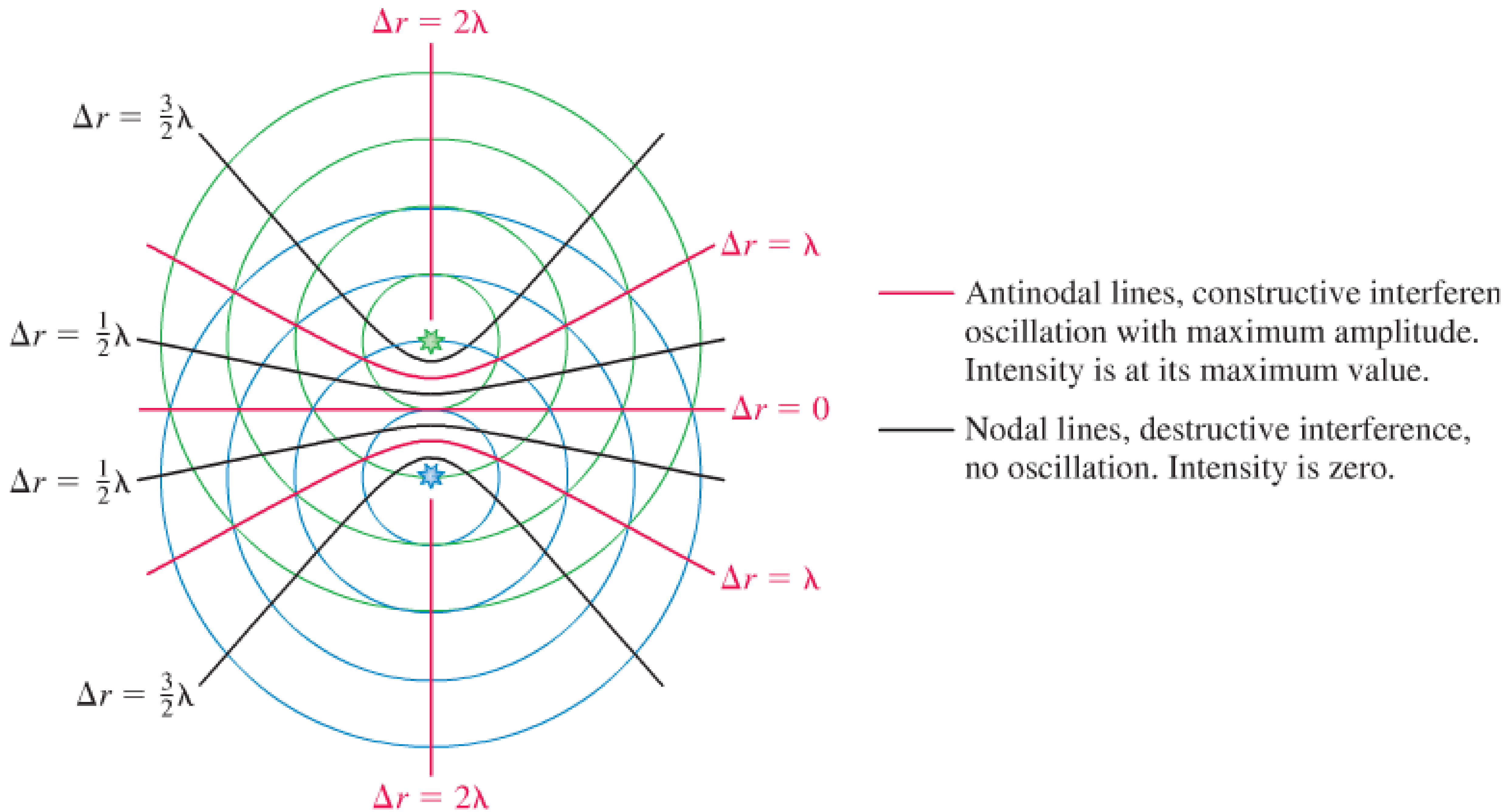


- Points of constructive interference. A crest is aligned with a crest, or a trough with a trough.
- Points of destructive interference. A crest is aligned with a trough of another wave.

Interference also occurs with spherical waves. Again we look for places where crests or troughs align. The phase difference is now

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0$$

Nodal and Anti-Nodal Lines



Simulation: http://phet.colorado.edu/sims/wave-interference