

Doppler Effect

The Doppler effect is the change in frequency of a wave detected by an observer moving relative to the source of the wave.



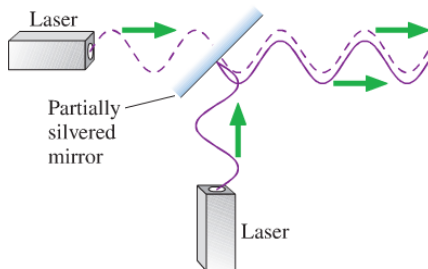
- It is named after Austrian physicist Christian Doppler who described it in 1842. Simulation:
<http://astro.unl.edu/classaction/animations/light/dopplershift.html>
- The detected frequency f , depends on the frequency of the source, f_s , the velocity of the wave, v , and the velocity of the source relative to the observer, v_s (going away is positive)

$$f = \frac{f_s}{1 + v_s/v}$$

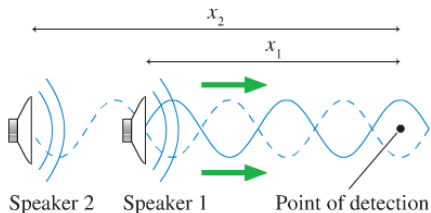
- Applications of the Doppler effect range through sirens, weather radar and determining the speed of stars.

Interference in 1-D

(a) Two overlapped light waves



(b) Two overlapped sound waves



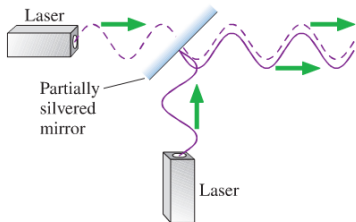
We assume sinusoidal waves of the same frequency and amplitude traveling to the right along the x -axis. The displacements are

$$D_1(x_1, t) = a \sin(kx_1 - \omega t + \phi_{10}) = a \sin \phi_1$$

$$D_2(x_2, t) = a \sin(kx_2 - \omega t + \phi_{20}) = a \sin \phi_2$$

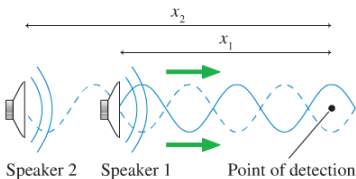
Interference in 1-D - Constructive Interference

(a) Two overlapped light waves



These two waves are **in phase** and will give **constructive interference**. If they are perfectly in phase and of equal amplitude a , this will lead to a combined amplitude $A = 2a$.

(b) Two overlapped sound waves



These two waves are **out of phase** and will give **destructive interference**. If they are 180° out of phase and of equal amplitude a , this will lead to a combined amplitude $A = a - a = 0$.

Interference in 1-D - Phase Differences

Remember our mathematical description of the two waves:

$$D_1(x_1, t) = a \sin(kx_1 - \omega t + \phi_{10}) = a \sin \phi_1$$

$$D_2(x_2, t) = a \sin(kx_2 - \omega t + \phi_{20}) = a \sin \phi_2$$

Let's now concentrate on the phases (arguments of the sin)

$$\phi_1 = kx_1 - \omega t + \phi_{10}$$

$$\phi_2 = kx_2 - \omega t + \phi_{20}$$

The **phase difference** is then

$$\Delta\phi = \phi_2 - \phi_1$$

Interference in 1-D - Phase Differences

Let's express the phase difference another way

$$\begin{aligned}\Delta\phi &= (kx_2 - \omega t + \phi_{20}) - (kx_1 - \omega t + \phi_{10}) \\ &= k(x_2 - x_1) + (\phi_{20} - \phi_{10}) \\ &= 2\pi \frac{\Delta x}{\lambda} + \Delta\phi_0\end{aligned}$$

There are two distinct contributions: the path length difference (Δx term) and the inherent phase difference ($\Delta\phi_0$ term).

Maximum Constructive Interference

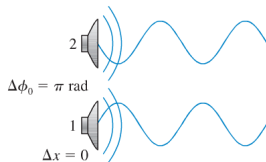
$$\Delta\phi = 2\pi \frac{\Delta x}{\lambda} + \phi_0 = m \cdot 2\pi \text{ rad}, m = 0, 1, 2, 3, \dots$$

Maximum Destructive Interference

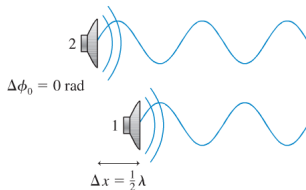
$$\Delta\phi = 2\pi \frac{\Delta x}{\lambda} + \phi_0 = \left(m + \frac{1}{2}\right) \cdot 2\pi \text{ rad}, m = 0, 1, 2, 3, \dots$$

Interference in 1-D - Phase Differences

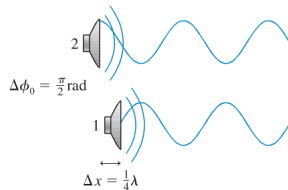
(a) The sources are out of phase.



(b) Identical sources are separated by half a wavelength.



(c) The sources are both separated and partially out of phase.



Mathematics of Interference in 1-D

- The displacement resulting from two waves is

$$\begin{aligned} D &= D_1 + D_2 = a \sin(kx_1 - \omega t + \phi_{10}) + a \sin(kx_2 - \omega t + \phi_{20}) \\ &= a \sin \phi_1 + a \sin \phi_2 \end{aligned}$$

- We can use the trig identity

$$\sin \phi_1 + \sin \phi_2 = 2 \cos \left[\frac{1}{2}(\phi_1 - \phi_2) \right] \sin \left[\frac{1}{2}(\phi_1 + \phi_2) \right]$$

- To write:

$$D = \underbrace{\left[2a \cos \frac{\Delta\phi}{2} \right]}_{\text{amplitude of new wave}} \sin(kx_{ave} - \omega t + (\phi_0)_{ave})$$

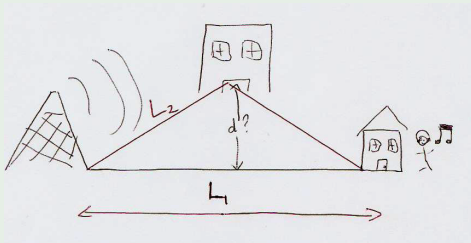
amplitude of new wave

(gives us back constructive and destructive phase-differences found earlier)

Example of EM Interference

Listening to AM Radio

Suppose you are listening to AM650 (650kHz) and you live 23km from the radio tower. There is another building halfway between you and the tower and radio waves are bouncing off of that building. How far off to the side is the building if destructive interference occurs between the direct and reflected waves? (assume equal amplitudes and no phase shift on reflection)



(i.e., What is d ??)

Example of EM Interference

- Direct path: $L_1 = 23$ km
- Indirect path:

$$\begin{aligned} L_2 &= 2\sqrt{\left(\frac{L_1}{2}\right)^2 + d^2}, \text{Pythagoras} \\ &= \sqrt{L_1^2 + 4d^2} \end{aligned}$$

- Path difference: $L_2 - L_1 = \sqrt{L_1^2 + 4d^2} - L_1$
- We want destructive interference:

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \phi_0 = \left(m + \frac{1}{2}\right) \cdot 2\pi \text{ rad}, m = 0, 1, 2, 3, \dots$$

Example of EM Interference

- Destructive interference:

$$\frac{\Delta x}{\lambda} 2\pi = (2m + 1)\pi$$

$$\Delta x = \lambda \left(m + \frac{1}{2}\right)$$

$$L_2 - L_1 = \lambda \left(m + \frac{1}{2}\right) = \sqrt{L_1^2 + 4d^2} - L_1$$

- Take the minimum d as $m = 0$

$$\frac{\lambda}{2} = \sqrt{L_1^2 + 4d^2} - L_1$$

$$L_1 + \frac{\lambda}{2} = \sqrt{L_1^2 + 4d^2}$$

$$L_1^2 + L_1\lambda + \frac{\lambda^2}{4} = L_1^2 + 4d^2$$

Example of EM Interference

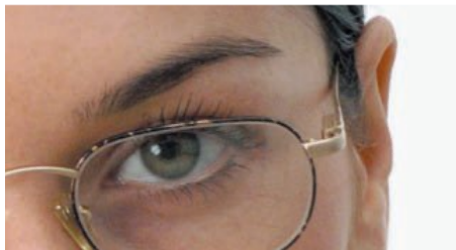
- Now solve for d :

$$\begin{aligned}L_1^2 + L_1\lambda + \frac{\lambda^2}{4} &= L_1^2 + 4d^2 \\ \cancel{L_1^2} + L_1\lambda + \frac{\lambda^2}{4} &= \cancel{L_1^2} + 4d^2 \\ d &= \frac{1}{2} \sqrt{L_1\lambda + \frac{\lambda^2}{4}}\end{aligned}$$

- Plugging in the numbers:

$$\begin{aligned}\lambda &= \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{650000 \text{ Hz}} = 462 \text{ m} \\ d &= 1.6 \text{ km}\end{aligned}$$

Application: anti-reflective coating



Air

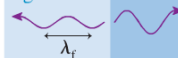
1. Incident wave approaches the first surface.



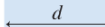
2. Part of the wave reflects back with a phase shift of π rad, part continues on into the film.



3. Part of the transmitted wave reflects at the second surface, part continues on into the glass.



4. The two reflected waves are overlapped and interfere.



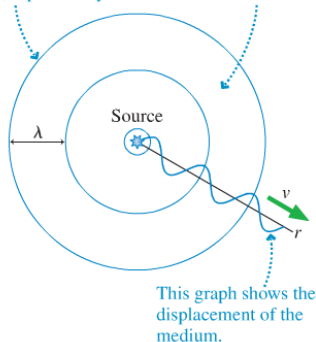
Thin film
Index n

Glass

Interference in 2 or 3 Dimensions

The wave fronts are crests, separated by λ .

Troughs are halfway between wave fronts.



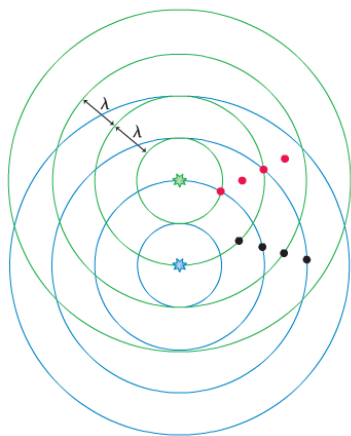
Working in 2 or 3 dimensions is not very different from working in 1-D:

$$D(r, t) = a \sin(kr - \omega t + \phi_0)$$

where r is the distance measured outwards from the source. Essentially we just replace x everywhere with a radial coordinate r ...

Interference in 2 or 3 Dimensions

Two in-phase sources emit circular or spherical waves.

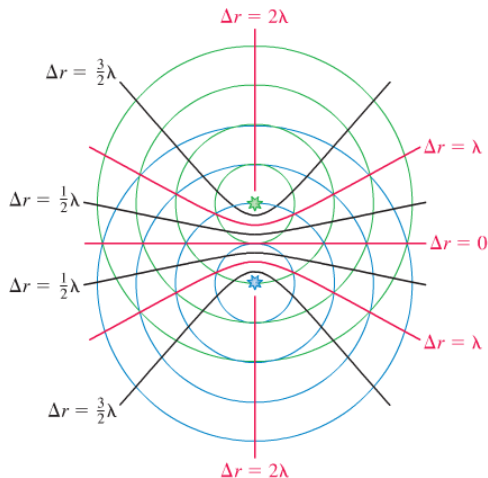


- Points of constructive interference.
A crest is aligned with a crest, or a trough with a trough.
- Points of destructive interference.
A crest is aligned with a trough of another wave.

Interference also occurs with spherical waves. Again we look for places where crests or troughs align. The phase difference is now

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0$$

Nodal and Anti-Nodal Lines



- Antinodal lines, constructive interference oscillation with maximum amplitude. Intensity is at its maximum value.
- Nodal lines, destructive interference, no oscillation. Intensity is zero.

Simulation: <http://phet.colorado.edu/sims/wave-interference>