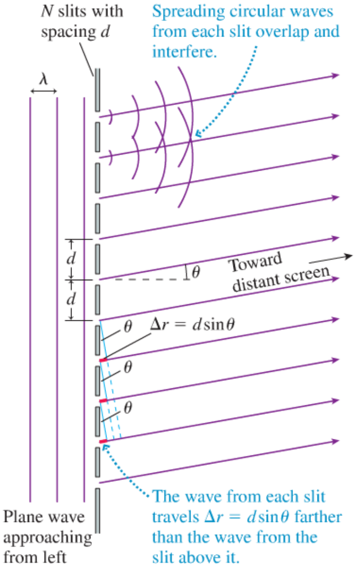
The Diffraction Grating



If one extends the double slit to large number of slits very closely spaced, one gets what is called a diffraction grating. $d \sin \theta$. Maxima are still at

$$d \sin \theta_m = m\lambda, m = 0, 1, 2, 3, \ldots$$

The difference is that the fringes are thinner and brighter.

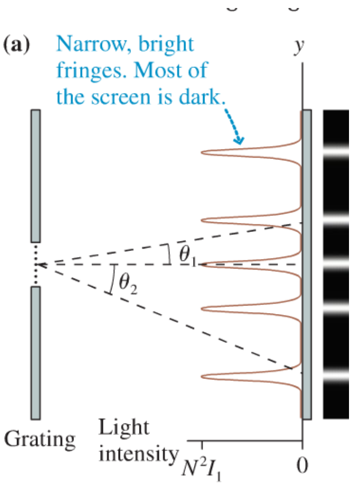
The Diffraction Grating

- Lines of high intensity occur only where the wavefronts from all the slits interfere constructively. Therefore the maxima are very intense and very narrow.
- The angle from the middle of the grating to the maxima is given by

$$d \sin\theta_m = m\lambda, m = 0, 1, 2, 3, \ldots$$

- The distance from the central maximum to the next maximum is given by

$$y_m = L \tan\theta_m$$

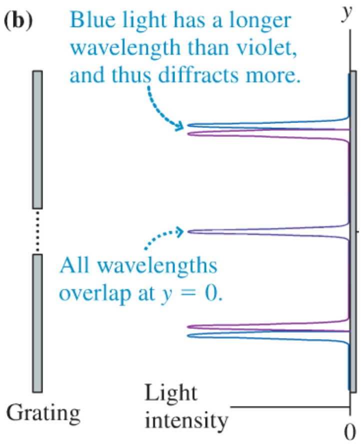


The Diffraction Grating

- The angles to the maxima are not small. Therefore, the small angle approximation cannot be used. The distance on the screen to the bright lines is given by

$$y_m = L \tan\left[\sin^{-1}\left(\frac{m\lambda}{d}\right)\right]$$

- The distances to the maxima provide a good way of determining wavelengths of light.
- Diffraction gratings are essential components of optical spectrometers.



The Diffraction Grating

Why are the fringes thinner and brighter?

- The brightness comes from the fact that constructive interference is now happening from more sources:

$$I_{max} = N^2 I_1$$

- One way to think about the narrowness is conservation of energy. If the bright fringes will be so much brighter (gain a square in the formula above) then they must be narrower too.

The Diffraction Grating

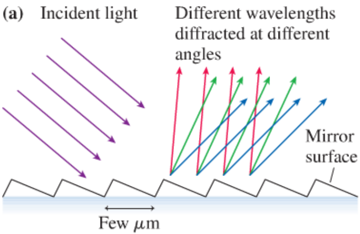
Why are the fringes thinner and brighter?
Another way to think of it is to look at the geometry again. Imagine we introduce a small phase difference between adjacent slits, say 1.1λ instead of 1λ

- With two adjacent slits, this does not affect the intensity very much
- Now imagine the interference of 2 slits which are separated by 5d (non-adjacent slits). $5 \times 1.1\lambda = 5.5\lambda$. Now these two slits are destructively interfering!
- Total width of the central max is (for large number of slits)

$$2\theta_{min} = \frac{2\lambda}{Nd}$$

Measuring θ_{min} can be another way to determine an unknown λ

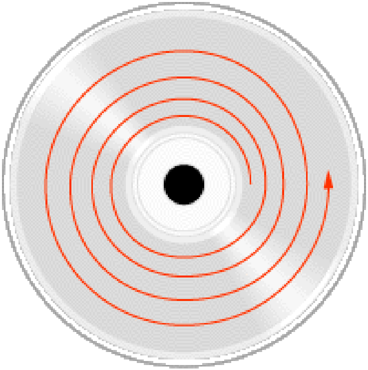
Reflection Gratings

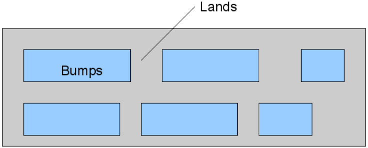


A reflection grating can be made by cutting parallel grooves in a mirror surface. These can be very precise, for scientific use, or mass produced in plastic.

- Many common gratings are actually reflection gratings rather than transmission gratings.
- A mirror with thousands of narrow parallel grooves makes a grating which reflects light instead of transmitting it, but the math is the same.
- A CD is an excellent example.

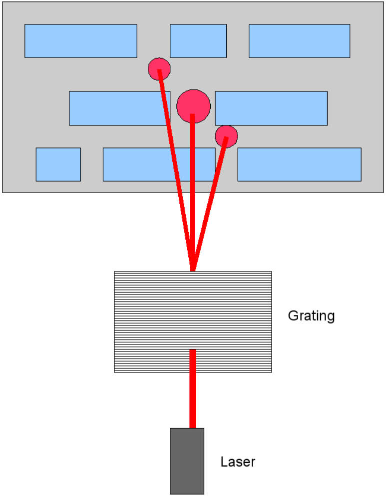
Compact Discs





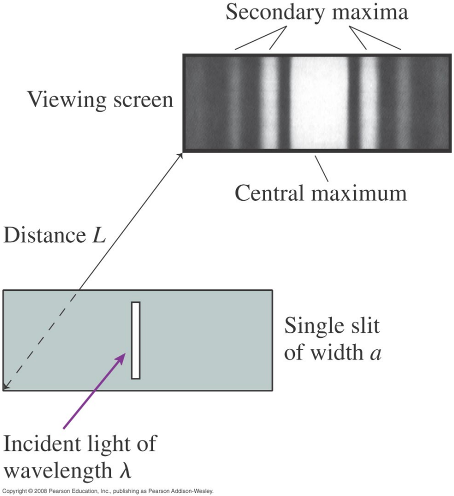
- A CD is a reflective surface covered in small bumps/pits. Each second of music requires $\sim 10^6$ bumps.
- From above there are "pits" in an aluminum layer, but CD is read from below, so "bumps" are seen.
- A CD player shines a laser on the bumps in a spiral track and detects the reflected laser beam.
- The intensity of the reflected beam changes as the bumps and lands pass by. Height of a bump is about $\lambda/4$, making a path difference of $\lambda/2$ and a phase change of π.

Compact Discs Tracking System



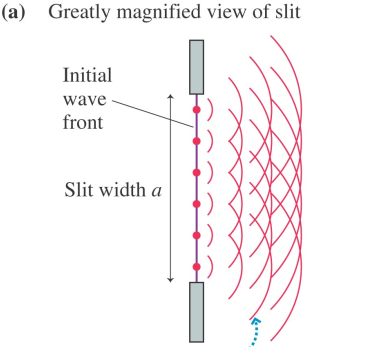
A diffraction grating is used. The central maximum reads the bumps while the 1st-order maxima are used as tracking beams (they should not fluctuate).

Single Slit Diffraction



- It is rather strange to talk about thousands of slits before talking about 1. However, thousands are actually a little easier.
- A single slit diffraction pattern involves a wide central maximum flanked by weaker secondary maxima and dark fringes.
- It would appear that we have only one light source in this case, so how do we understand the interference?
- We have to go back to Huygen's principle.

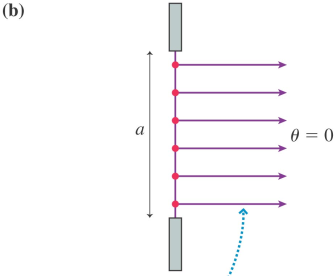
Single Slit Diffraction



The wavelets from each point on the initial wave front overlap and interfere, creating a diffraction pattern on the screen.

- A wave front passes through a narrow slit (width *a*). Note that narrow is important.
- Each point on the wave-front emits a spherical wave
- One slit becomes the source of many interfering wavelets.
- A single slit creates a diffraction pattern on the screen.

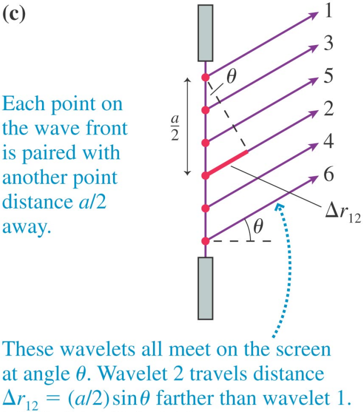
Why the Wide Central Maximum?



The wavelets going straight forward all travel the same distance to the screen. Thus they arrive in phase and interfere constructively to produce the central maximum.

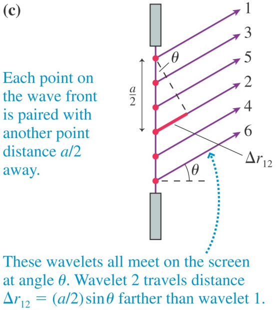
- Wavelets from any part of the slit have to travel approximately the same distance to reach the center of the screen.
- A set of in-phase wavelets therefore produce constructive interference at the center of the screen.

Why the Dark Bands?



- Consider the path-lengths well away from the centre axis
- For any wavelet it is possible to find a partner which is $a/2$ away.
- If the path difference between partners happens to be $\lambda/2$ then this pair will create total destructive intereference. A dark band will be created.
- For any given wavelength there will be an angle for which this condition is true! There will always be dark bands, as long as a is greater than λ and the slit is narrow.

The Mathematics of the Dark Bands



- The path difference between 1 and 2 is

$$\Delta r_{12} = \frac{a}{2}\sin\theta_1 = \frac{\lambda}{2}$$

- What about the other angles for destructive interference? The general formula becomes

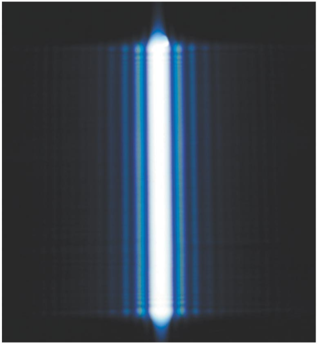
$$a\sin\theta_p = p\lambda, p = 1, 2, 3, \ldots$$

  The small angle approximation means we can write

$$\theta_p = p\frac{\lambda}{a}, p = 1, 2, 3, \ldots$$

- But if a is small then θ_p is large and the small angle approximation is not valid.

The Width of the Bands



- It can be useful to express the fringe position in distance rather than angle.
- The position on the screen is given by $y_p = L \tan \theta_p$. This leads to

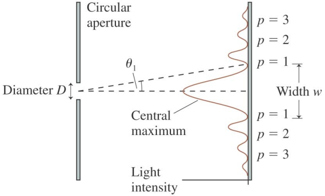
$$y_p = \frac{p\lambda L}{a}, p = 1, 2, 3, \ldots$$

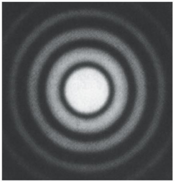
- The width of the central maximum is give by twice the distance to the first dark fringe

$$w = \frac{2\lambda L}{a}$$

It is important to note that: 1) the width grows if the screen is farther away 2) A thinner slit makes a wider central maximum.

Circular Aperture Diffraction



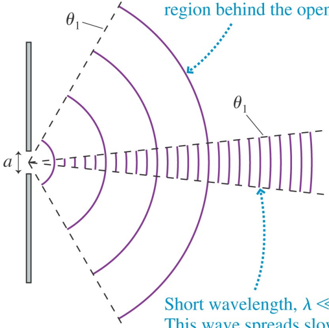


$$\theta_1 = \frac{1.22\lambda}{D}$$

And the width of the central maximum is

$$w = 2y_1 = 2L\tan\theta_1 \approx \frac{2.44\lambda L}{D}$$

The Wave and Ray Models of Light



- The factor that determines how much a wave spreads out is λ/a
- With water or sound we see diffraction in our everyday lives because the wavelength is roughly the same as the macroscopic openings and structures we see around us.
- We will only notice the spreading of light with apertures of roughly the same scale as the wavelength of light.

The Wave and Ray Models of Light

Sometimes we treat light like a stream of particles, sometimes like a wave and sometimes like a ray. Does light travel in a straight line or not? The answer depends on the circumstances.

Choosing a Model of Light

- When light passes through openings $<$ 1mm in size, diffraction effects are usually important. Treat light as a wave.
- When light passes through openings $>$ 1mm in size, treat it as a ray.