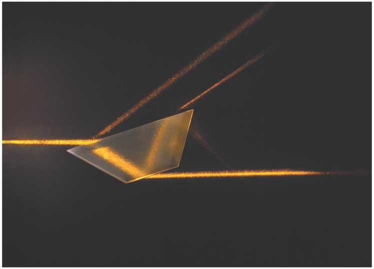
Refraction (23.3)



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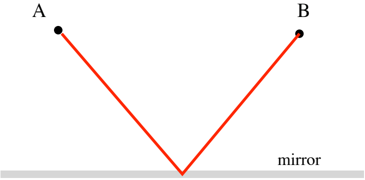
Two things happen when light hits the boundary between transparent materials

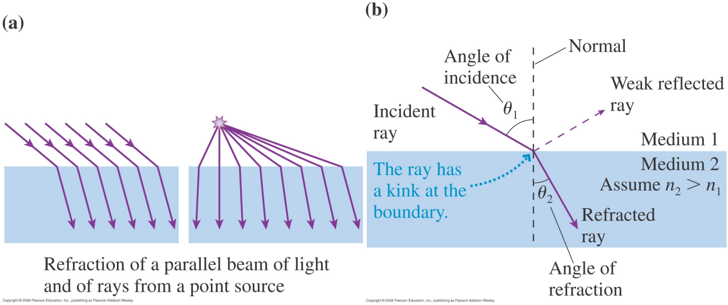
1. Part of the light reflects from the surface
2. Part of the light is transmitted through the second medium with a change of direction. This is called refraction

Fermat's Principle for Reflection



- When light travels from A to B what path will it take?
- Answer: The shortest.
- This is also the path that takes the least time.
- For reflection, the shortest path is the path of least time and this is consistent with the law of reflection.
- Fermat's principle says: Light travelling between two points takes the path of least time.

Refraction



- Simplify by drawing a single ray
- The angle between the incoming ray and the normal is the angle of incidence. In medium 1, use θ_1.
- The angle on the transmitted side from the normal is the angle of refraction. In medium 2, use θ_2.
- Snell's Law tell us that

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

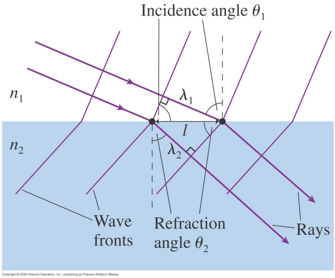
Index of Refraction

We have already mentioned the index of refraction a couple of times in the course...but a quick reminder:

$$n = \frac{c}{v_{medium}}$$

Knowing this true meaning of the index of refraction allows us to predict Snell's Law. When a wave changes to a medium of higher n then it slows-down and the wavelength gets shorter (frequency stays the same). So, how do we draw that?

Index of Refraction

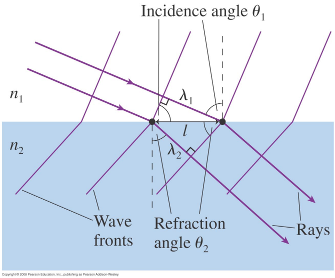


- Wave fronts are crests of waves
- The wavelength in a medium is

$$\lambda = \frac{\lambda_0}{n}$$

- Wave fronts are perpendicular to rays
- In each medium the wave fronts are parallel to each other.

Index of Refraction



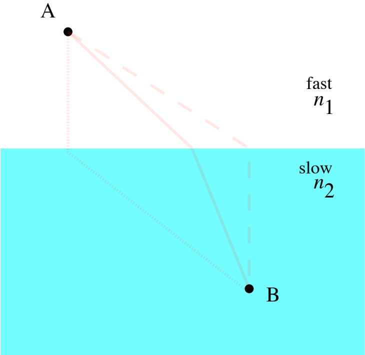
- Using upper and lower triangles:

$$l = \frac{\lambda_1}{\sin\theta_1} \qquad\qquad l = \frac{\lambda_2}{\sin\theta_2}$$

- Setting these equal to each other and using $\lambda_1 = \lambda_0/n_1$, $\lambda_2 = \lambda_0/n_2$ gives

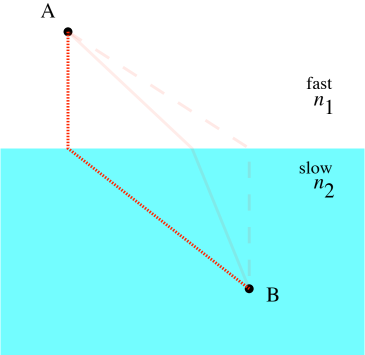
$$\frac{\lambda_0}{n_1 \sin\theta_1} = \frac{\lambda_0}{n_2 \sin\theta_2}$$
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Fermat's Principle for Refraction



- Fermat's principle: Light takes the path of least time when it goes between two points.
- The principle applies to light going between two media as well.
- Which path would have the shortest time?

Fermat's Principle for Refraction



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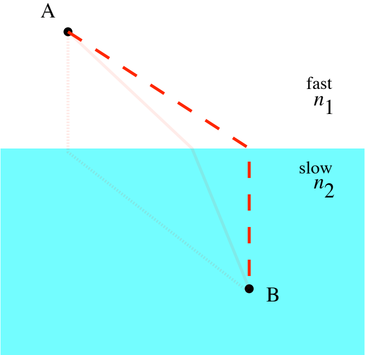
Fermat's Principle for Refraction



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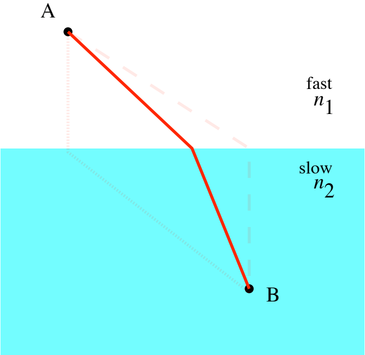
Fermat's Principle for Refraction



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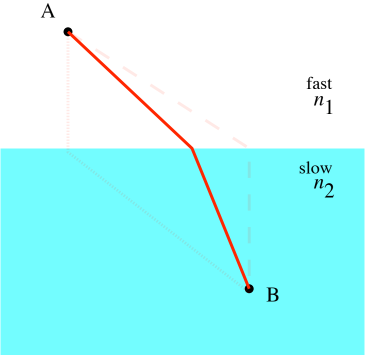
Fermat's Principle for Refraction

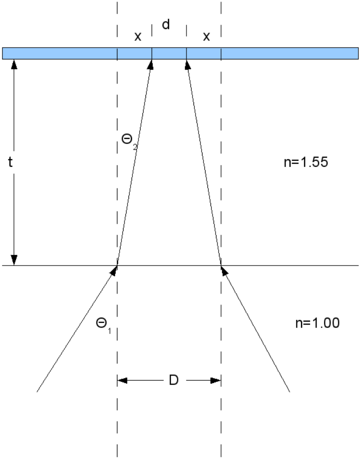


- It turns out that Fermat's Principle is consistent with Snell's law.
- See problem CP23.80 in the textbook.

Fermat's Principle, Philisophical Reflection

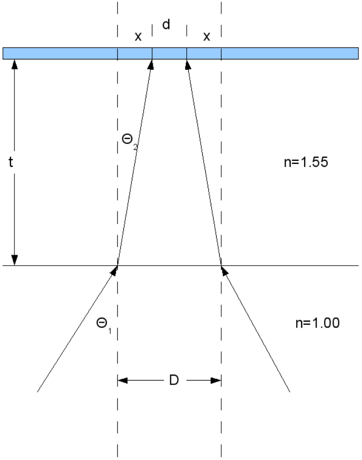
- Some physicists wax poetic about the beauty of Fermat's Principle
- Perhaps they think it's more "fundamental" than Snell's law.
- Warning: {beginning philosophy}
- But on reflection, they are really answers to two different questions.
- Snell's law tells you which way a ray of light will go when it enters another medium.
- Fermat's principle tells you which path a ray of light took if you know the starting and ending points.
- It's nice that they are consistent, but they are not interchangeable.
- {end of philosophy}

Example: Refraction and CDs



The laser beam that reads information from a CD has a diameter $D = 0.737$ mm where it strikes the underside of the disk and forms a converging cone with half-angle $\theta_1 = 27^\circ$. It then travels through $t = 1.2$ mm of transparent plastic with $n = 1.55$ before reaching the reflective information layer near the top surface. What is the beam diameter d at the information layer?

Example: Refraction and CDs



- We can see that

$$d = D - 2x, \; x = t \tan \theta_2$$

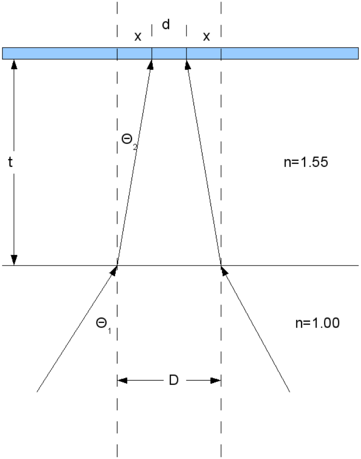
- Snell's Law gives

$$\theta_2 = \sin^{-1}\left(\frac{\sin \theta_1}{n}\right)$$

- Substituting

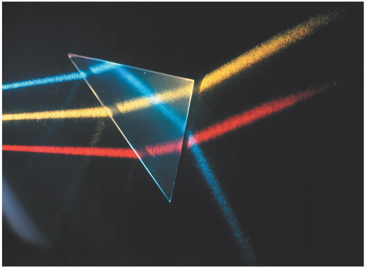
$$\begin{aligned} d &= D - 2t \tan\left[\sin^{-1}\left(\frac{\sin \theta_1}{n}\right)\right] \\ &= .737 \text{ mm} - \\ &\quad (2)(1.2 \text{ mm}) \tan\left[\sin^{-1}\left(\frac{\sin 27^\circ}{1.55}\right)\right] \\ &= 1.8 \; \mu\text{m} \end{aligned}$$

Example: Refraction and CDs



- The bumps in a CD are about 0.6 μm wide, 0.9 to 3.3 μm long and 0.12 μm deep.
- The beam needs to be narrowed in order to work!
- Crucial for controlling noise. An original beam only microns across would be disrupted by dust only microns across (typical dust is 1 to 100 μm). Now dust on the surface must be millimetre-scale to blot out information.

Total Internal Reflection (23.3)



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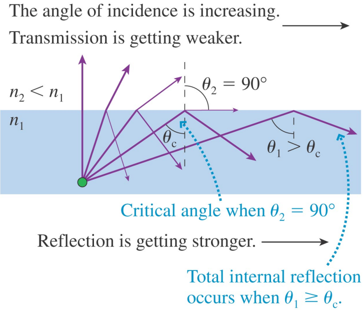

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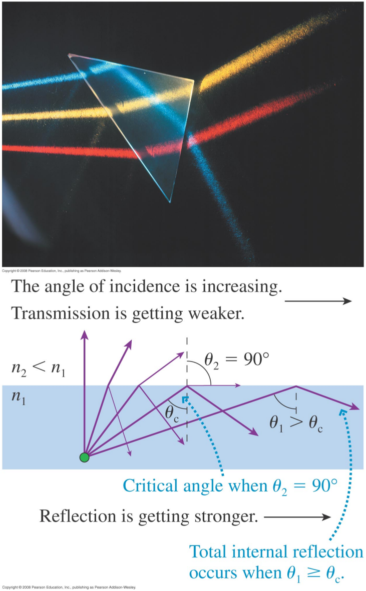
- If light strikes a boundary in which it transitions from a high index of refraction to a lower one, it can undergo Total Internal Reflection (TIR).
- The figure on the left shows several rays leaving a source inside a high-n medium. As the angle of incidence gets larger the angle of refraction gets closer and closer to 90°.
- When the angle of refraction (θ_2) is exactly 90 degrees we reach the critical angle. Above the critical angle there is no transmitted light.

Total Internal Reflection (23.3)



- Snell's Law at the critical angle gives

$$n_1 \sin \theta_c = n_2 \sin 90°$$

- Solving for θ_c gives

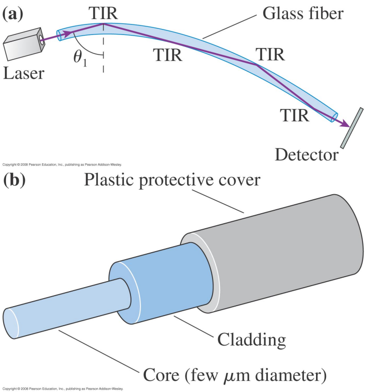
$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

- An example for glass:

$$\theta_c = \sin^{-1}\left(\frac{1.00}{1.50}\right) = 42°$$

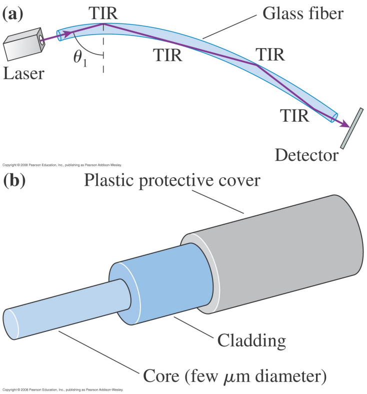
- There is no TIR if $n_2 > n_1$

Fibre Optics



- Fiber optics is an important application of TIR
- Shine a laser beam into the end of a glass tube at an incident angle close to 90°.
- Let the light bounce down the "light-pipe" until it reaches the end.
- They are covered in lower-index cladding to prevent light leakage (e.g., scratches).

Fibre Optics



- Tremendous advantages for transmitting information:
  - Less expensive than copper
  - Thinner than copper
  - Different wavelengths can carry different information (e.g., light-path to Fermilab)
  - No cross-talk between fibers
  - Lower power (less degradation)
  - No fire hazard.