Tutorial 1 Problems

A selection of the following problems were done:

Workbook (2nd edition), Chapter 20:
1, 2, 3, 4, 7, 13, 17, 20, 22, 25

20 Traveling Waves

20.1 The Wave Model

1. a. In your own words, define what a *transverse wave* is.

In a transverse wave, the quantity that is oscillating, such as the particles in a string, oscillates in a direction that is transverse to the direction of propagation of the wave.

b. Give an example of a wave that, from your own experience, you know is a transverse wave. What observations or evidence tells you this is a transverse wave?

Vibrations of a bass guitar string are a form of transverse wave. You can see that the oscillation is perpendicular to the string.

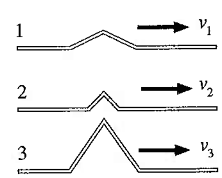
2. a. In your own words, define what a *longitudinal wave* is.

In a longitudinal wave, the oscillations are parallel to the direction of propagation of the wave.

b. Give an example of a wave that, from your own experience, you know is a longitudinal wave. What observations or evidence tells you this is a longitudinal wave?

Waves on a slinky created by pushing and pulling along the length of the slinky are longitudinal. You can observe that the slinky remains straight, with no transverse disturbance as the wave propagates.

3. Three wave pulses travel along the same string. Rank in order, from largest to smallest, their wave speeds v_1, v_2, and v_3.



Order: $V_1 = V_2 = V_3$

Explanation:

Wave speed is independent of wave amplitude.

20.2 One-Dimensional Waves

4. A wave pulse travels along a string at a speed of 200 cm/s. What will be the speed if:
Note: Each part below is independent and refers to changes made to the original string.

a. The string's tension is doubled?

$v = \sqrt{\frac{T}{\mu}}$ so $v' = \sqrt{\frac{2T}{\mu}} = \sqrt{2}\sqrt{\frac{T}{\mu}} = \sqrt{2}v$ or $v' \doteq 1.4v$

$\boxed{v' = 280 \text{ cm/s}}$

b. The string's mass is quadrupled (but its length is unchanged)?

$\mu = \frac{m}{L}$; $v' = \sqrt{\frac{T}{4\mu}} = \frac{1}{2}\sqrt{\frac{T}{\mu}} = \frac{v}{2}$

so $\boxed{v' = 100 \text{ cm/s}}$

c. The string's length is quadrupled (but its mass is unchanged)?

$\mu = \frac{m}{L}$ so $\mu' = \frac{m}{4L} = \mu/4$ $\quad v' = \sqrt{\frac{T}{\mu/4}} = 2\sqrt{\frac{T}{\mu}}$

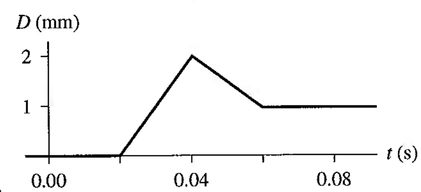
$\boxed{v' = 400 \text{ cm/s}}$

d. The string's mass and length are both quadrupled?

$\mu' = \frac{4m}{4L} = \frac{m}{L}$ so the speed is unchanged.

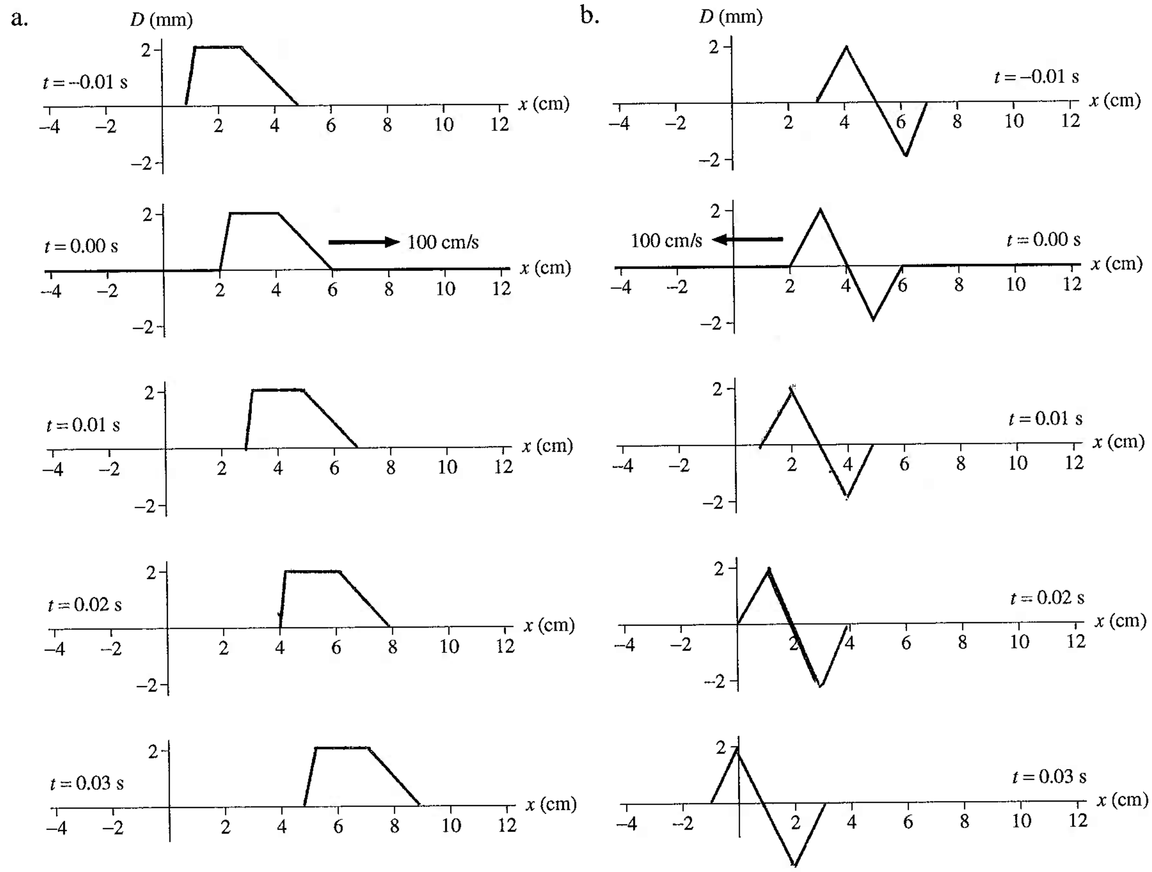
$\boxed{v' = 200 \text{ cm/s}}$

5. This is a history graph showing the displacement as a function of time at one point on a string. Did the displacement at this point reach its maximum of 2 mm *before* or *after* the interval of time when the displacement was a constant 1 mm? Explain how you interpreted the graph to answer this question.

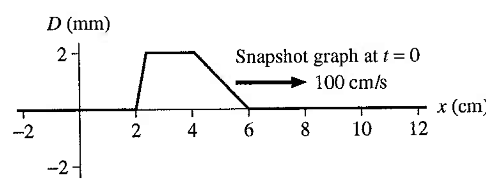


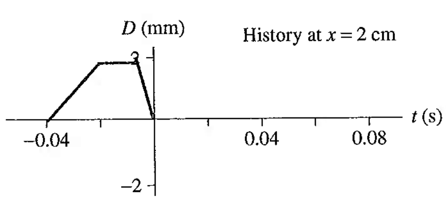
The constant 1mm displacement interval appears at later times. Therefore, the displacement at this point reached 2mm before it settled at 1mm.

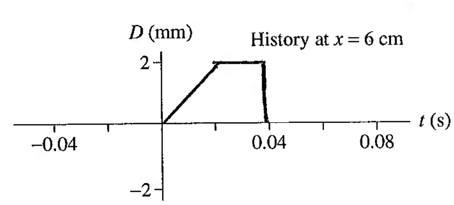
6. Each figure below shows a snapshot graph at time $t = 0$ s of a wave pulse on a string. The pulse on the left is traveling to the right at 100 cm/s; the pulse on the right is traveling to the left at 100 cm/s. Draw snapshot graphs of the wave pulse at the times shown next to the axes.



7. This snapshot graph is taken from Exercise 6a. On the axes below, draw the *history* graphs $D(x = 2\text{ cm}, t)$ and $D(x = 6\text{ cm}, t)$, showing the displacement at $x = 2$ cm and $x = 6$ cm as functions of time. Refer to your graphs in Exercise 6a to see what is happening at different instants of time.

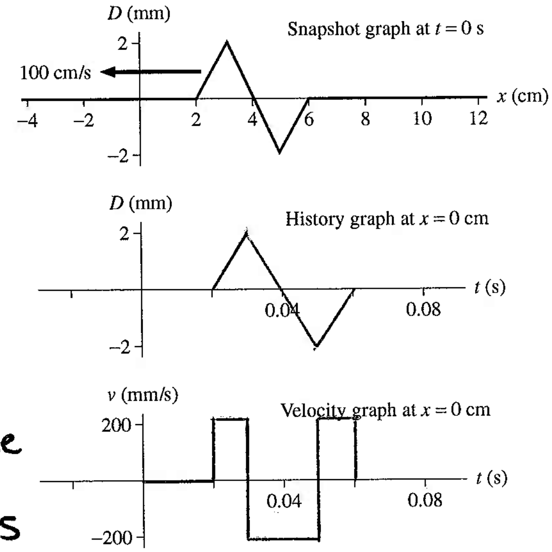






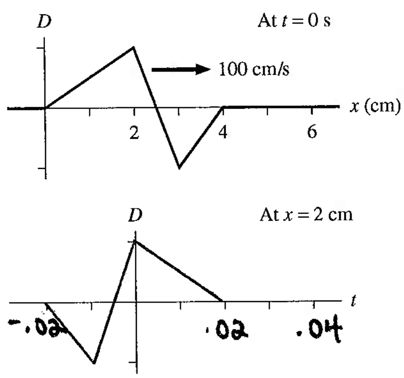
8. This snapshot graph is from Exercise 6b.
   a. Draw the history graph $D(x = 0\text{ cm}, t)$ for this wave at the point $x = 0$ cm.
   b. Draw the *velocity*-versus-time graph for the piece of the string at $x = 0$ cm. Imagine painting a dot on the string at $x = 0$ cm. What is the velocity of this dot as a function of time as the wave passes by?
   c. As a wave passes through a medium, is the speed of a particle in the medium the same as or different from the speed of the wave through the medium? Explain.

The speeds are different. The particle's speed oscillates while the wave speed remains constant.

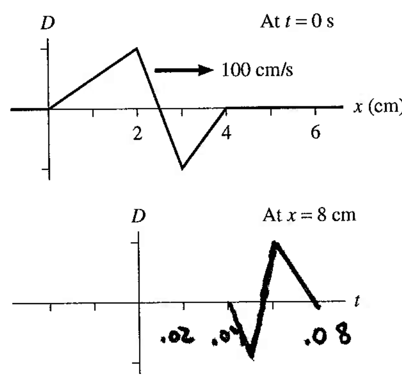


9. Below are four snapshot graphs of wave pulses on a string. For each, draw the history graph at the specified point on the x-axis. No time scale is provided on the t-axis, so you must determine an appropriate time scale and label the t-axis appropriately.

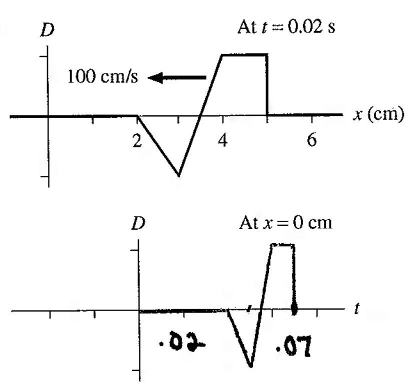
a.



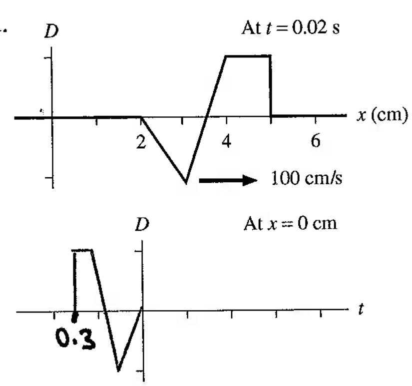
b.



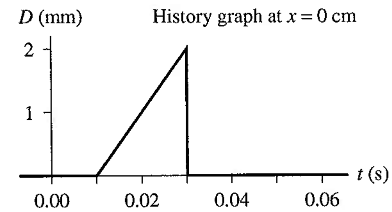
c.



d.



10. A history graph $D(x = 0\text{ cm}, t)$ is shown for the $x = 0$ cm point on a string. The pulse is moving to the right at 100 cm/s.



a. Does the $x = 0$ cm point on the string rise quickly and then fall slowly, or rise slowly and then fall quickly? Explain.

It rises slowly and then falls quickly. (The slow rise appears at earlier times.)

b. At what time does the leading edge of the wave pulse arrive at $x = 0$ cm? 0.1s

c. At $t = 0$ s, how far is the leading edge of the wave pulse from $x = 0$ cm? Explain.

$\Delta x = v\Delta t$; $\boxed{1\text{cm}} = (100\frac{\text{cm}}{\text{s}})(0.1\text{s})$

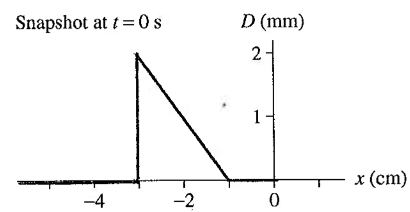
d. At $t = 0$ s, is the leading edge to the right or to the left of $x = 0$ cm? left

e. At what time does the trailing edge of the wave pulse leave $x = 0$ cm? 0.3s

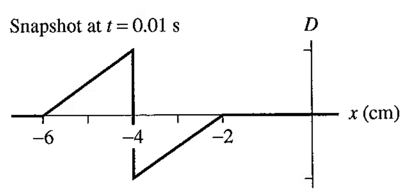
f. At $t = 0$ s, how far is the trailing edge of the pulse from $x = 0$ cm? Explain.

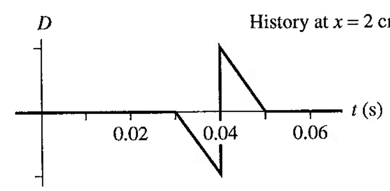
$(100\frac{\text{cm}}{\text{s}})(0.3\text{s}) = \boxed{3\text{cm}}$

g. By referring to the answers you've just given, draw a snapshot graph $D(x, t = 0\text{ s})$ showing the wave pulse on the string at $t = 0$ s.



11. These are a history graph *and* a snapshot graph for a wave pulse on a string. They describe the same wave from two perspectives.





a. In which direction is the wave traveling? Explain.

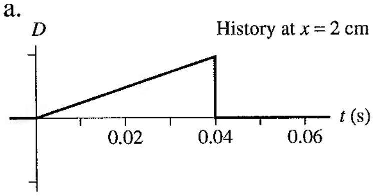
The wave is traveling to the right. It reaches the 2cm point at later times, starting from the left.

b. What is the speed of this wave?

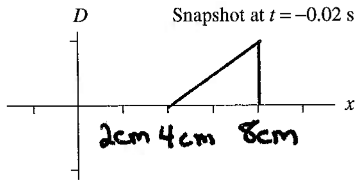
$200\frac{\text{cm}}{\text{s}}$ The leading edge reaches the 2cm point at $t = 0.03$s. At $t = 0.01$s, it was 4cm to the left at −2cm. $v = \frac{\Delta x}{\Delta t} = \frac{4\text{cm}}{(0.03\text{s} - 0.01\text{s})} = 200\frac{\text{cm}}{\text{s}}$.

12. Below are two history graphs for wave pulses on a string. The speed and direction of each pulse are indicated. For each, draw the snapshot graph at the specified instant of time. No distance scale is provided, so you must determine an appropriate scale and label the x-axis appropriately.

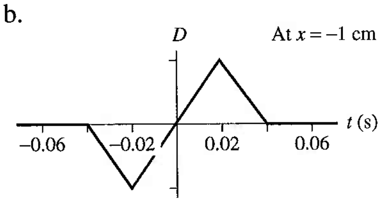
a.



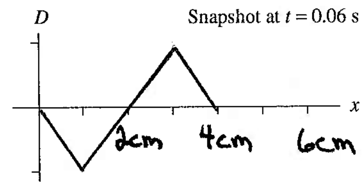
100 cm/s to the left



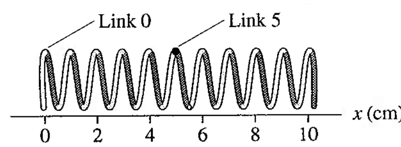
b.

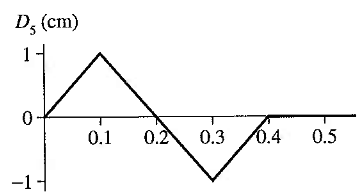


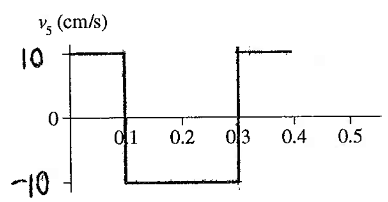
50 cm/s to the left



13. A horizontal Slinky is at rest on a table. A wave pulse is sent along the Slinky, causing the top of link 5 to move *horizontally* with the displacement shown in the graph.







a. Is this a transverse or a longitudinal wave? Explain.

Longitudinal, because the particle motion is parallel to the wave motion.

b. What is the position of link 5 at $t = 0.1$ s? 6cm

What is the position of link 5 at $t = 0.2$ s? 5cm

What is the position of link 5 at $t = 0.3$ s? 4cm

Note: *Position*, not displacement.

c. Draw a velocity-versus-time graph of link 5. Add an appropriate scale to the vertical axis.

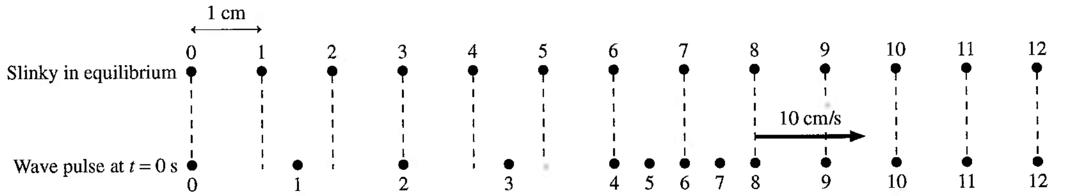
d. Can you determine, from the information given, whether the wave pulse is traveling to the right or to the left? If so, give the direction and explain how you found it. If not, why not?

You can't tell from the motion of only one point. It could be moving to the left if propagated by pulling on the right end or moving to the right if propagated by pushing on the left end.

e. Can you determine, from the information given, the speed of the wave? If so, give the speed and explain how you found it. If not, why not?

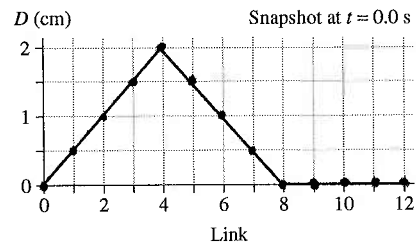
You cannot determine the speed without more information about the motion of other points.

14. We can use a series of dots to represent the positions of the links in a Slinky. The top set of dots shows a Slinky in equilibrium with a 1 cm spacing between the links. A wave pulse is sent down the Slinky, traveling to the right at 10 cm/s. The second set of dots shows the Slinky at $t = 0$ s. The links are numbered, and you can measure the displacement Δx of each link.



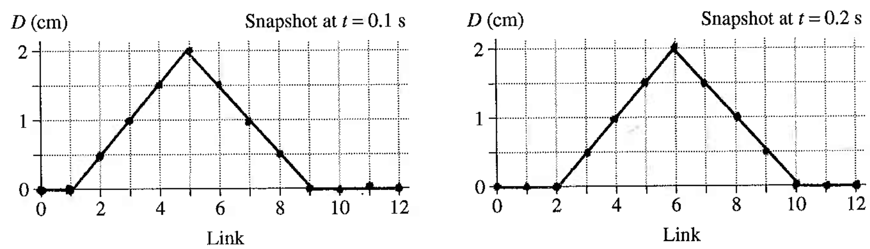
a. Draw a snapshot graph showing the displacement of each link at $t = 0$ s. There are 13 links, so your graph should have 13 dots. Connect your dots with lines to make a continuous graph.

b. Is your graph a "picture" of the wave or a "representation" of the wave? Explain.

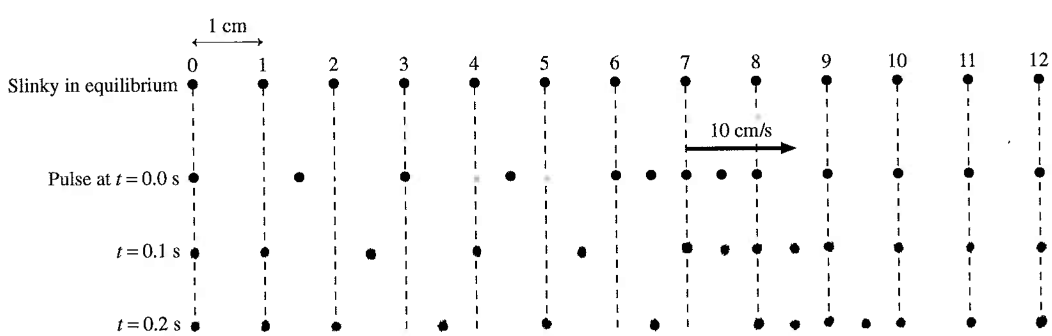


The graph is a representation of the wave. The displacement is not perpendicular to the length.

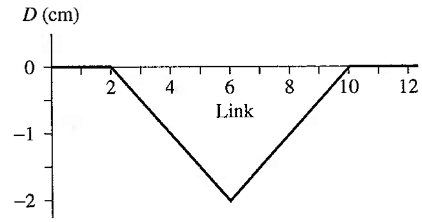
c. Draw graphs of displacement versus the link number at $t = 0.1$ s and $t = 0.2$ s.

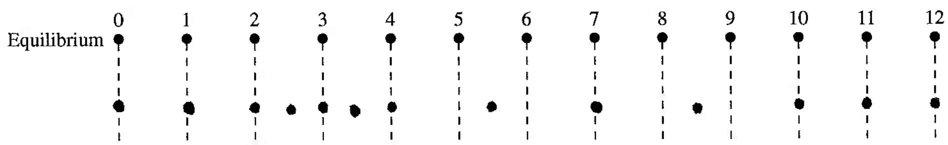


d. Now draw dot pictures of the links at $t = 0.1$ s and $t = 0.2$ s. The equilibrium positions and the $t = 0$ s picture are shown for reference.

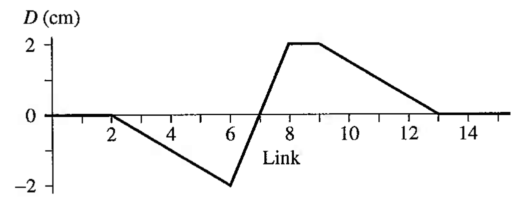


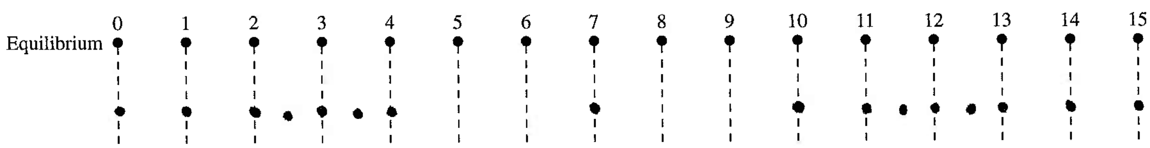
15. The graph shows displacement versus the link number for a wave pulse on a Slinky. Draw a dot picture showing the Slinky at this instant of time. A picture of the Slinky in equilibrium, with 1 cm spacings, is given for reference.





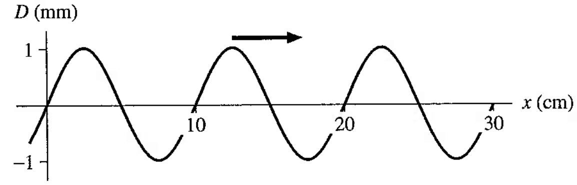
16. The graph shows displacement versus the link number for a wave pulse on a Slinky. Draw a dot picture showing the Slinky at this instant of time. A picture of the Slinky in equilibrium, with 1 cm spacings, is given for reference.



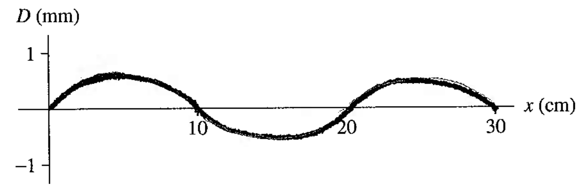


20.3 Sinusoidal Waves

17. The figure shows a sinusoidal traveling wave. Draw a graph of the wave if:



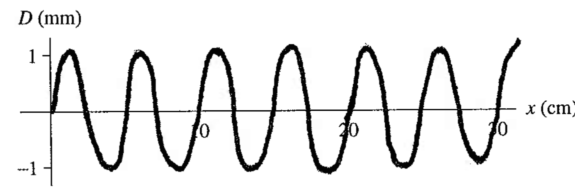
a. Its amplitude is halved and its wavelength is doubled.



b. Its speed is doubled and its frequency is quadrupled.

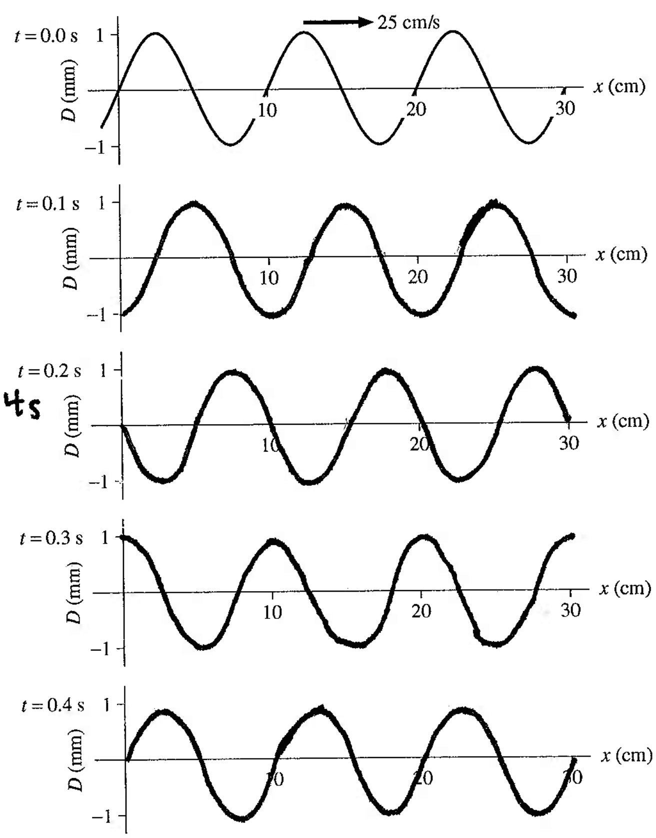
$v = f\lambda$ or $\lambda = \frac{v}{f}$

$\lambda' = \frac{2v}{4f} = \frac{1}{2}\lambda$



18. The wave shown at time $t = 0$ s is traveling to the right at a speed of 25 cm/s.

a. Draw snapshot graphs of this wave at times $t = 0.1$ s, $t = 0.2$ s, $t = 0.3$ s, and $t = 0.4$ s.



b. What is the wavelength of the wave?

10cm

c. Based on your graphs, what is the period of the wave?

The shape repeats every 0.4s so the period is $T = \boxed{0.4s}$

d. What is the frequency of the wave?

$\frac{1}{T} = f = 2.5\,Hz$

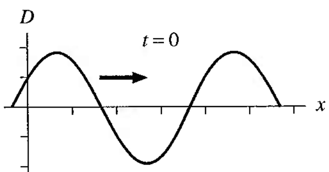
e. What is the value of the product λf?

$\lambda f = (10cm)(2.5s^{-1}) = 25\frac{cm}{s}$

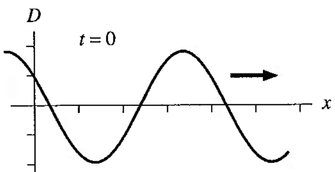
f. How does this value of λf compare to the speed of the wave?

λf is the speed of the wave.

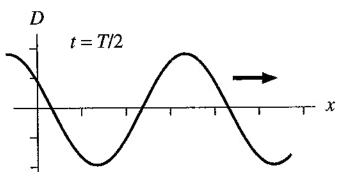
19. Three waves traveling to the right are shown below. The first two are shown at $t = 0$, the third at $t = T/2$. What are the phase constants ϕ_0 of these three waves?



$\phi_0 = \frac{\pi}{6} = 30°$



$\phi_0 = \frac{5\pi}{6} = 150°$

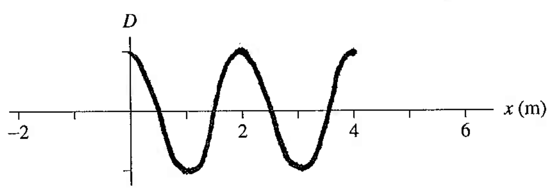


$\phi_0 = \frac{-\pi}{6} = -30°$

Note: Knowing the displacement $D(0,0)$ is a *necessary* piece of information for finding ϕ_0 but is not by itself enough. The first two waves above have the same value for $D(0,0)$ but they do *not* have the same ϕ_0. You must also consider the overall shape of the wave.

20. A sinusoidal wave with wavelength 2 m is traveling along the x-axis. At $t = 0$ s the wave's phase at $x = 2$ m is $\pi/2$ rad.

a. Draw a snapshot graph of the wave at $t = 0$ s.



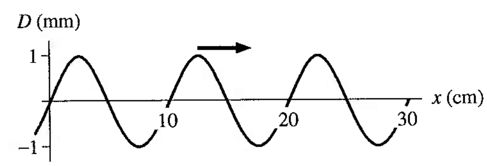
b. At $t = 0$ s, what is the phase at $x = 0$ m? $\pi/2$

c. At $t = 0$ s, what is the phase at $x = 1$ m? π

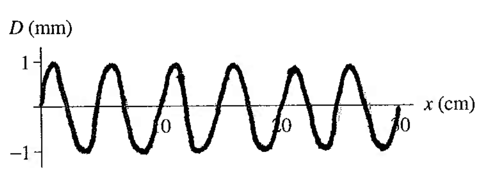
d. At $t = 0$ s, what is the phase at $x = 3$ m? π

Note: No calculations are needed. Think about what the phase *means* and utilize your graph.

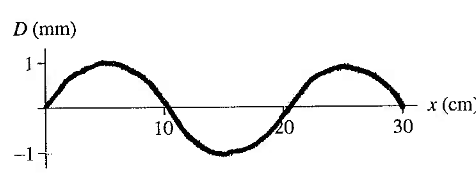
21. Consider the wave shown. Redraw this wave if:



a. Its wave number is doubled.

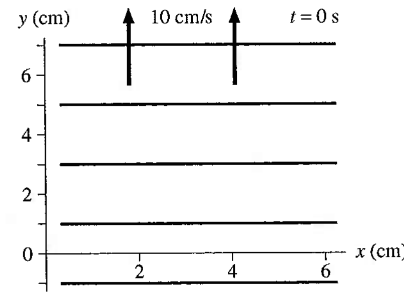


b. Its wave number is halved.



20.4 Waves in Two and Three Dimensions

22. A wave-front diagram is shown for a sinusoidal plane wave at time $t = 0$ s. The diagram shows only the xy-plane, but the wave extends above and below the plane of the paper.



a. What is the wavelength of this wave? 2cm

b. At $t = 0$ s, for which values of y is the wave a crest?

-1, 1, 3, 5, 7cm

c. At $t = 0$ s, for which values of y is the wave a trough?

0, 2, 4, 6 cm

d. Can you tell if this is a transverse or a longitudinal wave? If so, which is it and how did you determine it? If not, why not?

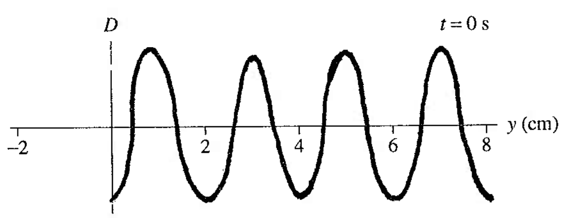
You cannot tell from this diagram. The location of the crests does not tell you the direction of their displacement.

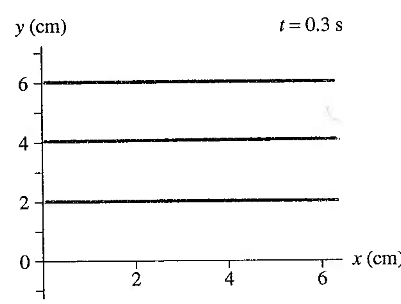
e. How does the displacement at the point $(x, y, z) = (6, 5, 0)$ compare to the displacement at the point $(2, 5, 0)$? Is it more, less, the same, or is there no way to tell? Explain.

The displacements are the same because their y-values are the same. The wave front is defined by constant y-values.

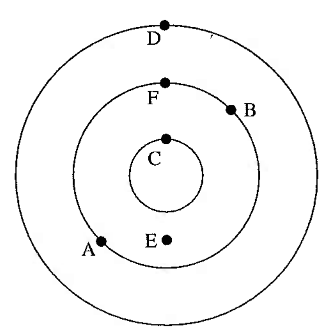
f. On the left axes below, draw a snapshot graph $D(y, t = 0\text{ s})$ along the y-axis at $t = 0$ s.

g. On the right axes below, draw a wave-front diagram at time $t = 0.3$ s.





23. These are the wave fronts of a circular wave. What is the phase difference between:



a. Points A and B? 0

b. Points C and D? 4π (720°)

c. Points E and F? π (180°)

20.5 Sound and Light

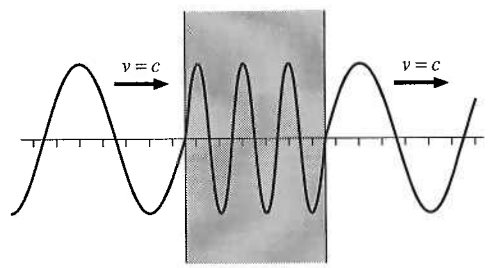
24. Rank in order, from largest to smallest, the wavelengths λ_1 to λ_3 for sound waves having frequencies $f_1 = 100$ Hz, $f_2 = 1000$ Hz, and $f_3 = 10{,}000$ Hz.

Order: $\lambda_1 > \lambda_2 > \lambda_3$

Explanation:

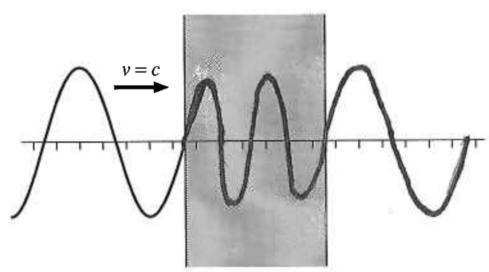
$v = f\lambda \approx$ constant for these frequencies, so large f implies small λ.

25. A light wave travels from vacuum, through a transparent material, and back to vacuum. What is the index of refraction of this material? Explain.



$n = 3$. The frequency does not change, so that the change in wavelength also tells the change in speed. Here $\lambda \Rightarrow \lambda/3$ so $v = \frac{c}{3}$ and $n = 3$

26. A light wave travels from vacuum, through a transparent material whose index of refraction is $n = 2.0$, and back to vacuum. Finish drawing the snapshot graph of the light wave at this instant.



20.6 Power, Intensity, and Decibels

27. A laser beam has intensity I_0.

a. What is the intensity, in terms of I_0, if a lens focuses the laser beam to $\frac{1}{10}$ its initial diameter?

$$\frac{I_1}{I_0} = \frac{r_0^2}{r_1^2} = \frac{r_0^2}{(r_0/10)^2} = 100 \quad \boxed{I_1 = 100I_0}$$

b. What is the intensity, in terms of I_0, if a lens defocuses the laser beam to 10 times its initial diameter?

$$\frac{I_1}{I_0} = \frac{r_0^2}{r_1^2} = \frac{r_0^2}{(10r_0)^2} = \frac{1}{100} \quad \boxed{I_1 = \frac{I_0}{100}}$$

28. Sound wave A delivers 2 J of energy in 2 s. Sound wave B delivers 10 J of energy in 5 s. Sound wave C delivers 2 mJ of energy in 1 ms. Rank in order, from largest to smallest, the sound powers P_A, P_B, and P_C of these three sound waves.

Order: $P_C = P_B > P_A$

Explanation:

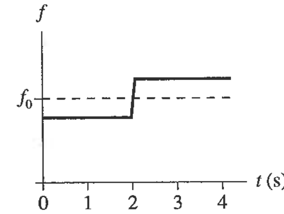
$$P_A = \frac{2J}{2s} = 1W;\ P_B = \frac{10J}{5s} = 2W;\ P_C = \frac{2mJ}{1ms} = 2W$$

29. A giant chorus of 1000 male vocalists is singing the same note. Suddenly 999 vocalists stop, leaving one soloist. By how many decibels does the sound intensity level decrease? Explain.

The sound decreases by 30 decibels. Assuming they are all singing at the same level, the intensity drops to $\frac{1}{1000}$ or $\frac{1}{10^3}$, which is 30 decibels.

20.7 The Doppler Effect

30. You are standing at $x = 0$ m, listening to a sound that is emitted at frequency f_0. The graph shows the frequency you hear during a four-second interval. Which of the following describes the sound source?

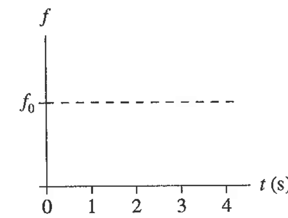


i. It moves from left to right and passes you at $t = 2$ s.
ii. It moves from right to left and passes you at $t = 2$ s.
iii. It moves toward you but doesn't reach you. It then reverses direction at $t = 2$ s.
iv. It moves away from you until $t = 2$ s. It then reverses direction and moves toward you but doesn't reach you.

Explain your choice.

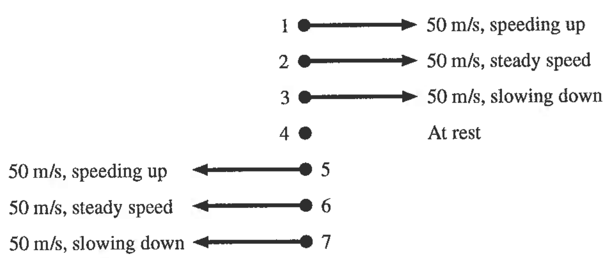
The initial shift is to a lower frequency so the source is initially moving away until $t = 2s$, then the shift is to higher frequency due to movement towards you.

31. You are standing at $x = 0$ m, listening to a sound that is emitted at frequency f_0. At $t = 0$ s, the sound source is at $x = 20$ m and moving toward you at a steady 10 m/s. Draw a graph showing the frequency you hear from $t = 0$ s to $t = 4$ s. Only the shape of the graph is important, not the numerical values of f.



It takes 2s for the source to reach you.
$\Delta t = \frac{\Delta x}{v} = \frac{20m}{10 m/s} = 2s.$

32. You are standing at $x = 0$ m, listening to seven identical sound sources. At $t = 0$ s, all seven are at $x = 343$ m and moving as shown below. The sound from all seven will reach your ear at $t = 1$ s.



Rank in order, from highest to lowest, the seven frequencies f_1 to f_7 that you hear at $t = 1$ s.

Order: $f_5 = f_6 = f_7 > f_4 > f_3 = f_2 = f_1$

Explanation:

The frequency depends upon the source's speed, not its acceleration. If we assume that $x = 343m$ is to the right, then 5-7 are moving towards you and 1-3 are moving away from you, all at equal speeds.