Tutorial 10 Problems

A selection of the following problems were done:

Workbook (2nd edition)
Chapter 32:
23, 29, 30

Textbook (2nd edition)
Chapter 32:
42, 74, 76

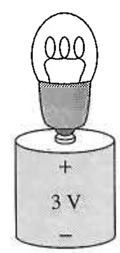
Chapter 33:
13

32 Fundamentals of Circuits

32.1 Circuit Elements and Diagrams

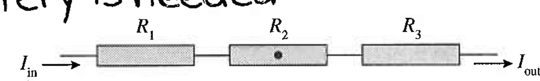
32.2 Kirchhoff's Laws and the Basic Circuit

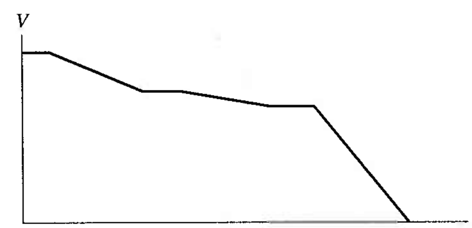
1. The tip of a flashlight bulb is touching the top of a 3 V battery. Does the bulb light? Why or why not?



No. This is not a complete circuit. Both the side and the base of the bulb would have to be connected to the battery. A connection from the outer metal case of the bulb to the negative terminal of the battery is needed.

2. Current I_{in} flows into three resistors connected together one after the other. The graph shows the value of the potential as a function of distance.





a. Is I_{out} greater than, less than, or equal to I_{in}? Explain.

Equal. This is due to the conservation of current.

b. Rank in order, from largest to smallest, the three resistances R_1, R_2, and R_3.

Order: $R_3 > R_1 > R_2$

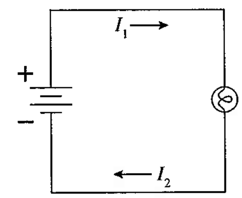
Explanation:

$R = \frac{\Delta V}{I}$ and the current is the same through each resistor: $\Delta V_3 > \Delta V_1 > \Delta V_2$

c. Is there an electric field at the point inside R_2 that is marked with a dot? If so, in which direction does it point? If not, why not?

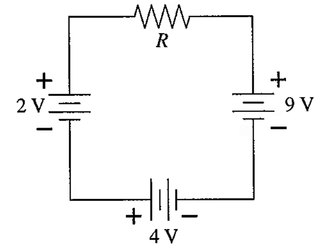
Yes. The resistor is just a conductor with a larger resistance than the wires in the circuit. The electric field points to the right.

3. A flashlight bulb is connected to a battery and is glowing. Is current I_2 greater than, less than, or equal to current I_1? Explain.



$I_1 = I_2$ due to conservation of current.

4. a. In which direction does current flow through resistor R?



Right to left.

b. Which end of R is more positive? Explain.

The right end. The current always enters the resistor on the high potential side of the resistor.

c. If this circuit were analyzed in a clockwise direction, what numerical value would you assign to ΔV_R? Why?

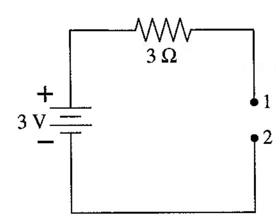
$\Delta V_R = +3V$

$\Delta V_R = +9V - 4V - 2V = +3V$

d. What value would ΔV_R have if the circuit were analyzed in a counterclockwise direction?

$\Delta V_R = -3V$

5. The wire is broken on the right side of this circuit. What is the potential difference ΔV_{12} between points 1 and 2? Explain.



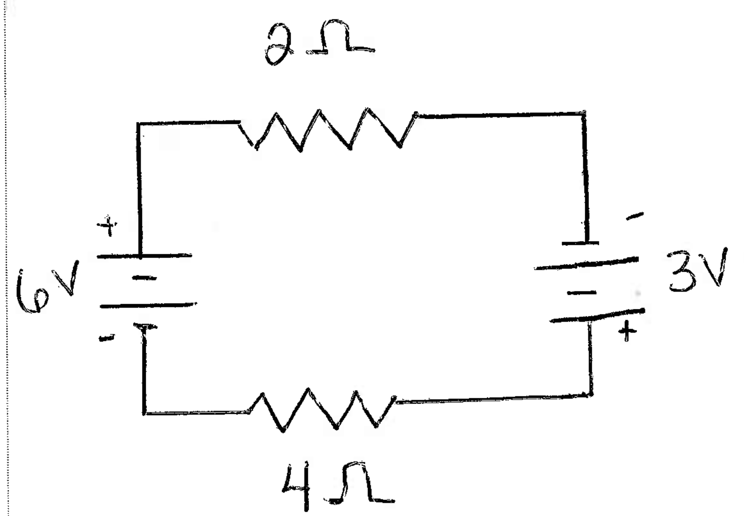
$\Delta V_{12} = 3V$

There is no current through the resistor so there is no potential difference across the resistor.

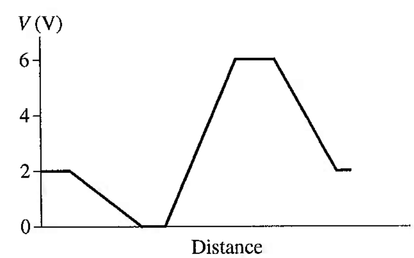
6. Draw a circuit for which the Kirchhoff loop law equation is

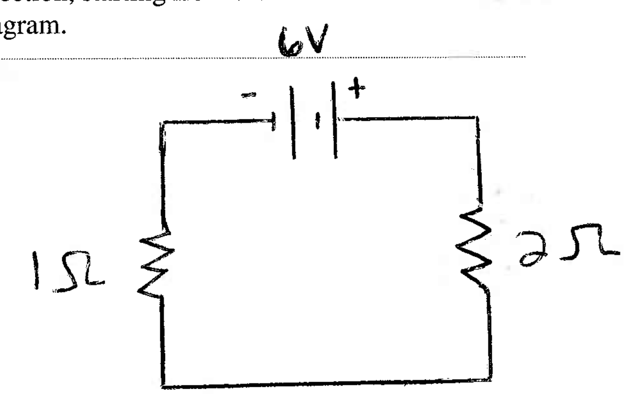
$$6\text{V} - I\cdot 2\Omega + 3\text{V} - I\cdot 4\Omega = 0$$

Assume that the analysis is done in a clockwise direction.



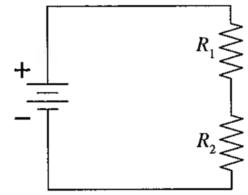
7. The current in a circuit is 2.0 A. The graph shows how the potential changes when going around the circuit in a clockwise direction, starting from the lower left corner. Draw the circuit diagram.





32.3 Energy and Power

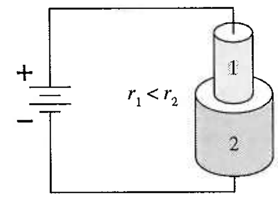
8. This circuit has two resistors, with $R_1 > R_2$. Which of the two resistors dissipates the larger amount of power? Explain.



R_1 dissipates more power.

$P_R = I^2 R$ The same current flows through each resistor.

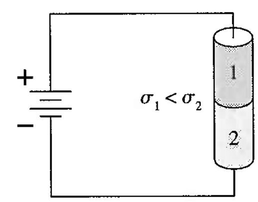
9. Two conductors of equal lengths are connected to a battery by ideal wires. The conductors are made of the same material but have different radii. Which of the two conductors dissipates the larger amount of power? Explain.



Conductor 1. Since $r_1 < r_2$; then $A_1 < A_2$ and $R_1 > R_2$.

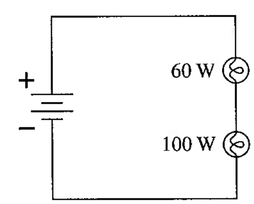
$R = \frac{\rho L}{A}$ Both have the same ρ and the same L. $P_R = I^2 R$

10. Two conductors of equal lengths are connected to a battery by ideal wires. The conductors have the same radii but are made of different materials and have different conductivities σ. Which of the two conductors dissipates the larger amount of power? Explain.



Conductor 1. Resistivity is the reciprocal of conductivity $\rho = \frac{1}{\sigma}$.
So if $\sigma_1 < \sigma_2$ then $\rho_1 > \rho_2$ and $R_1 > R_2$.
Therefore $(P_R)_1 > (P_R)_2$
$R = \frac{\rho L}{A}$ and $P_R = I^2 R$

11. A 60 W lightbulb and a 100 W lightbulb are placed one after the other in a circuit. The battery's emf is large enough that both bulbs are glowing. Which one glows more brightly? Explain.



Here the currents are the same so $P = I^2 R$ and the 60 W bulb has more resistance. (The Wattage ratings are determined by putting equal voltages across the bulbs not equal currents.)

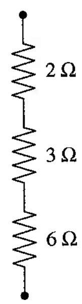
Therefore, the 60 W bulb glows more brightly.

32.4 Series Resistors

32.5 Real Batteries

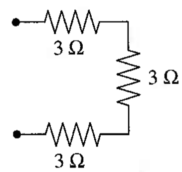
12. What is the equivalent resistance of each group of resistors?

a.



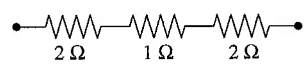
R_{eq} = 11 Ω

b.



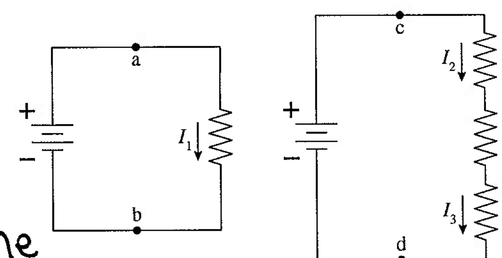
R_{eq} = 9 Ω

c.



R_{eq} = 5 Ω

13. The figure shows two circuits. The two batteries are identical and the four resistors all have exactly the same resistance.



a. Is ΔV_{ab} larger than, smaller than, or equal to ΔV_{cd}? Explain.

$\Delta V_{ab} = \Delta V_{cd}$ Points a and c are at the same potential as the positive electrode of the battery. Likewise b and d are at the same potential as the negative electrode.

b. Rank in order, from largest to smallest, the currents I_1, I_2, and I_3.

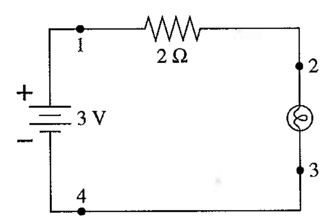
Order: $I_1 > I_2 = I_3$

Explanation:

I_2 and I_3 are equal due to conservation of current.

$I_1 = \frac{\Delta V}{R}$; $I_2 = I_3 = \frac{\Delta V}{3R}$

14. The lightbulb in this circuit has a resistance of 1 Ω.



a. What are the values of:

ΔV_{12} −2V

ΔV_{23} −1V

ΔV_{34} 0V

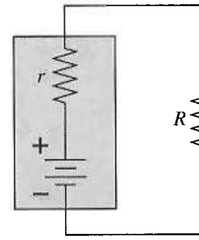
b. Suppose the bulb is now removed from its socket. Then what are the values of:

ΔV_{12} 0V

ΔV_{23} −3V

ΔV_{34} 0V

15. If the value of R is increased, does ΔV_{bat} increase, decrease, or stay the same? Explain.



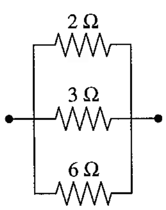
ΔV_{bat} will increase because

$$\Delta V_{bat} = \frac{R}{R+r}\,\mathcal{E}$$

32.6 Parallel Resistors

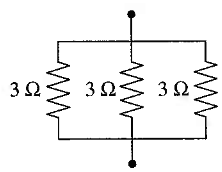
16. What is the equivalent resistance of each group of resistors?

a.



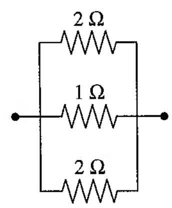
$R_{eq} =$ 1 Ω

b.



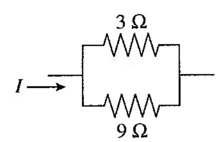
$R_{eq} =$ 1 Ω

c.



$R_{eq} =$ 0.5 Ω

17. a. What fraction of current I goes through the 3 Ω resistor?

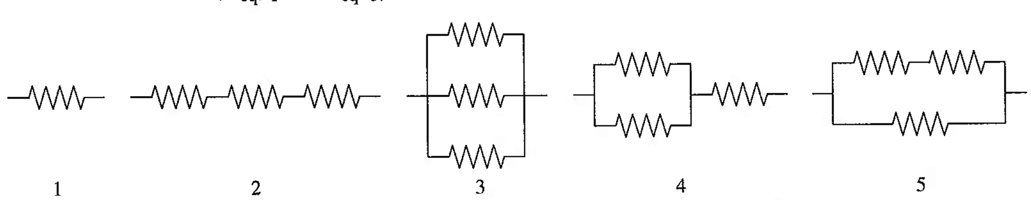


$\frac{3}{4}$

b. If the 9 Ω resistor is replaced with a larger resistor, will the fraction of current going through the 3 Ω resistor increase, decrease, or stay the same?

Increase.

18. The figure shows five combinations of identical resistors. Rank in order, from largest to smallest, the equivalent resistances $(R_{eq})_1$ to $(R_{eq})_5$.



Order: $(R_{eq})_2 > (R_{eq})_4 > (R_{eq})_1 > (R_{eq})_5 > (R_{eq})_3$

Explanation:

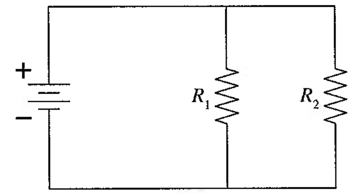
$(R_{eq})_1 = R \quad (R_{eq})_2 = 3R \quad (R_{eq})_3 = \frac{R}{3} \quad (R_{eq})_4 = \frac{3R}{2}$

$(R_{eq})_5 = \frac{2R}{3}$

32.7 Resistor Circuits

32.8 Getting Grounded

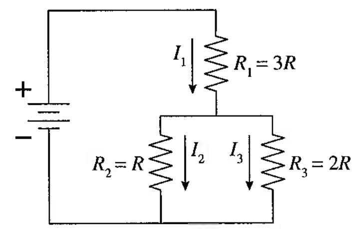
19. The circuit shown has a battery and two resistors, with $R_1 > R_2$. Which of the two resistors dissipates the larger amount of power? Explain your reasoning.



R_2 dissipates more power.

$P = \frac{(\Delta V)^2}{R}$ and both resistors have the same potential drop because they are connected in parallel.

20. Rank in order, from largest to smallest, the three currents I_1 to I_3.

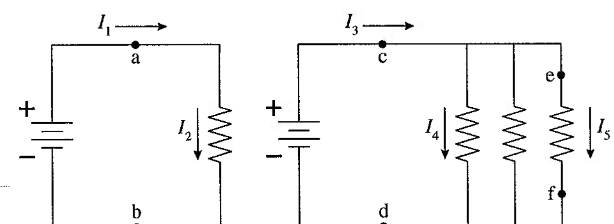


Order: $I_1 > I_2 > I_3$

Explanation:

By Kirchhoff's Junction Rule $I_1 = I_2 + I_3$. Also I_3 is less than I_2 because $R_3 > R_2$ and they have the same potential difference because they are in parallel. $I = \frac{\Delta V}{R}$

21. The two batteries are identical and the four resistors all have exactly the same resistance.



a. Compare ΔV_{ab}, ΔV_{cd}, and ΔV_{ef}. Are they all the same? If not, rank them in decreasing order. Explain your reasoning.

They are all the same. Points a, c, and e are all at the same potential as the positive electrode on the battery. Points b, d, and f are all at the same potential as the negative electrode.

b. Rank in order, from largest to smallest, the five currents I_1 to I_5.

Order: $I_3 > I_1 = I_2 = I_4 = I_5$

Explanation:

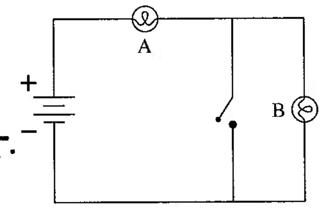
By conservation of current $I_1 = I_2$.

Also $I_1 = \frac{\Delta V_{bat}}{R}$ $I_3 = \frac{\Delta V_{bat}}{R/3} = 3I_1$

$I_4 = \frac{\Delta V_{bat}}{R} = I_5$

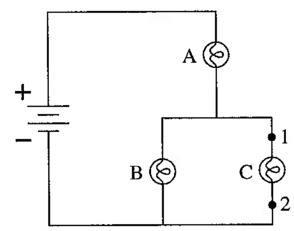
Exercises 22–28: Assume that all wires are ideal (zero resistance) and that all batteries are ideal (constant potential difference).

22. Initially bulbs A and B are glowing. Then the switch is closed. What happens to each bulb? Does it get brighter, stay the same, get dimmer, or go out? Explain your reasoning.



Bulb A gets brighter and bulb B goes out. When the switch is closed all the current travels along this zero resistance path rather than through bulb B. With less resistance in the circuit the current is larger so bulb A burns brighter.

23. a. Bulbs A, B, and C are identical. Rank in order, from most to least, the brightnesses of the three bulbs.



Order: A > B = C

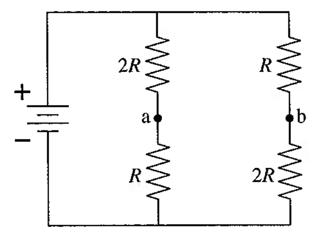
Explanation:

All current flowing from the battery must go through bulb A. Then the current splits at the junction with half going through B and half going through C. With the same resistance the larger current means more brightness.

b. Suppose a wire is connected between points 1 and 2. What happens to each bulb? Does it get brighter, stay the same, get dimmer, or go out? Explain.

Bulbs B and C go out because now there is a zero resistance wire along which all the current will flow. Now the total resistance in the circuit is less, the current is more, and bulb A will burn brighter.

24. a. Consider the points a and b. Is the potential difference $\Delta V_{ab} = 0$? If so, why? If not, which point is more positive?

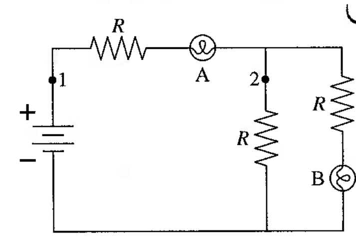


No, because the voltage drop across the 2R resistor is greater than the voltage drop across the R resistor. V=IR
Point B is more positive.

b. If a wire is connected between points a and b, does a current flow through it? If so, in which direction—to the right or to the left? Explain.

To the left. A wire connecting points a and b forces them to be at the same potential. However, since the current through each R resistor is twice that of each 2R resistor, the junction rule applied at a and b shows that current flows to the right.

25. Bulbs A and B are identical. Initially both are glowing.



a. Bulb A is removed from its socket. What happens to bulb B? Does it get brighter, stay the same, get dimmer, or go out? Explain.

It goes out. With bulb A out of its socket the circuit is not complete. There is no closed path from the battery through the circuit.

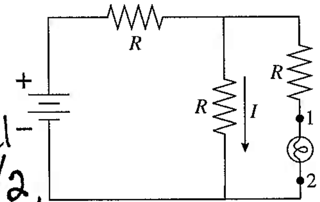
b. Bulb A is replaced. Bulb B is then removed from its socket. What happens to bulb A? Does it get brighter, stay the same, get dimmer, or go out? Explain.

Bulb A gets dimmer. The current in the circuit decreases because the equivalent resistance of the circuit increases.

c. The circuit is restored to its initial condition. A wire is then connected between points 1 and 2. What happens to the brightness of each bulb?

Bulb A goes out because there is now a zero resistance path instead of going through A. Bulb B gets brighter because now the resistance of the circuit is less and the current from the battery through bulb B is more.

26. Initially the lightbulb is glowing. It is then removed from its socket.



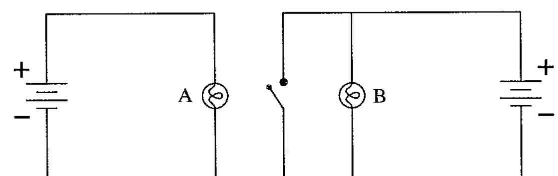
a. What happens to the current I when the bulb is removed? Does it increase, stay the same, or decrease? Explain.

Assume $R_{bulb} << R$. Initially, the two parallel-resistors have an equivalent resistance $R/2$, which when combined with the series resistor makes the circuit $R_{eq} = 3R/2$ and thus the current through the battery $I_{batt} = \Delta V/(3R/2) = \frac{2}{3}\frac{\Delta V}{R}$. Since the current is split between the parallel resistors, $I_{batt} = \frac{1}{3}\frac{\Delta V}{R}$. With the bulb gone, $I_{batt} = I = \frac{\Delta V}{2R}$, an increase.

b. What happens to the potential difference ΔV_{12} between points 1 and 2? Does it increase, stay the same, decrease, or become zero? Explain.

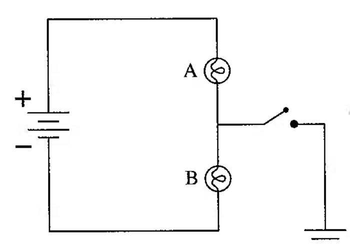
Increases. Now there is no potential difference across the resistor that is in series with the bulb because there is no current through the resistor. So the potential difference between 1 and 2 is now the potential difference of the parallel group.

27. Bulbs A and B are identical and initially both are glowing. Then the switch is closed. What happens to each bulb? Does its brightness increase, stay the same, decrease, or go out? Explain.



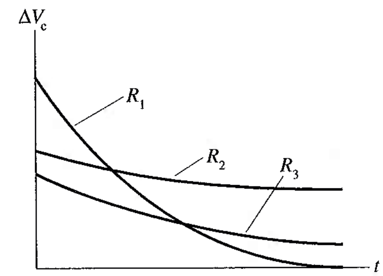
Stay the same. After the current passes through bulb A it will return to the battery. The negative electrode of the battery is at a lower potential than the positive electrode of the other battery.

28. Bulbs A and B are identical and initially both are glowing. Then the switch is closed. What happens to each bulb? Does its brightness increase, stay the same, decrease, or go out? Explain.



Stays the same. The ground wire is not part of a complete circuit so no current will flow down it. The current through the bulbs remains the same.

32.9 *RC* Circuits



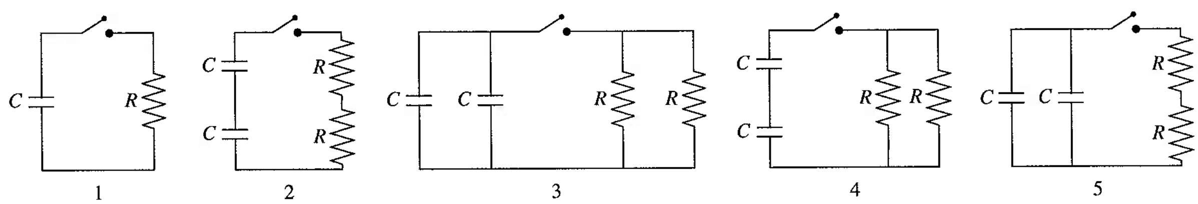
29. The graph shows the voltage as a function of time on a capacitor as it is discharged (separately) through three different resistors. Rank in order, from largest to smallest, the values of the resistances R_1 to R_3.

Order: $R_2 > R_3 > R_1$

Explanation:

$\tau = RC$ The decay time is longest for the R_2 curve and shortest for the R_1 curve.

30. The capacitors in each circuit are discharged when the switch closes at $t = 0$ s. Rank in order, from largest to smallest, the time constants τ_1 to τ_5 with which each circuit will discharge.



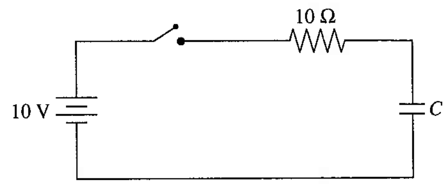
Order: $\tau_5 > \tau_1 = \tau_2 = \tau_3 > \tau_4$

Explanation:

$\tau_1 = RC$ $\tau_2 = (2R)(\frac{C}{2}) = \tau_1$ $\tau_3 = (\frac{R}{2})(2C) = \tau_1$

$\tau_4 = (\frac{R}{2})(\frac{C}{2}) = \frac{1}{4}\tau_1$ $\tau_5 = (2R)(2C) = 4\tau_1$

31. The charge on the capacitor is zero when the switch closes at $t = 0$ s.



a. What will be the current in the circuit after the switch has been closed for a long time? Explain.

After the switch has been closed a long time the capacitor will be fully charged and the current will decrease to zero.

(Actually $I = 0$ at $t \Rightarrow \infty$.)

b. Immediately after the switch closes, before the capacitor has had time to charge, the potential difference across the capacitor is zero. What must be the potential difference across the resistor in order to satisfy Kirchhoff's loop law? Explain.

10V. With a zero potential difference across the capacitor $\Delta V_C + \Delta V_R = \Delta V_{batt}$ so

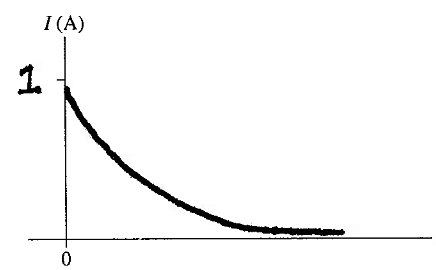
$\Delta V_R = \Delta V_{batt} - \Delta V_C$

$\Delta V_R = 10V - 0V = 10V$

c. Based on your answer to part b, what is the current in the circuit immediately after the switch closes?

$$I = \frac{\Delta V_R}{R} = \frac{10V}{10\Omega} = 1A$$

d. Sketch a graph of current versus time, starting from just before $t = 0$ s and continuing until the switch has been closed a long time. There are no numerical values for the horizontal axis, so you should think about the *shape* of the graph.



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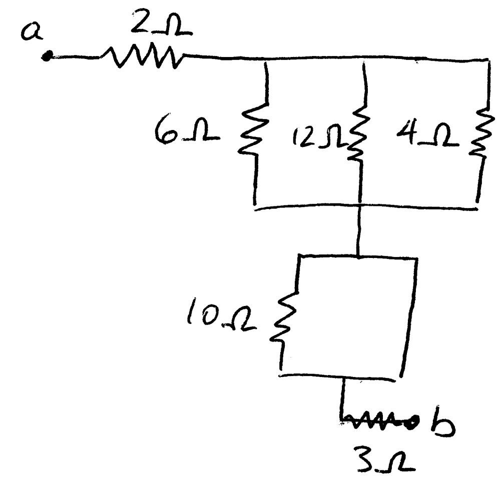
Chapter 32 Exercises and Problems

#42



Redraw the circuit. You can add lengths of ideal zero resistance wire anywhere that wires join.

In the new drawing, note that the 6Ω, 12Ω, and 4Ω resistors are in parallel. Note also that the 10Ω resistor is in parallel with a 0Ω wire, shorting out the 10Ω resistor; we can safely ignore that resistor.

The resistance of the 6Ω, 12Ω, 4Ω parallel combination is

$$R_p = \left(\frac{1}{6\Omega} + \frac{1}{12\Omega} + \frac{1}{4\Omega}\right)^{-1} = 2\Omega$$

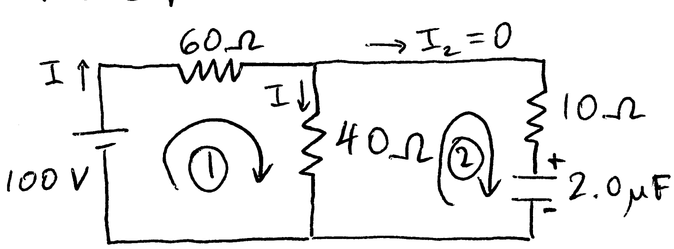
We now have

The effective resistance is just that of 2Ω, 3Ω, and R_p in series, so

$$R_{eff} = 2\Omega + 2\Omega + 3\Omega = 7\Omega.$$

#74

a) After a long time, the capacitor fills up with charge and does not allow any more current through the 10Ω resistor. The capacitor acts like a break in the circuit. Use Kirchhoff's voltage loop rule for the loops shown. For loop ①:



$$100\,V - I(60\Omega) - I(40\Omega) = 0$$

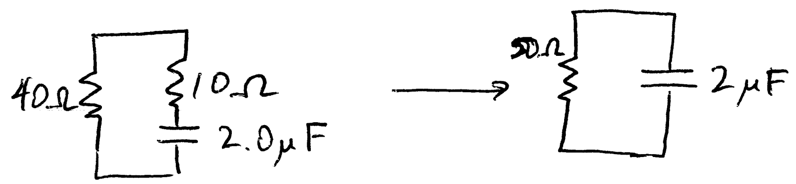
$$\Rightarrow I = \frac{100V}{100\Omega} = 1A.$$

For loop ②,

$$I(40\Omega) - \cancelto{0}{I_2}(10\Omega) - \frac{Q}{2.0\,\mu F} = 0$$

$$\Rightarrow Q = I(40\Omega)(2\times10^{-6}\,F) = 8\times10^{-5}\,C.$$

b) When the switch is open, the battery and 60Ω resistor are no longer part of the circuit. We are left with just a capacitor (charged up) that discharges across the 40Ω and 10Ω resistors.



Recall that a discharging capacitor follows the relation

$$Q = Q_0 e^{-t/RC}.$$

We want

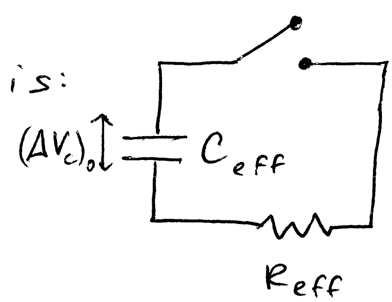
$$Q = 0.10\,Q_0 = Q_0 e^{-t/RC}$$

$$\Rightarrow \frac{-t}{RC} = \ln(+0.10)$$

$$\Rightarrow t = -RC\ln(0.1) = -(50\Omega)(2\times10^{-6}\,F)\ln(0.1) = 2.3\times10^{-4}\,s.$$

#76

The equivalent circuit for this problem is:



where $C_{eff} = \left(\frac{1}{60\,\mu F} + \frac{1}{60\,\mu F}\right)^{-1} + 20\,\mu F$

$$= 50\,\mu F$$

and $R_{eff} = \left(\frac{1}{30\Omega} + \frac{1}{20\Omega}\right)^{-1} + 8\Omega = 20\Omega.$

All current passes through the 8Ω resistor, so we can take the current through the entire equivalent circuit to be the same as the current through the 8Ω resistor.

$$(\Delta V_c)_0 = \frac{Q_0}{C_{eff}} \Rightarrow Q_0 = C_{eff}(\Delta V_c)_0.$$

$$I = I_0 e^{-t/\tau},$$

where $I_0 = \frac{Q_0}{\tau}$ and $\tau = R_{eff} C_{eff}$.

$$\tau = (20\,\Omega)(50\times10^{-6}\,F) = 1\times10^{-3}\,s.$$

$$I_0 = \frac{C_{eff}(\Delta V_c)_0}{\tau} = \frac{(50\times10^{-6}\,F)(10\,V)}{1\times10^{-3}\,s} = 0.50\,A$$

We want to know when the current decayed to half its initial value, so

$$I = \tfrac{1}{2} I_0 = I_0 e^{-t/\tau}$$

$$\Rightarrow \frac{-t}{\tau} = \ln\tfrac{1}{2}$$

$$\Rightarrow t = -\tau \ln\tfrac{1}{2} = -(1\times10^{-3}\,s)(\ln\tfrac{1}{2}) = 6.9\times10^{-4}\,s.$$

We actually didn't need to calculate I_0 at all.

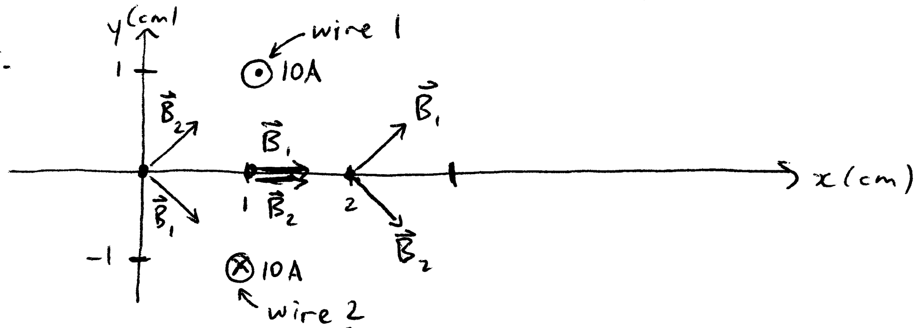
Chapter 33 Exercises and Problems

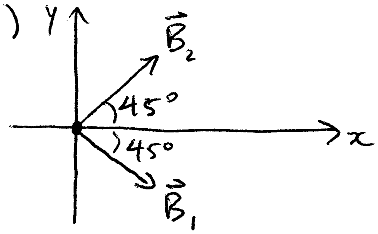
#13.

The magnetic field due to current through a long, straight wire is

$$\vec{B} = \frac{\mu_0 I}{2\pi d}$$ (tangent to a circle around the wire in the right-hand direction).

Draw the vectors.



a) 

Notice that the magnitudes of $\vec{B}_1$ and $\vec{B}_2$ are the same!

$$B_1 = \frac{\mu_0 I}{2\pi d} = \frac{(4\pi \times 10^{-7}\, T\cdot m/A)(10A)}{2\pi[(1cm)^2 + (1cm)^2]^{1/2}}$$

$$= \frac{2\times 10^{-6}\, T\cdot m}{[(0.01m)^2 + (0.01m)^2]^{1/2}} = \sqrt{2}\times 10^{-4}\, T = B_2.$$

Now split the fields into components.

$$B_{1x} = B_1 \cos 45° = \sqrt{2}\times 10^{-4}\,T \left(\frac{1}{\sqrt{2}}\right) = 1.0\times 10^{-4}\,T.$$

$$B_{1y} = B_1 \sin 45° = 1.0 \times 10^{-4}\, T.$$

$$\vec{B}_1 = B_{1x}\hat{i} + (-B_{1y})\hat{j}.$$

$$B_{2x} = B_2 \cos 45° = 1.0\times 10^{-4}\,T$$

$$B_{2y} = B_2 \sin 45° = 1.0\times 10^{-4}\,T.$$

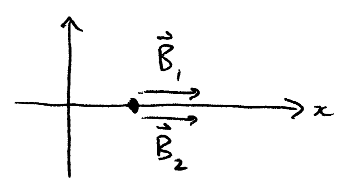
$$\vec{B}_2 = B_{2x}\hat{i} + B_{2y}\hat{j}.$$

$$\Rightarrow \vec{B} = \vec{B}_1 + \vec{B}_2 = (1.0\times10^{-4}T + 1.0\times 10^{-4}\,T)\hat{i} + (-1.0\times10^{-4}T + 1.0\times10^{-4}\,T)\hat{j}$$

$$= 2.0\times 10^{-4}\,T\,\hat{i}.$$

b) Again, the magnitudes $B_1 = B_2$, but this time, both point in the $\hat{i}$ direction.

$$B_1 = \frac{\mu_0 I}{2\pi d} = \frac{(4\pi\times 10^{-7}\,T\cdot m/A)(10A)}{2\pi(0.01\,m)} = 2.0\times 10^{-4}\,T = B_2.$$



$$\vec{B} = \vec{B}_1 + \vec{B}_2 = B_1\hat{i} + B_2\hat{i} = 4.0\times 10^{-4}\,T\,\hat{i}.$$

c) The calculation is exactly the same as in part a) except that $\vec{B}_1$ and $\vec{B}_2$ have swapped directions.

$$\vec{B} = 2.0\times 10^{-4}\,T\,\hat{i}.$$