

# Tutorial 6 Problems

A selection of the following problems were done:

**Midterm 1 (all)**

**Workbook (2nd edition)**

Chapter 28:

3

**Textbook (2nd edition)**

Chapter 27:

33, 47, 55

Chapter 28:

30

# 27

# The Electric Field

## 27.1 Electric Field Models

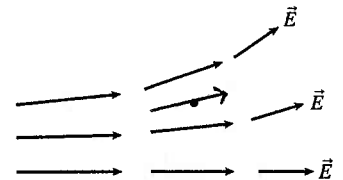
## 27.2 The Electric Field of Multiple Point Charges

1. You've been assigned the task of determining the magnitude and direction of the electric field at a point in space. Give a step-by-step procedure of how you will do so. List any objects you will use, any measurements you will make, and any calculations you will need to perform. Make sure that your measurements do not disturb the charges that are creating the field.

A tiny, positive test charge will be placed at the point in space and the force will be measured. From the force measurement and the charge, the electric field will be calculated using  $E = \frac{F}{q}$ . The direction of the field will be the same as the direction of the force because  $q$  is positive.

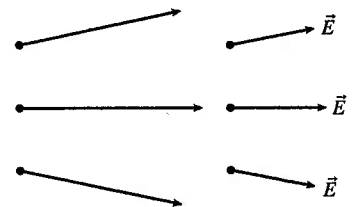
2. Is there an electric field at the position of the dot? If so, draw the electric field vector on the figure. If not, what would you need to do to create an electric field at this point?

Yes there is an electric field at this position.



3. This is the electric field in a region of space.
  - a. Explain the information that is portrayed in this diagram.

There is some source of positive charge on the left side. The longer vectors indicate a relatively larger electric field on the left, closest to the charge.

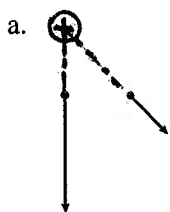


- b. If field vectors were drawn at the same six points but each was only half as long, would the picture represent the same electric field or a different electric field? Explain.

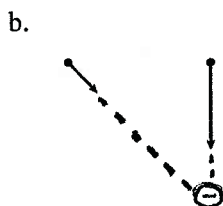
It could represent the same electric field because the length of the field vector only represents relative magnitude of the electric field at that point.

4. Each figure shows two vectors. Can a point charge create an electric field that looks like this at these two points? If so, add the charge to the figure. If not, why not?

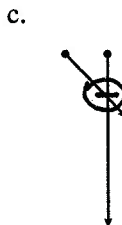
**Note:** The dots are the points to which the vectors are attached. There are no charges at these points.



Yes. Point 2 is further away from charge.



Yes.

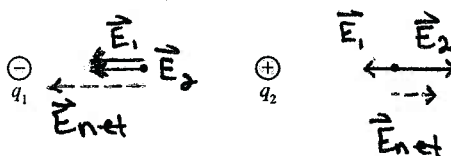
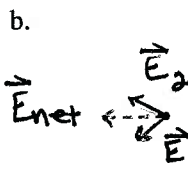
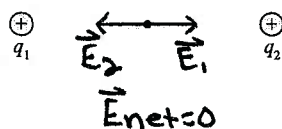
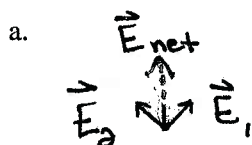


Yes. The field vector shows the field only at the dot.

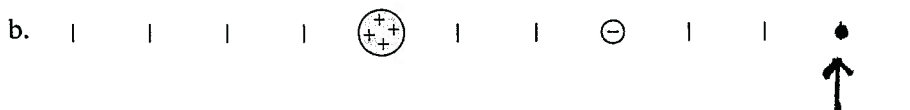


No. The field due to a point charge cannot go in two directions.

5. At each of the dots, use a **black** pen or pencil to draw and label the electric fields  $\vec{E}_1$  and  $\vec{E}_2$  due to the two point charges. Make sure that the *relative* lengths of your vectors indicate the strength of each electric field. Then use a **red** pen or pencil to draw and label the net electric field  $\vec{E}_{\text{net}}$ .

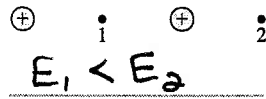


6. For each of the figures, use dots to mark any point or points (other than infinity) where  $\vec{E} = \vec{0}$ .

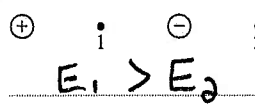


7. Compare the electric field strengths  $E_1$  and  $E_2$  at the two points labeled 1 and 2. For each, is  $E_1 > E_2$ , is  $E_1 = E_2$ , or is  $E_1 < E_2$ ?

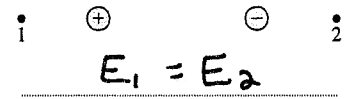
a.



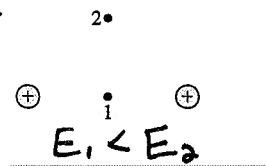
b.



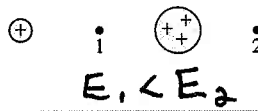
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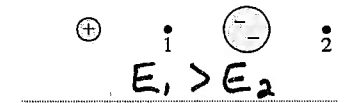
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e.

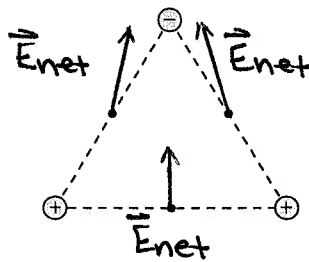


f.

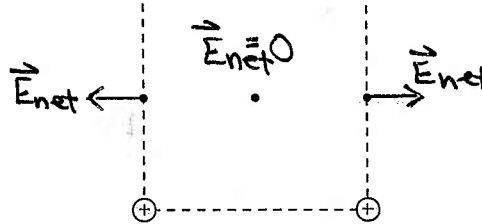


8. For each figure, draw and label the net electric field vector  $\vec{E}_{\text{net}}$  at each of the points marked with a dot or, if appropriate, label the dot  $\vec{E}_{\text{net}} = \vec{0}$ . The lengths of your vectors should indicate the magnitude of  $\vec{E}$  at each point.

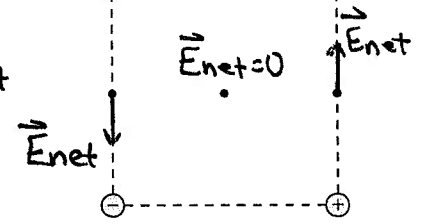
a.



b.

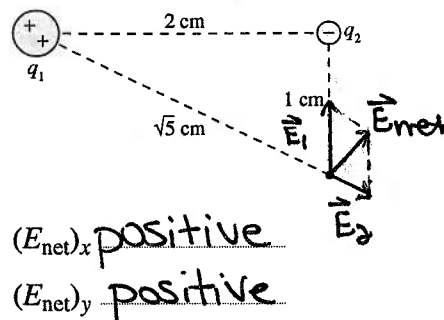


c.

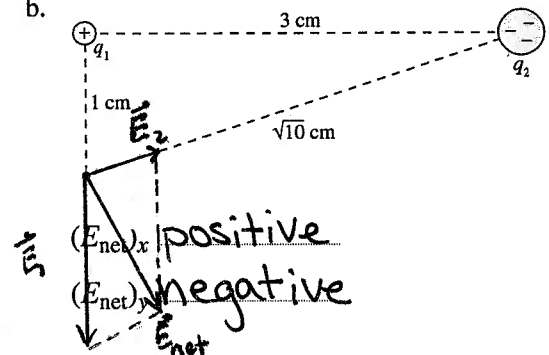


9. At the position of the dot, draw field vectors  $\vec{E}_1$  and  $\vec{E}_2$  due to  $q_1$  and  $q_2$ , and the net electric field  $\vec{E}_{\text{net}}$ . Then, in the blanks, state whether the  $x$ - and  $y$ -components of  $\vec{E}_{\text{net}}$  are positive or negative.

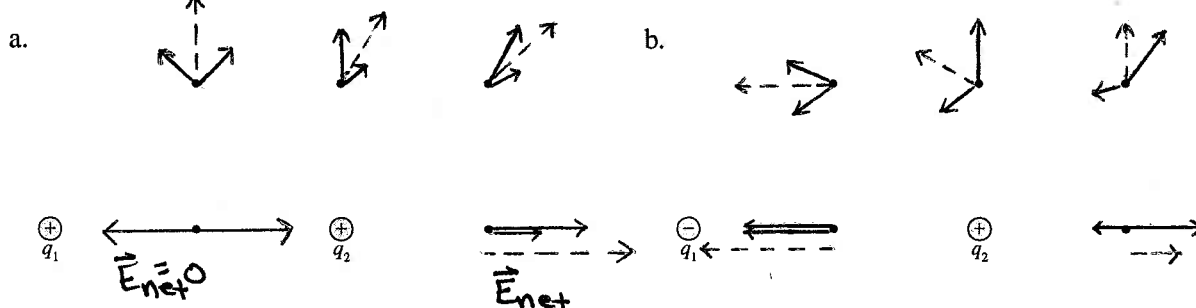
a.



b.

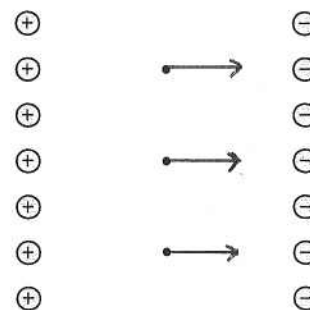


10. Use a **black** pen or pencil to draw the two electric fields  $\vec{E}_1$  and  $\vec{E}_2$  at each dot. Then use a **red** pen or pencil to draw  $\vec{E}_{\text{net}}$ . The lengths of your vectors should indicate the magnitude of  $\vec{E}$  at each point.

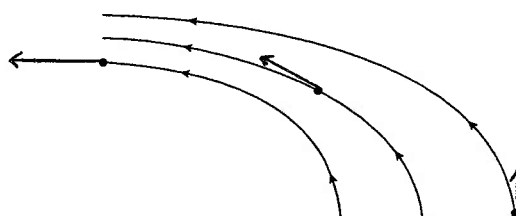


11. Draw the electric field vector at the three points marked with a dot.

Hint: Think of the charges as horizontal positive/negative pairs, then use superposition.



12. The figure shows the electric field lines in a region of space. Draw the electric field vectors at the three dots.

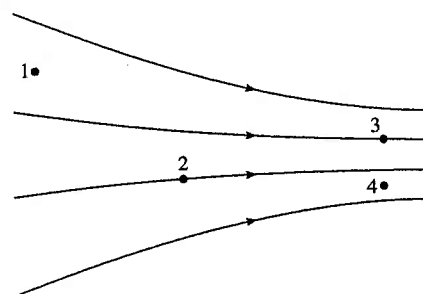


13. The figure shows the electric field lines in a region of space. Rank in order, from largest to smallest, the electric field strengths  $E_1$  to  $E_4$  at points 1 to 4.

Order:  $E_3 = E_4 > E_2 > E_1$

Explanation:

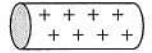
The electric field strength is larger in the region where the field lines are closer together ( $E_3$  and  $E_4$ ) and smaller where the field lines are farther apart.



## 27.3 The Electric Field of a Continuous Charge Distribution

14. A small segment of wire contains 10 nC of charge.

a. The segment is shrunk to one-third of its original length. What is the ratio  $\lambda_f/\lambda_i$ , where  $\lambda_i$  and  $\lambda_f$  are the initial and final linear charge densities?



$$\frac{\lambda_f}{\lambda_i} = \frac{(Q_f/L_f)}{(Q_i/L_i)} \quad \text{But } Q_i = Q_f \Rightarrow \frac{\lambda_f}{\lambda_i} = \frac{L_i}{L_f} = 3$$

b. Suppose the original segment of wire is stretched to 10 times its original length. How much charge must be added to the wire to keep the linear charge density unchanged?

10 times the original amount of charge would give a constant linear charge density. So the amount of charge to add to the original is 9 times the original charge.

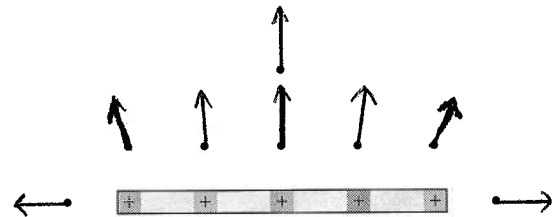
15. A wire has initial linear charge density  $\lambda_i$ . The wire is stretched in length by 50%, and one-third of the charge is removed. What is the ratio  $\lambda_f/\lambda_i$ , where  $\lambda_f$  is the final linear charge density?

$$\lambda_i = \frac{Q_i}{L_i} \quad L_f = 1.5 L_i \quad Q_f = \frac{Q_i}{3}$$

$$\lambda_f = \frac{Q_f}{L_f} = \frac{(Q_i/3)}{(1.5 L_i)} = \frac{1}{(3)(1.5)} \frac{Q_i}{L_i} = \frac{1}{4.5} \lambda_i$$

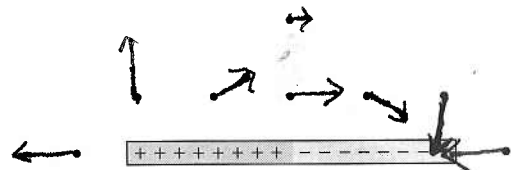
$$\frac{\lambda_f}{\lambda_i} = \frac{1}{4.5}$$

16. The figure shows a uniformly charged positive wire. Five small, equally-spaced segments of charge are shown. Use these five segments to estimate the wire's electric field—both magnitude and direction—at each point in space marked with a dot. Draw each  $\vec{E}$  on the figure.

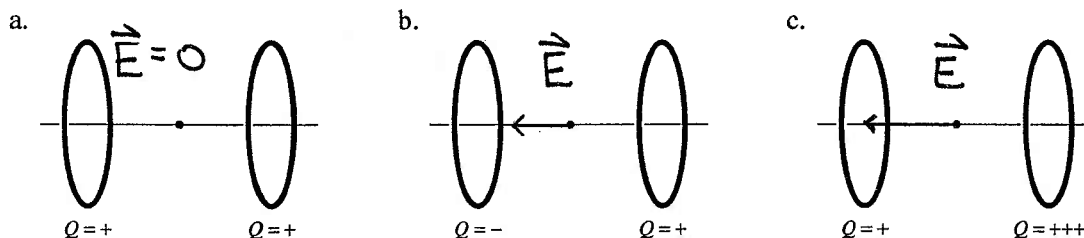


17. Equal-length, equally charged positive and negative wires are placed end-to-end. Draw the electric field at each of the dots.

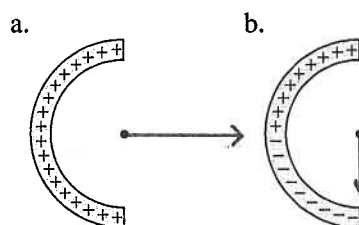
Hint: Think about the superposition of the fields of a positive and a negative wire.



18. Two rings of charge face each other. The total charge on each ring is indicated beneath it. Draw the electric field vector on the axis of the rings at the midpoint between them (at the dot), or label the point  $\vec{E} = \vec{0}$ .

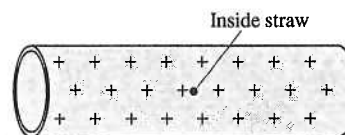


19. The figure shows two charged rods bent into a semicircle. For each, draw the electric field vector at the “center” of the semicircle.



20. A hollow soda straw is uniformly charged. What is the electric field at the center (inside) of the straw? Explain.

The electric field at the center is zero. We can think of the straw as being made up of as many rings of positive charge. At the center of the ring adding all field vectors gives a resultant electric field equal to zero.



21. An electron experiences a force of magnitude  $F$  when it is 1 cm from a very long charged wire with linear charge density  $\lambda$ . If the charge density is doubled, at what distance from the wire will a proton experience a force of the same magnitude  $F$ ?

$$F = eE = e \left( \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r} \right)$$

If the charge density  $\lambda$  is doubled, then the distance  $r$  from the wire must also be doubled for the force to be the same.  $r = 2\text{ cm}$

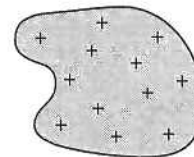
## 27.4 The Electric Fields of Rings, Disks, Planes, and Spheres

22. An irregularly-shaped area of charge has surface charge density  $\eta_i$ .

Each dimension ( $x$  and  $y$ ) of the area is reduced by a factor of 3.163.

- a. What is the ratio  $\eta_f/\eta_i$ , where  $\eta_f$  is the final surface charge density?

$$A_f = \frac{A_i}{3.163^2} \quad \frac{\eta_f}{\eta_i} = \frac{(Q/A_f)}{(Q/A_i)} = \frac{A_i}{A_f} = 3.163^2 = 10.00$$



- b. Compare the final force on a electron very far away to the initial force on the same electron.

$$F_f = eE_f = e \frac{\eta_f}{2\epsilon_0} \quad F_i = eE_i = e \frac{\eta_i}{2\epsilon_0} \quad \frac{F_f}{F_i} = \frac{\eta_f}{\eta_i} = 9.99$$

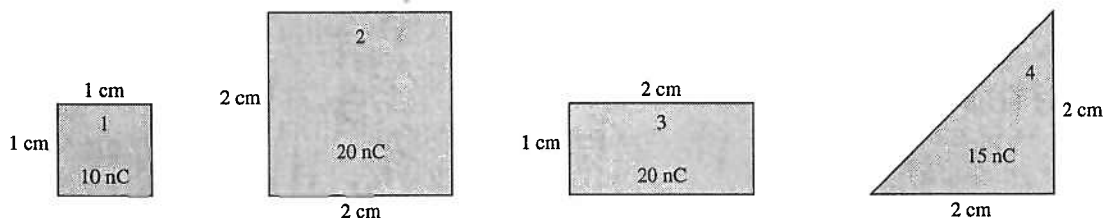
$$F_f = 9.99 F_i$$

23. A circular disk has surface charge density  $8 \text{ nC/cm}^2$ . What will be the surface charge density if the radius of the disk is doubled?

$$\eta_i = \frac{Q}{A_i} = \frac{Q}{\pi r_i^2} = 8 \frac{\text{nC}}{\text{cm}^2}$$

$$\eta_a = \frac{Q}{A_2} = \frac{Q}{\pi r_2^2} = \frac{Q}{\pi (2r_i)^2} = \frac{Q}{4\pi r_i^2} = \frac{1}{4} \eta_i = 2 \frac{\text{nC}}{\text{cm}^2}$$

24. Rank in order, from largest to smallest, the surface charge densities  $\eta_1$  to  $\eta_4$  of surfaces 1 to 4.



Order:  $\eta_1 = \eta_3 > \eta_4 > \eta_2$

Explanation:

$$\eta_1 = \frac{Q_1}{A_1} = \frac{10 \text{ nC}}{(1 \text{ cm} \times 1 \text{ cm})} = 10 \frac{\text{nC}}{\text{cm}^2} \quad \eta_2 = \frac{20 \text{ nC}}{(2 \text{ cm} \times 2 \text{ cm})} = 5 \frac{\text{nC}}{\text{cm}^2}$$

$$\eta_3 = \frac{20 \text{ nC}}{(1 \text{ cm} \times 2 \text{ cm})} = 10 \frac{\text{nC}}{\text{cm}^2} \quad \eta_4 = \frac{15 \text{ nC}}{\frac{1}{2}(2 \text{ cm} \times 2 \text{ cm})} = 7.5 \frac{\text{nC}}{\text{cm}^2}$$

25. A sphere of radius  $R_i$  has charge  $Q_i$ . What happens to the electric field strength at  $r = 2R_i$  if:

- a. The quantity of charge is halved?

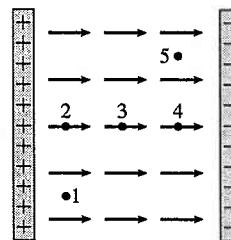
$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad \text{If } Q \text{ is halved, then } E \text{ is also halved.}$$

- b. The radius of the sphere is halved?

The field outside a sphere is the same as that of a point charge  $Q$  located at the center of the sphere. So if the radius of the sphere changes, the field remains the same outside the sphere at the distance  $r = 2R_i$ .

## 27.5 The Parallel-Plate Capacitor

26. Rank in order, from largest to smallest, the electric field strengths  $E_1$  to  $E_5$  at each of these points.



Order:  $E_1 = E_2 = E_3 = E_4 = E_5$

Explanation:

The electric field is constant everywhere between the plates. This is indicated by the electric field vectors which are all the same length and in the same direction.

27. A parallel-plate capacitor is constructed of two square plates, size  $L \times L$ , separated by distance  $d$ . The plates are given charge  $\pm Q$ . What is the ratio  $E_f/E_i$  of the final electric field strength  $E_f$  to the initial electric field strength  $E_i$  if:

a.  $Q$  is doubled?

$$\frac{E_f}{E_i} = \frac{\eta_f/\epsilon_0}{\eta_i/\epsilon_0} = \frac{\eta_f}{\eta_i} = \frac{Q_f/A_f}{Q_i/A_i} \quad \text{If } Q \text{ is doubled (} A = \text{constant)} \\ \frac{E_f}{E_i} = \frac{Q_f}{Q_i} = 2$$

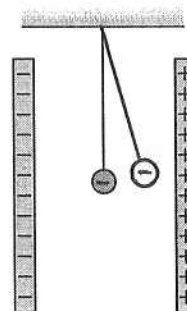
b.  $L$  is doubled?

$$\text{If } L \text{ is doubled then } A_f = 4A_i \text{ (} Q = \text{constant)} \quad \frac{E_f}{E_i} = \frac{A_i}{A_f} = \frac{A_i}{4A_i} = \frac{1}{4}$$

c.  $d$  is doubled?

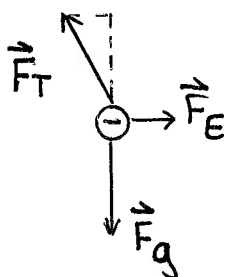
$$E \text{ does not depend on } d. \quad \frac{E_f}{E_i} = 1$$

28. A ball hangs from a thread between two vertical capacitor plates. Initially, the ball hangs straight down. The capacitor plates are charged as shown, then the ball is given a small negative charge. The ball moves to one side, but not enough to touch a capacitor plate.



a. Draw the ball and thread in the ball's new equilibrium position.

b. In the space below, draw a free-body diagram of the ball when in its new position.

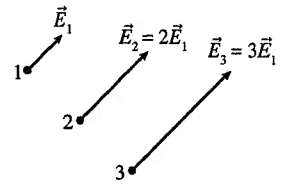


## 27.6 Motion of a Charged Particle in an Electric Field

## 27.7 Motion of a Dipole in an Electric Field

29. A small positive charge  $q$  experiences a force of magnitude  $F_1$  when placed at point 1. In terms of  $F_1$ :

- a. What is the force on charge  $q$  at point 3?  $3F_1$   
 b. What is the force on a charge  $3q$  at point 1?  $3F_1$   
 c. What is the force on a charge  $2q$  at point 2?  $4F_1$   
 d. What is the force on a charge  $-2q$  at point 2?  $-4F_1$



30. A small object is released from rest in the center of the capacitor. For each situation, does the object move to the right, to the left, or remain in place? If it moves, does it accelerate or move at constant speed?

- a. Positive object.

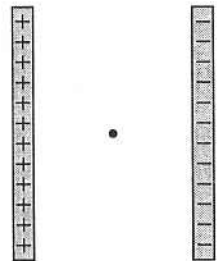
Accelerates to the right.

- b. Negative object.

Accelerates to the left.

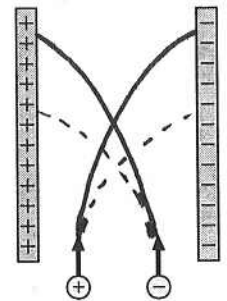
- c. Neutral object.

Remains in place.

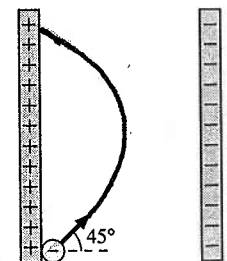


31. Positively and negatively charged objects, with equal masses and equal quantities of charge, enter the capacitor in the directions shown.

- a. Use solid lines to draw their trajectories on the figure if their initial velocities are fast.  
 b. Use dashed lines to draw their trajectories on the figure if their initial velocities are slow.



32. An electron is launched from the positive plate at a  $45^\circ$  angle. It does not have sufficient speed to make it to the negative plate. Draw its trajectory on the figure.



33. A proton and an electron are released from rest in the center of a capacitor.  
 a. Compare the forces on the two charges. Are they equal, or is one larger? Explain.

The forces on the two charges are equal.  
 $F = qE$  They each have the same amount of charge and are placed in the same field.

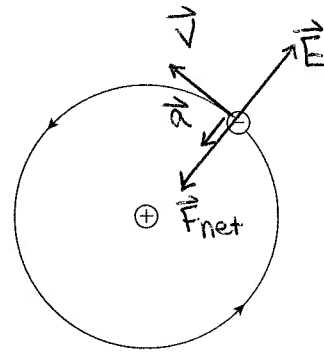
- b. Compare the accelerations of the two charges. Are they equal, or is one larger? Explain.

The acceleration of the electron is larger because the electron has smaller mass.  $a = \frac{F}{m}$

34. The figure shows an electron orbiting a proton in a hydrogen atom.  
 a. What force or forces act on the electron?

The electric force.

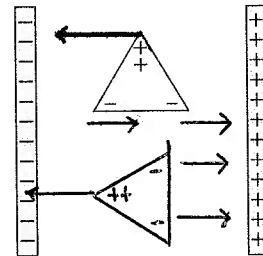
- b. Draw and label the following vectors on the figure: the electron's velocity  $\vec{v}$  and acceleration  $\vec{a}$ , the net force  $\vec{F}_{\text{net}}$  on the electron, and the electric field  $\vec{E}$  at the position of the electron.



35. Does a charged particle always move in the direction of the electric field? If so, explain why. If not, give an example that is otherwise.

No. If the charged particle has an initial velocity component in a perpendicular direction then it would travel in a different direction than the field. For example, a charge could move in circular motion as in problem #34 above.

36. Three charges are placed at the corners of a triangle. The ++ charge has twice the quantity of charge of the two - charges; the net charge is zero.  
 a. Draw the force vectors on each of the charges.  
 b. Is the triangle in equilibrium? No If not, draw the equilibrium orientation directly beneath the triangle that is shown.  
 c. Once in the equilibrium orientation, will the triangle move to the right, move to the left, rotate steady, or be at rest? Explain.



In equilibrium the triangle will remain in place because the net force is zero and the net torque is zero.

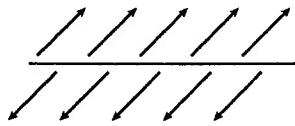
# 28

# Gauss's Law

## 28.1 Symmetry

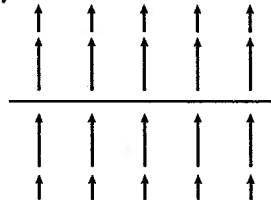
1. An infinite plane of charge is seen edge on. The sign of the charge is not given. Do the electric fields shown below have the same symmetry as the charge? If not, why not?

a.



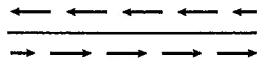
No. The field is not reflected in a plane coming out of the page.

b.



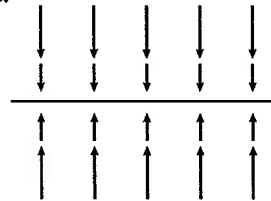
No. The field is not symmetric under a reflection in a plane coming out of the page.

c.



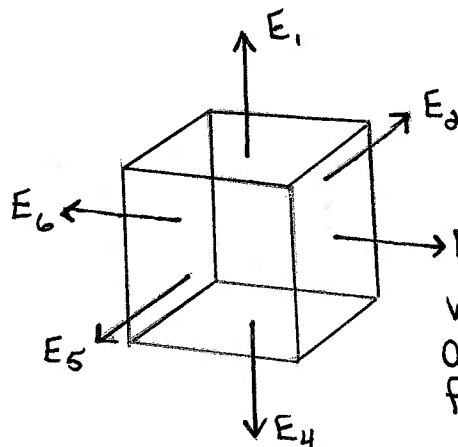
No. The field is not reflected up and down.

d.

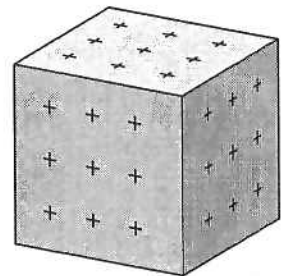


Yes. This field has the same symmetry as the charge.

2. Suppose you had a uniformly charged cube. Can you use symmetry alone to deduce the shape of the cube's electric field? If so, sketch and describe the field shape. If not, why not?



Choose a Gaussian surface in the shape of a cube. The electric field at each face will have the same magnitude and be perpendicular to that face.

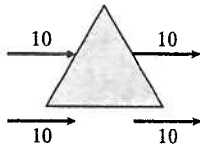


## 28.2 The Concept of Flux

3. The figures shown below are cross sections of three-dimensional closed surfaces. They have a flat top and bottom surface above and below the plane of the page. However, the electric field is everywhere parallel to the page, so there is no flux through the top or bottom surface. The electric field is uniform over each face of the surface. The field strength, in N/C, is shown.

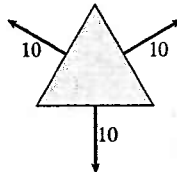
For each, does the surface enclose a net positive charge, a net negative charge, or no net charge?

a.



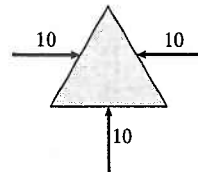
$$Q_{\text{net}} = 0$$

b.



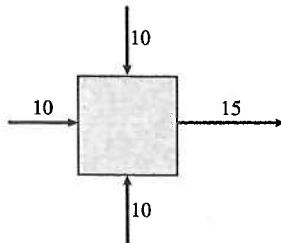
$$Q_{\text{net}} = +$$

c.



$$Q_{\text{net}} = -$$

d.



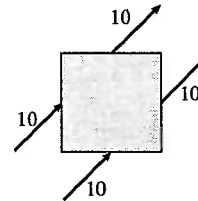
$$Q_{\text{net}} = -$$

e.



$$Q_{\text{net}} = +$$

f.

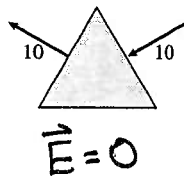


$$Q_{\text{net}} = 0$$

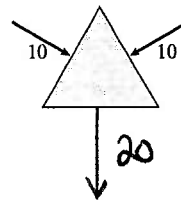
4. The figures shown below are cross sections of three-dimensional closed surfaces. They have a flat top and bottom surface above and below the plane of the page, but there is no flux through the top or bottom surface. The electric field is uniform over each face of the surface. The field strength, in N/C, is shown.

Each surface contains no net charge. Draw the missing electric field vector (or write  $\vec{E} = \vec{0}$ ) in the proper direction. Write the field strength beside it.

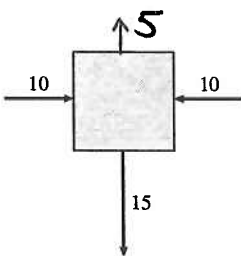
a.



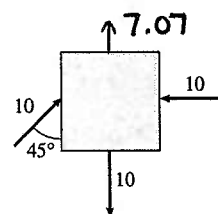
b.



c.

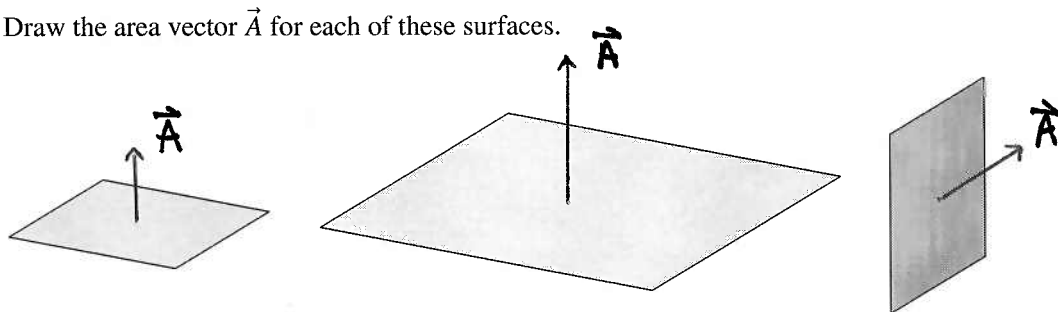


d.



## 28.3 Calculating Electric Flux

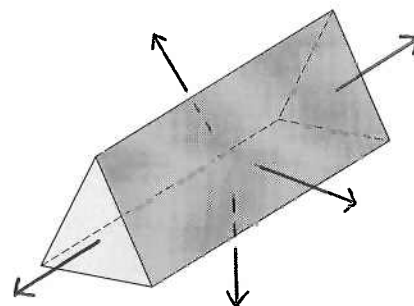
5. Draw the area vector  $\vec{A}$  for each of these surfaces.



6. How many area vectors are needed to characterize this closed surface?

5

Draw them.

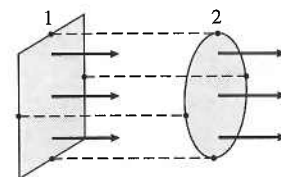


7. The diameter of the circle equals the edge length of the square. Is the electric flux  $\Phi_1$  through the square larger than, smaller than, or equal to the electric flux  $\Phi_2$  through the circle? Explain.

Because  $A_1 > A_2$  and  $E_1 = E_2$

$$\Phi_1 > \Phi_2$$

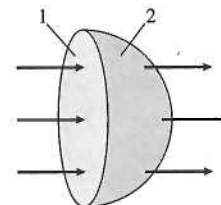
$$\Phi_1 = E_1 A_1, \Phi_2 = E_2 A_2$$



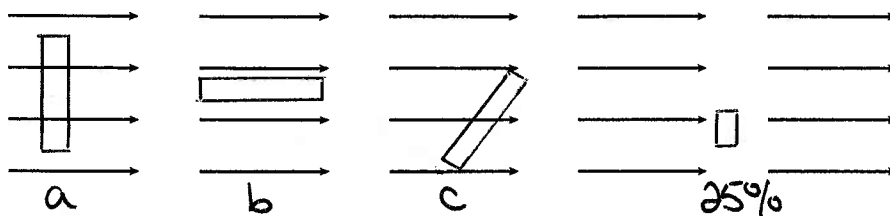
8. Is the electric flux  $\Phi_1$  through the circle larger than, smaller than, or equal to the electric flux  $\Phi_2$  through the hemisphere? Explain.

Any flux into surface 1 must come out of surface 2.

$$\Phi_1 = \Phi_2$$



9. A uniform electric field is shown below.



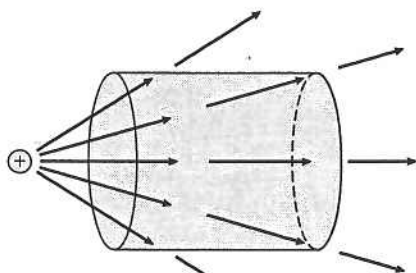
Draw and label an *edge view* of three square surfaces, all the *same size*, for which

- The flux is maximum.
- The flux is minimum.
- The flux has half the value of the flux through square 1.

Give the tilt angle of any squares not perpendicular to the field lines.

10. Is the net electric flux through each of the closed surfaces below positive (+), negative (-), or zero (0)?

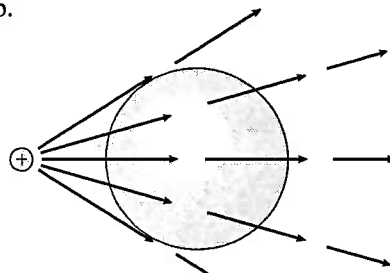
a.



$$\Phi = 0$$

All flux lines that flow in also flow out.

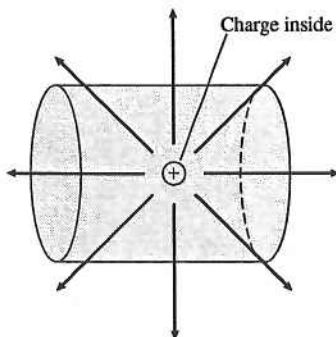
b.



$$\Phi = 0$$

All flux lines that flow into the surface also flow out.

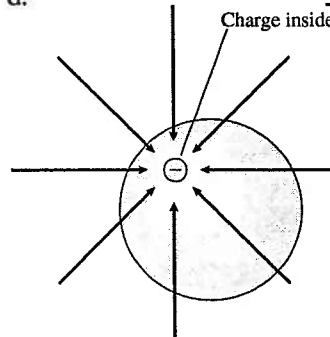
c.



$$\Phi = +$$

Flux only flows out.

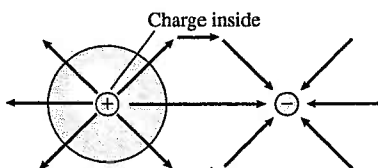
d.



$$\Phi = -$$

Flux only flows in.

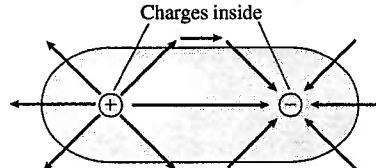
e.



$$\Phi = +$$

Flux only flows out.

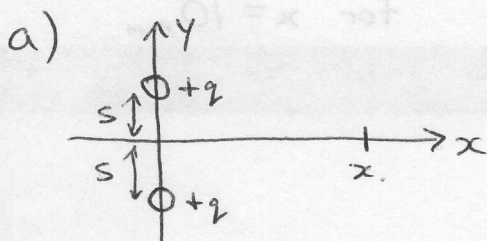
f.



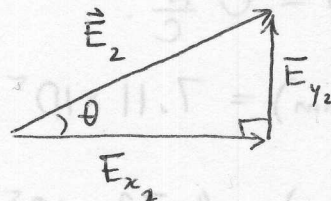
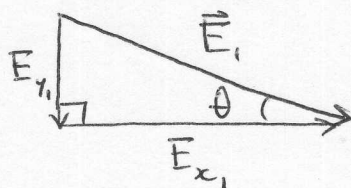
$$\Phi = 0$$

The amount of flux into the closed surface is equal to the amount of flux out.

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Chapter 27 Exercises and Problems  
#33



Find components of electric field for each charge.

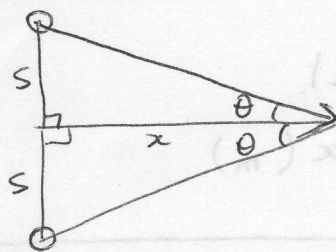


$$E_1 = \frac{kq_1}{r_1^2} = \frac{kq}{s^2 + x^2}$$

$$E_2 = \frac{kq_2}{r_2^2} = \frac{kq}{(-s)^2 + x^2} = \frac{kq}{s^2 + x^2}$$

magnitudes of electric fields

The directions of the  $\vec{E}$ -fields are found using the problem's geometry:



$$\tan \theta = \frac{s}{x}$$

$$\cos \theta = \frac{x}{\sqrt{x^2 + s^2}}$$

$$\sin \theta = \frac{s}{\sqrt{x^2 + s^2}}$$

$$\vec{E}_1 = E_{x1} \hat{i} + (E_{y1}) \hat{j} = E_1 \cos \theta \hat{i} - E_1 \sin \theta \hat{j}$$

$$\vec{E}_2 = E_{x2} \hat{i} + E_{y2} \hat{j} = E_2 \cos \theta \hat{i} + E_2 \sin \theta \hat{j}$$

The total  $\vec{E}$ -field is

$$\vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 = (E_1 + E_2) \cos \theta \hat{i} + (E_2 - E_1) \sin \theta \hat{j}$$

$$= \frac{2kq}{s^2 + x^2} \frac{x}{\sqrt{x^2 + s^2}} \hat{i} + \left( \frac{kq}{s^2 + x^2} - \frac{kq}{s^2 + x^2} \right) \frac{s}{\sqrt{x^2 + s^2}} \hat{j}$$

$$= \frac{2kqx}{(s^2 + x^2)^{3/2}} \hat{i} + 0 \hat{j}$$

$$b) \vec{E} = \frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1 \times 10^{-9} \text{ C})(10 \times 10^{-3} \text{ m})}{[(6 \times 10^{-3} \text{ m})^2 + (10 \times 10^{-3} \text{ m})^2]^{3/2}} \hat{i} = 1.13 \times 10^5 \frac{\text{N}}{\text{C}} \hat{i}$$

for  $x = 10 \text{ mm}$ .

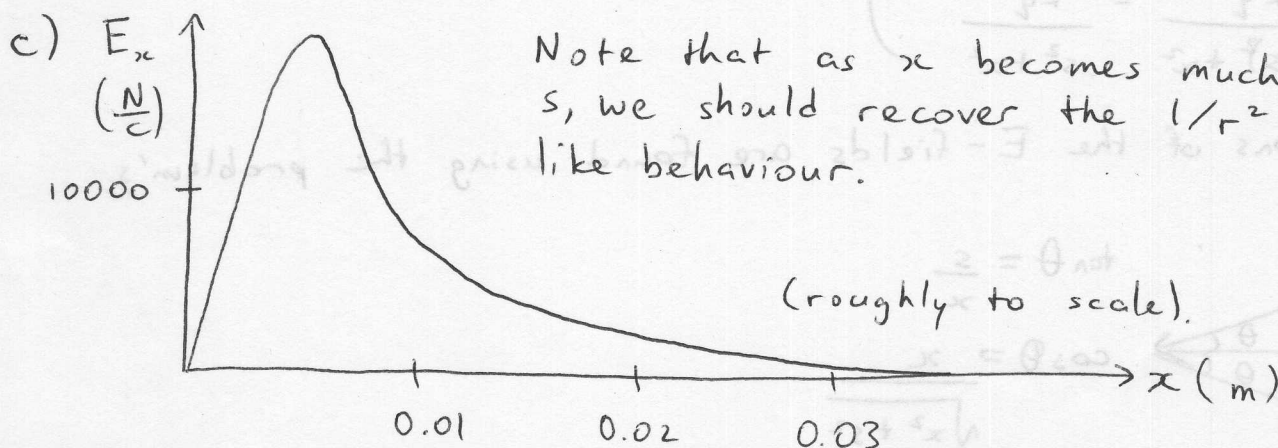
Similarly, for  $x = 0, 2, 4, 6 \text{ mm}$ , we get

$$\vec{E}(x=0) = 0 \frac{\text{N}}{\text{C}}.$$

$$\vec{E}(x=2 \text{ mm}) = 7.11 \times 10^5 \frac{\text{N}}{\text{C}}.$$

$$\vec{E}(x=4 \text{ mm}) = 4.79 \times 10^5 \frac{\text{N}}{\text{C}}.$$

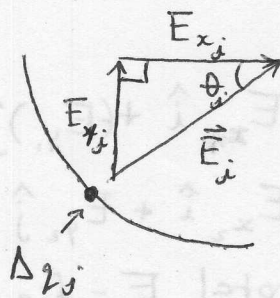
$$\vec{E}(x=6 \text{ mm}) = 2.94 \times 10^5 \frac{\text{N}}{\text{C}}.$$



#47

a) Split the rod into many small pieces of charge and find  $\vec{E}$  due to one piece.

$$E_j = \frac{k \Delta q_j}{R^2} \Rightarrow \begin{cases} E_{x_j} = E_j \cos \theta_j \\ E_{y_j} = E_j \sin \theta_j \end{cases}$$



$$E_{x_j} = \frac{k \Delta q_j}{R^2} \cos \theta_j, \quad E_{y_j} = \frac{k \Delta q_j}{R^2} \sin \theta_j$$

$$b) \vec{E}_{\text{net}} = \sum_j (E_{x_j} \hat{i} + E_{y_j} \hat{j}) = \frac{k}{R^2} \left[ \sum_j (\Delta q_j \cos \theta_j \hat{i} + \Delta q_j \sin \theta_j \hat{j}) \right]$$

Convert to an integral. Note that the rod has a charge density  $\lambda$ , so multiply this by the tiny length  $\Delta s_j$  that the

charge occupies on the rod.

$$\Delta q_j = \lambda \Delta s_j = \lambda \underbrace{R \Delta \theta_j}_{\text{arc length.}}$$

$$\vec{E}_{\text{net}} = \frac{k}{R^2} \sum_j \left( \lambda R \Delta \theta_j \cos \theta_j \hat{i} + \lambda R \Delta \theta_j \sin \theta_j \hat{j} \right)$$

To convert to an integral, let  $\theta_j \rightarrow \theta$ ,  $\Delta \theta_j \rightarrow d\theta$ ,  $\sum_j \rightarrow \int$ :

$$\vec{E}_{\text{net}} = \frac{k\lambda}{R^2} \int_0^{\pi/2} \left( R \cos \theta d\theta \hat{i} + R \sin \theta d\theta \hat{j} \right)$$

$$\begin{aligned} \text{c) } \vec{E}_{\text{net}} &= \frac{k\lambda R}{R^2} \left[ \sin \theta \Big|_0^{\pi/2} \hat{i} + (-\cos \theta) \Big|_0^{\pi/2} \hat{j} \right] \\ &= \frac{k\lambda}{R} \left[ (1 - 0) \hat{i} + (-(-1) - (-1)) \hat{j} \right] \\ &= \frac{k\lambda}{R} (\hat{i} + \hat{j}) \end{aligned}$$

If the rod has charge  $Q$ , then

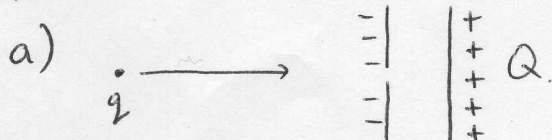
$$\lambda = \frac{Q}{\underbrace{\frac{2\pi R}{4}}_{\text{arc length of quarter circle}}}$$

$$= \frac{2Q}{\pi R}$$

$$k = \frac{1}{4\pi\epsilon_0}$$

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\epsilon_0} \frac{2Q}{\pi R^2} (\hat{i} + \hat{j}).$$

#55



$$E = \frac{1}{\epsilon_0} = \frac{Q}{\epsilon_0 A} = \frac{F}{q} = \frac{m_{\text{proton}} |a|}{q}$$

Find the acceleration required to slow a proton from  $v_i = 2.0 \times 10^6 \text{ m/s}$  to  $v_f = 2.0 \times 10^5 \text{ m/s}$ .

~~$$v_f = at + v_i$$~~
~~$$a = \frac{v_f - v_i}{t}$$~~

$$v_f^2 - v_i^2 = 2a\Delta x$$

$$a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{(2 \times 10^5 \frac{m}{s})^2 - (2 \times 10^6 \frac{m}{s})^2}{2(2 \times 10^{-2} m)}$$

$$= -9.9 \times 10^{13} \frac{m}{s^2}$$

The required charge density is

$$\eta = \frac{m_{proton} |a|}{q \epsilon_0} = \frac{(1.67 \times 10^{-27} kg)(9.9 \times 10^{13} \frac{N}{kg})}{1.6 \times 10^{-19} C} (8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2})$$

$$= 9.14 \times 10^{-6} \frac{C}{m^2}$$

b) With a larger charge density, the force is stronger, so the device works since it slows the proton more.

## Chapter 28 Exercises and Problems

#30

sign convention: flux into the object is negative.

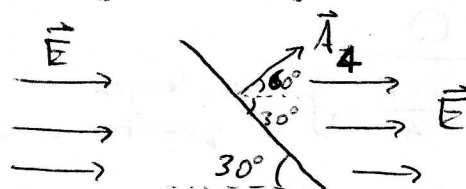
$$\Phi_1 = \vec{E} \cdot \vec{A}_1 = -EA_1 = -(400 \frac{N}{C})(4m)(2m) = -3200 \frac{N}{C} m^2$$

$$\Phi_2 = \vec{E} \cdot \vec{A}_2 = 0 \text{ since no E field lines go through surface 2.}$$

$$\Phi_4 = \vec{E} \cdot \vec{A}_4 = EA_4 \cos 60^\circ$$

$$= (400 \frac{N}{C})(4m)(4m) \cos 60^\circ$$

$$= 3200 \frac{N}{C} m^2$$



$$2m \text{ (vertical side)} \quad l = \frac{2m}{\sin 30^\circ} = 4m \text{ (hypotenuse)}$$

$$\Phi_3 = \Phi_5 = 0 \text{ since no E field components are perpendicular to the surfaces.}$$