Tutorial 7 Problems

A selection of the following problems were done:

Midterm 2 practice (all)

Workbook (2nd edition)
Chapter 28:
3, 13

Chapter 29:
3, 7, 10

Textbook (2nd edition)
Chapter 28:
19, 30, 38

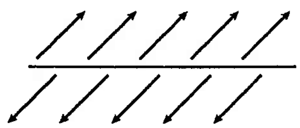
Chapter 29:
40, 71

28 Gauss's Law

28.1 Symmetry

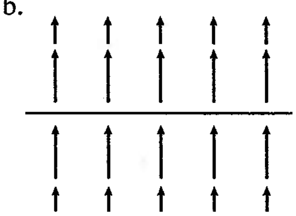
1. An infinite plane of charge is seen edge on. The sign of the charge is not given. Do the electric fields shown below have the same symmetry as the charge? If not, why not?

a.



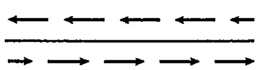
No. The field is not reflected in a plane coming out of the page.

b.



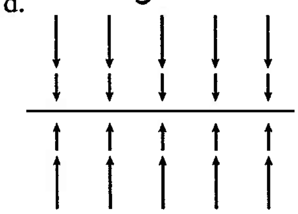
No. The field is not symmetric under a reflection in a plane coming out of the page.

c.



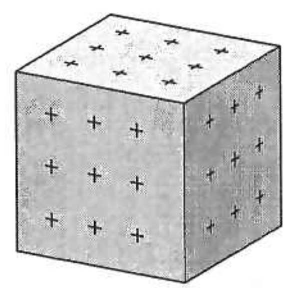
No. The field is not reflected up and down.

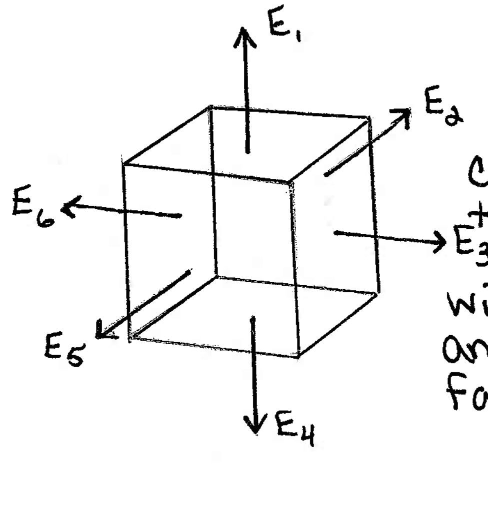
d.



Yes. This field has the same symmetry as the charge.

2. Suppose you had a uniformly charged cube. Can you use symmetry alone to deduce the shape of the cube's electric field? If so, sketch and describe the field shape. If not, why not?





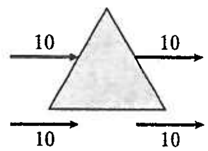
Choose a Gaussian surface in the shape of a cube. The electric field at each face will have the same magnitude and be perpendicular to that face.

28.2 The Concept of Flux

3. The figures shown below are cross sections of three-dimensional closed surfaces. They have a flat top and bottom surface above and below the plane of the page. However, the electric field is everywhere parallel to the page, so there is no flux through the top or bottom surface. The electric field is uniform over each face of the surface. The field strength, in N/C, is shown.

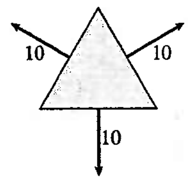
For each, does the surface enclose a net positive charge, a net negative charge, or no net charge?

a.



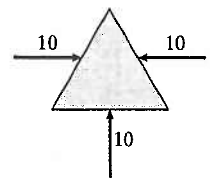
Q_{net} = 0

b.



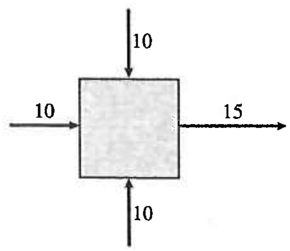
Q_{net} = +

c.



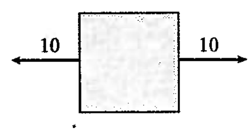
Q_{net} = −

d.



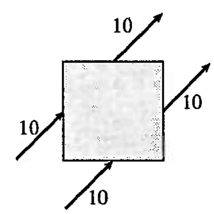
Q_{net} = −

e.



Q_{net} = +

f.

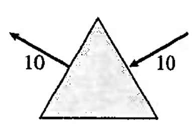


Q_{net} = 0

4. The figures shown below are cross sections of three-dimensional closed surfaces. They have a flat top and bottom surface above and below the plane of the page, but there is no flux through the top or bottom surface. The electric field is uniform over each face of the surface. The field strength, in N/C, is shown.

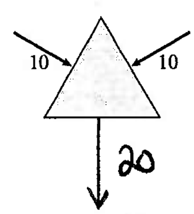
Each surface contains no net charge. Draw the missing electric field vector (or write $\vec{E} = \vec{0}$) in the proper direction. Write the field strength beside it.

a.

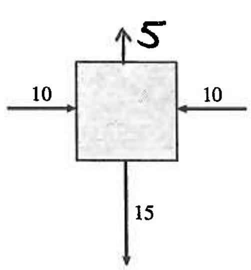


$\vec{E} = 0$

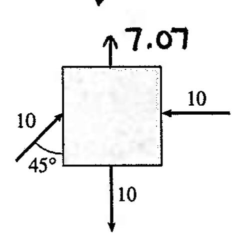
b.



c.

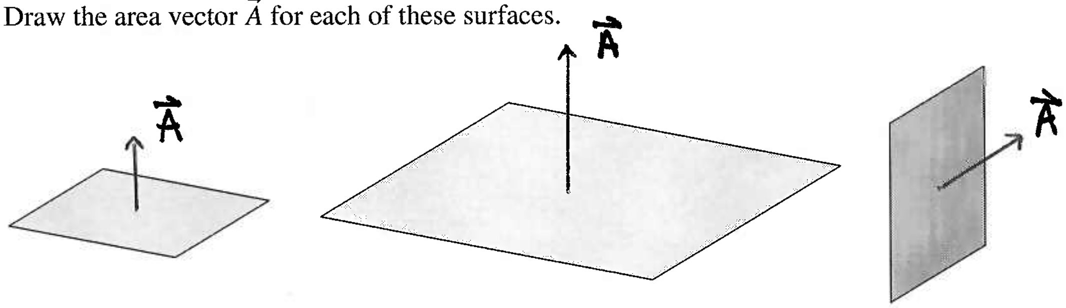


d.



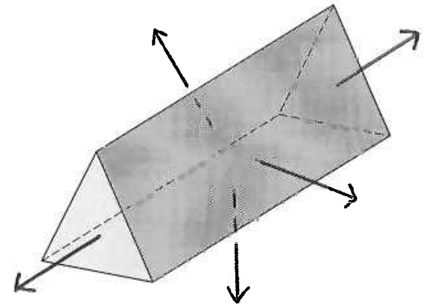
28.3 Calculating Electric Flux

5. Draw the area vector $\vec{A}$ for each of these surfaces.

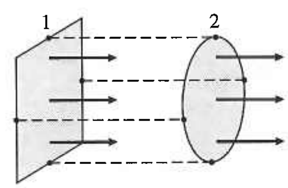


6. How many area vectors are needed to characterize this closed surface? 5

Draw them.



7. The diameter of the circle equals the edge length of the square. Is the electric flux Φ_1 through the square larger than, smaller than, or equal to the electric flux Φ_2 through the circle? Explain.

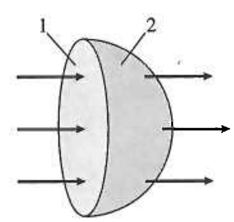


Because $A_1 > A_2$ and $E_1 = E_2$

$\Phi_1 > \Phi_2$

$\Phi_1 = E_1 A_1$ $\Phi_2 = E_2 A_2$

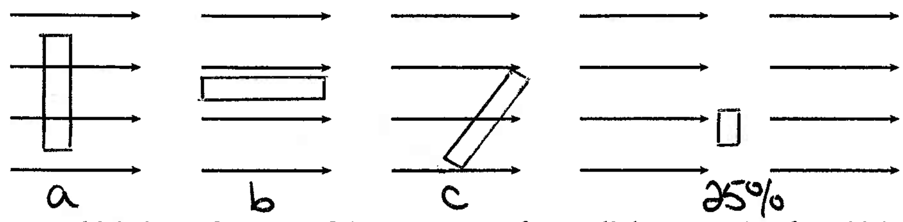
8. Is the electric flux Φ_1 through the circle larger than, smaller than, or equal to the electric flux Φ_2 through the hemisphere? Explain.



Any flux into surface 1 must come out of surface 2.

$\Phi_1 = \Phi_2$

9. A uniform electric field is shown below.



Draw and label an *edge view* of three square surfaces, all the *same size*, for which

a. The flux is maximum.

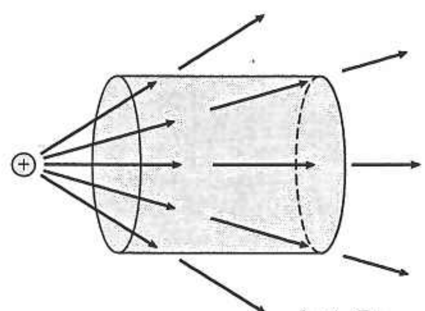
b. The flux is minimum.

c. The flux has half the value of the flux through square 1.

Give the tilt angle of any squares not perpendicular to the field lines.

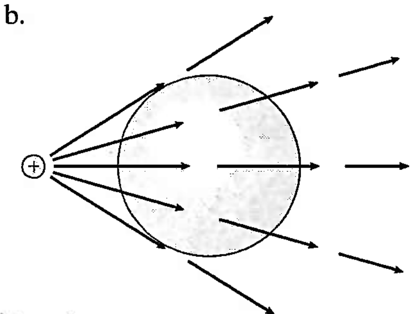
10. Is the net electric flux through each of the closed surfaces below positive (+), negative (–), or zero (0)?

a.



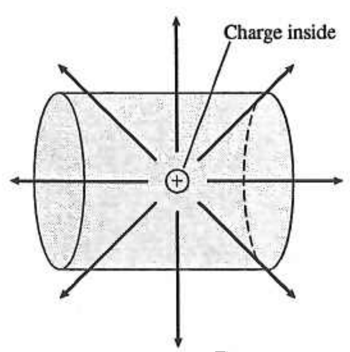
$\Phi =$ 0 All flux lines that flow in also flow out.

b.



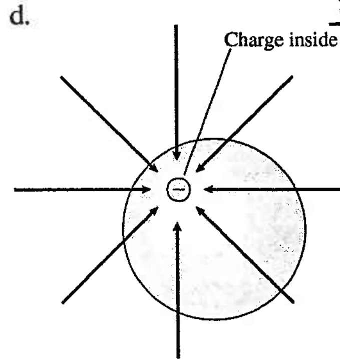
$\Phi =$ 0 All flux lines that flow into the surface also flow out.

c.



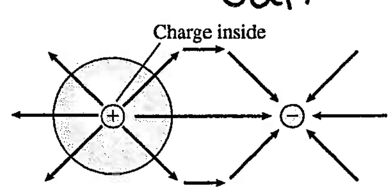
$\Phi =$ + Flux only flows out.

d.



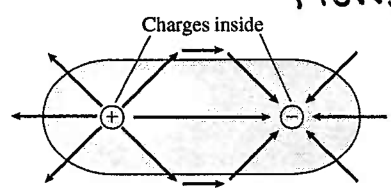
$\Phi =$ – Flux only flows in.

e.



$\Phi =$ + Flux only flows out.

f.



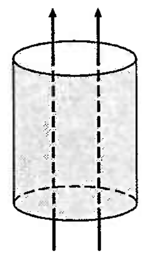
$\Phi =$ 0 The amount of flux into the closed surface is equal to the amount of flux out.

28.4 Gauss's Law

28.5 Using Gauss's Law

11. For each of the closed cylinders shown below, are the electric fluxes through the top, the wall, and the bottom positive (+), negative (−), or zero (0)? Is the net flux positive, negative, or zero?

a.



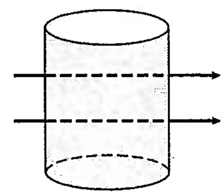
Φ_{top} = +

Φ_{wall} = 0

Φ_{bot} = −

Φ_{net} = 0

b.



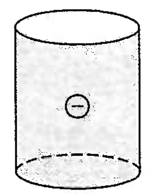
Φ_{top} = 0

Φ_{wall} = 0

Φ_{bot} = 0

Φ_{net} = 0

c.



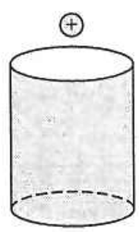
Φ_{top} = −

Φ_{wall} = −

Φ_{bot} = −

Φ_{net} = −

d.



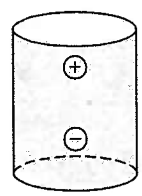
Φ_{top} = −

Φ_{wall} = +

Φ_{bot} = +

Φ_{net} = 0

e.



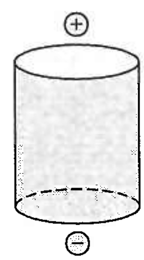
Φ_{top} = +

Φ_{wall} = 0

Φ_{bot} = −

Φ_{net} = 0

f.



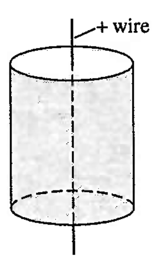
Φ_{top} = −

Φ_{wall} = 0

Φ_{bot} = +

Φ_{net} = 0

g.



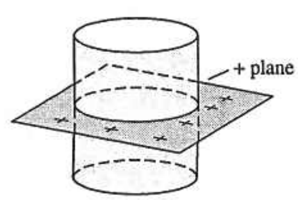
Φ_{top} = 0

Φ_{wall} = +

Φ_{bot} = 0

Φ_{net} = +

h.



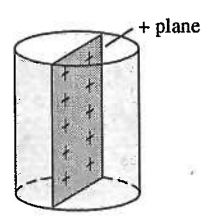
Φ_{top} = +

Φ_{wall} = 0

Φ_{bot} = +

Φ_{net} = +

i.



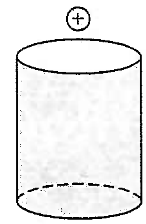
Φ_{top} = 0

Φ_{wall} = +

Φ_{bot} = 0

Φ_{net} = +

12. For this closed cylinder, $\Phi_{top} = -15\ \text{Nm}^2/\text{C}$ and $\Phi_{bot} = 5\ \text{Nm}^2/\text{C}$. What is Φ_{wall}?

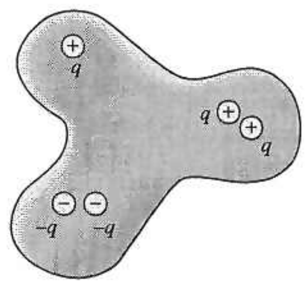


$\Phi_{wall} = 10\ \text{Nm}^2/\text{C}$

$\Phi_{top} + \Phi_{bot} + \Phi_{wall} = 0$

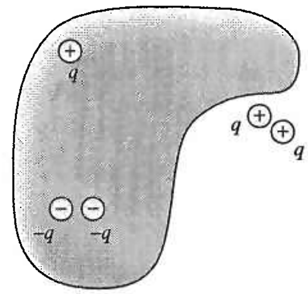
13. What is the electric flux through each of these surfaces? Give your answers as multiples of q/ε_0.

a.



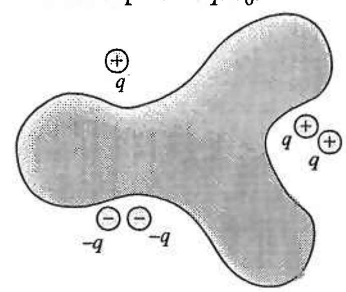
$\Phi_e =$ $+q/\varepsilon_0$

b.



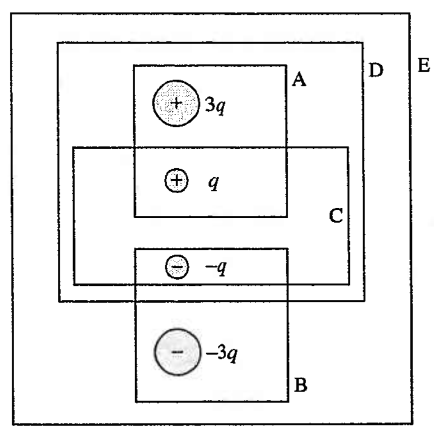
$\Phi_e =$ $-q/\varepsilon_0$

c.



$\Phi_e =$ 0

14. What is the electric flux through each of these surfaces? Give your answers as multiples of q/ε_0.



$\Phi_A =$ $+4q/\varepsilon_0$

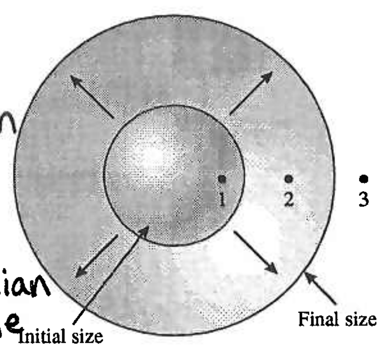
$\Phi_B =$ $-4q/\varepsilon_0$

$\Phi_C =$ 0

$\Phi_D =$ $+3q/\varepsilon_0$

$\Phi_E =$ 0

15. A charged balloon expands as it is blown up, increasing in size from the initial to final diameters shown. Do the electric fields at points 1, 2, and 3 increase, decrease, or stay the same? Explain your reasoning for each.

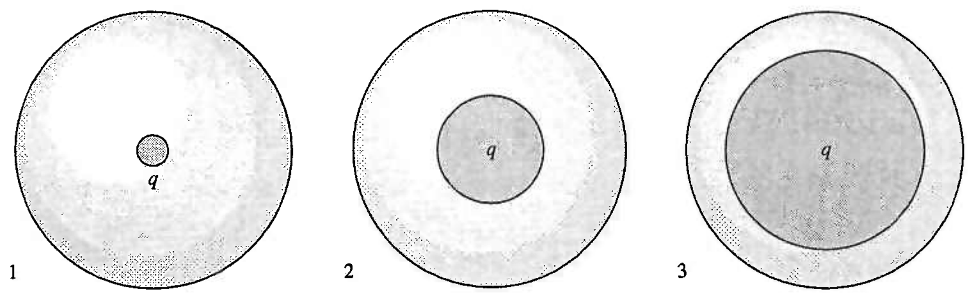


Point 1: Stays the same. A spherical Gaussian surface through 1 never encloses any charge, so the field at 1 is always zero.

Point 2: Decreases. As the balloon expands past point 2, the charge enclosed by a Gaussian surface through 2 decreases to zero, so the field decreases to zero.

Point 3: Stays the same. A spherical Gaussian surface through 3 always encloses all the charge on the balloon. The field at 3 is always as if the entire charge were located at the center of the balloon.

16. Three charges, all the same charge q, are surrounded by three spheres of equal radii.



a. Rank in order, from largest to smallest, the fluxes Φ_1, Φ_2, and Φ_3 through the spheres.

Order: $\Phi_1 = \Phi_2 = \Phi_3$

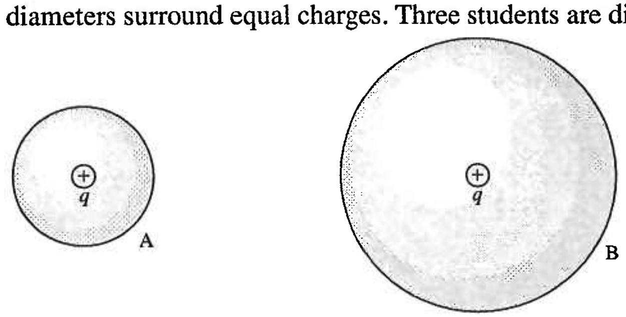
Explanation:
The flux is equal to the total charge enclosed by the surface divided by ϵ_0. Each surface encloses the same amount of charge.

b. Rank in order, from largest to smallest, the electric field strengths E_1, E_2, and E_3 on the surfaces of the spheres.

Order: $E_1 = E_2 = E_3$

Explanation:
The electric field outside a sphere of total charge q is the same as the field of a point charge q at the center.

17. Two spheres of different diameters surround equal charges. Three students are discussing the situation.



Student 1: The flux through spheres A and B are equal because they enclose equal charges.

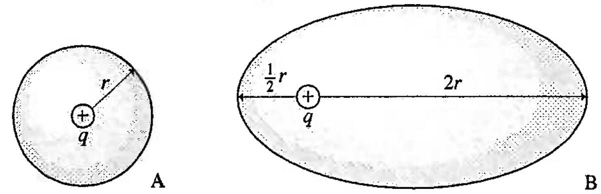
Student 2: But the electric field on sphere B is weaker than the electric field on sphere A. The flux depends on the electric field strength, so the flux through A is larger than the flux through B.

Student 3: I thought we learned that flux was about surface area. Sphere B is larger than sphere A, so I think the flux through B is larger than the flux through A.

Which of these students, if any, do you agree with? Explain.

Student 1. The area increases as r^2 and the electric field strength decreases as $1/r^2$ so the flux is the same through spheres A and B.

18. A sphere and an ellipsoid surround equal charges. Four students are discussing the situation.



Student 1: The fluxes through A and B are equal because the average radius is the same.

Student 2: I agree that the fluxes are equal, but it's because they enclose equal charges.

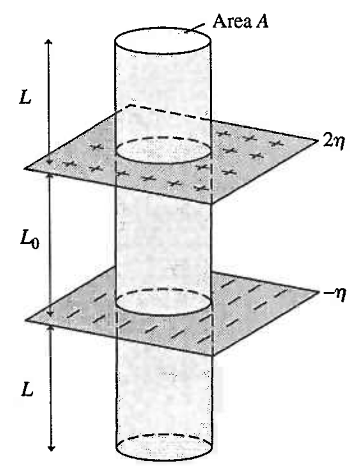
Student 3: The electric field is not perpendicular to the surface for B, and that makes the flux through B less than the flux through A.

Student 4: I don't think that Gauss's law even applies to a situation like B, so we can't compare the fluxes through A and B.

Which of these students, if any, do you agree with? Explain.

Student 2. As the area increases, the electric field decreases by the same factor so that the fluxes are equal.

19. Two parallel, infinite planes of charge have charge densities 2η and $-\eta$. A Gaussian cylinder with cross section A extends distance L to either side.



a. Is $\vec{E}$ perpendicular or parallel to the surface at the:

Top $\perp$ Bottom $\perp$ Wall $\parallel$

b. Is the electric field E_{top} emerging from the top surface stronger than, weaker than, or equal in strength to the field E_{bot} emerging from the bottom? Explain.

Equal. $E_{top} = \frac{2\eta}{\epsilon_0} - \frac{\eta}{\epsilon_0} = \frac{\eta}{\epsilon_0}$ up

$E_{bottom} = \frac{2\eta}{\epsilon_0} - \frac{\eta}{\epsilon_0} = \frac{\eta}{\epsilon_0}$ down

c. By inspection, write the electric fluxes through the three surfaces in terms of E_{top}, E_{bot}, E_{wall}, L, L_0, and A. (You may not need all of these.)

$\Phi_{top} = E_{top}A$ $\Phi_{bot} = E_{bot}A$ $\Phi_{wall} = 0$

d. How much charge is enclosed within the cylinder? Write Q_{in} in terms of η, L, L_0, and A.

$Q_{in} = (2\eta - \eta)A = \eta A$

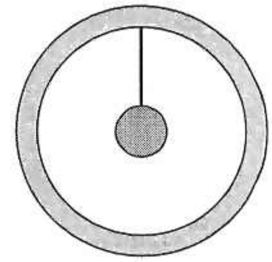
e. By combining your answers from parts b, c, and d, use Gauss's law to determine the electric field strength above the top plane. Show your work.

$\Phi_{top} = E_{top}A = \frac{Q_{in}}{\epsilon_0} = \frac{\eta A}{\epsilon_0}$

$E_{top} = \frac{\eta}{\epsilon_0}$

28.6 Conductors in Electrostatic Equilibrium

20. A small metal sphere hangs by a thread within a larger, hollow conducting sphere. A charged rod is used to transfer positive charge to the outer surface of the hollow sphere.



a. Suppose the thread is an insulator. After the charged rod touches the outer sphere and is removed, are the following surfaces positive, negative, or not charged?

The small sphere: not charged

The inner surface of the hollow sphere: not charged

The outer surface of the hollow sphere: positive

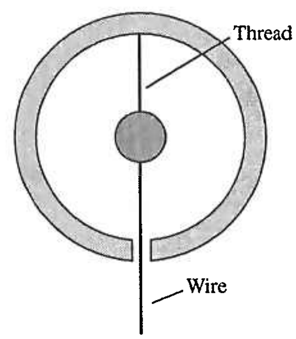
b. Suppose the thread is a conductor. After the charged rod touches the outer sphere and is removed, are the following surfaces positive, negative, or not charged?

The small sphere: not charged

The inner surface of the hollow sphere: not charged

The outer surface of the hollow sphere: positive

21. A small metal sphere hangs by an insulating thread within a larger, hollow conducting sphere. A conducting wire extends from the small sphere through, but not touching, a small hole in the hollow sphere. A charged rod is used to transfer positive charge to the wire. After the charged rod has touched the wire and been removed, are the following surfaces positive, negative, or not charged?

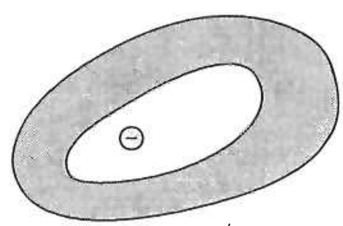


The small sphere: positive

The inner surface of the hollow sphere: negative

The outer surface of the hollow sphere: positive

22. A -10 nC point charge is inside a hole in a conductor. The conductor has no net charge.



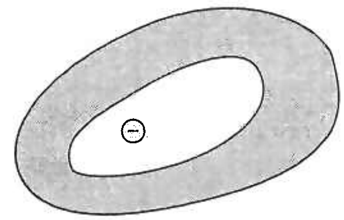
a. What is the total charge on the inside surface of the conductor?

+10nC

b. What is the total charge on the outside surface of the conductor?

-10nC

23. A -10 nC point charge is inside a hole in a conductor. The conductor has a net charge of $+10$ nC.



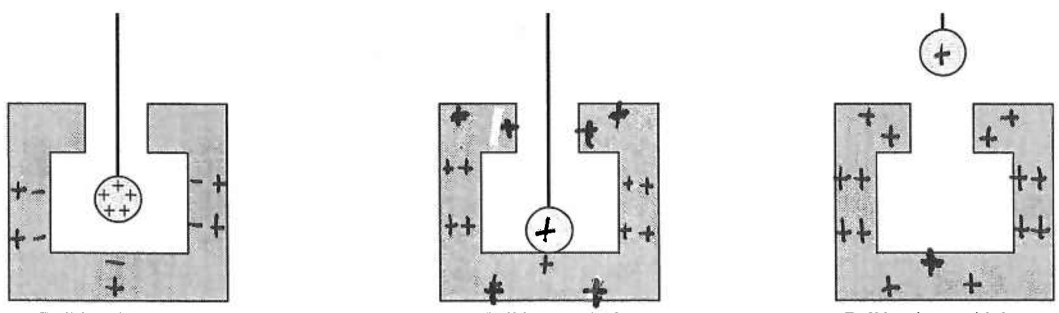
a. What is the total charge on the inside surface of the conductor?

+10 nC

b. What is the total charge on the outside surface of the conductor?

0

24. An insulating thread is used to lower a positively charged metal ball into a metal container. Initially, the container has no net charge. Use plus and minus signs to show the charge distribution on the ball at the times shown in the figure. (The ball's charge is already shown in the first frame.)

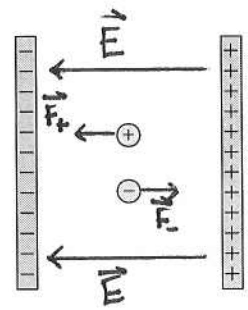


Ball hasn't touched | Ball has touched | Ball has been withdrawn

29 The Electric Potential

29.1 Electric Potential Energy

29.2 The Potential Energy of Point Charges



1. A positive point charge and a negative point charge are inside a parallel-plate capacitor. The point charges interact only with the capacitor, not with each other. Let the negative capacitor plate be the zero of potential energy for both charges.

   a. Use a **black** pen or pencil to draw the electric field vectors inside the capacitor.

   b. Use a **red** pen or pencil to draw the forces acting on the two charges.

   c. Is the potential energy of the *positive* point charge positive, negative, or zero? Explain.

   Positive. $U = qEs$ with q, E and s all positive. The positive charge will move in the direction of decreasing potential when released from rest.

   d. In which direction (right, left, up, or down) does the potential energy of the positive charge decrease? Explain.

   Left. When released from rest the positive charge moves toward the negative plate. Its kinetic energy increases and its potential energy decreases.

   e. In which direction will the positive charge move if released from rest? Use the concept of energy to explain your answer.

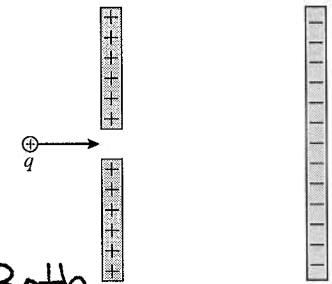
   Left. Kinetic energy increases as it accelerates toward the negative plate and potential energy decreases.

   f. Does your answer to part e agree with the force vector you drew in part b? Yes

   g. Repeat steps c to f for the *negative* point charge.

   c. negative. $U = qEs$ with q negative
   d. right. The potential energy becomes more negative as the charge moves to the right.
   e. right. It moves toward the direction of decreasing potential energy as its kinetic energy increases.
   f. yes.

2. A positive charge q is fired through a small hole in the positive plate of a capacitor. Does q speed up or slow down inside the capacitor? Answer this question twice:



a. First using the concept of force.

It speeds up. A positive charge inside the capacitor experiences an attractive force due to the negative plate and a repulsive force due to the positive plate. Both forces point toward the right.

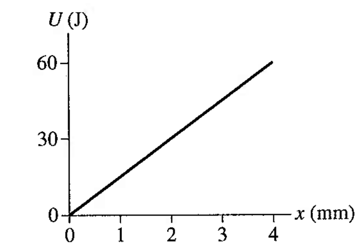
b. Second using the concept of energy.

The positive charge speeds up because it gains kinetic energy as it moves toward the negative plate in the direction of decreasing potential energy.

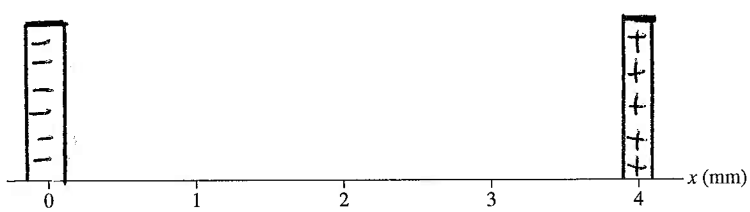
3. Charge $q_1 = 3$ nC is distance r from a positive point charge Q. Charge $q_2 = 1$ nC is distance $2r$ from Q. What is the ratio U_1/U_2 of their potential energies due to their interactions with Q?

$$\frac{U_1}{U_2} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_1 Q}{r}}{\frac{1}{4\pi\varepsilon_0}\frac{q_2 Q}{2r}} = \frac{q_1(2r)}{q_2(r)} = \frac{3\text{nC}(2r)}{1\text{nC}(r)} = 6$$

4. The figure shows the potential energy of a positively charged particle in a region of space.



a. What possible arrangement of source charges is responsible for this potential energy? Draw the source charges above the axis below.



b. With what kinetic energy should the charged particle be launched from $x = 0$ mm to have a turning point at $x = 3$ mm? Explain.

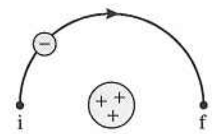
$-\Delta K = \Delta U$

$K_i - K_f = qEd \quad K_i = qE(0.003\text{m}) = 45\text{J}$ (from the graph)

c. How much kinetic energy does this charged particle of part b have as it passes $x = 2$ mm?

For each 1mm of travel toward the right there is a gain of 15J of potential energy and a loss of 15J of kinetic energy. As it passes x=2mm it has 15J of kinetic energy.

5. An electron ($q = -e$) completes half of a circular orbit of radius r around a nucleus with $Q = +3e$.



a. How much work is done on the electron as it moves from i to f? Give either a numerical value or an expression from which you could calculate the value if you knew the radius. Justify your answer.

Zero. The motion is always perpendicular to the force.

b. By how much does the electric potential energy change as the electron moves from i to f?

$\Delta U = U_f - U_i = 0$

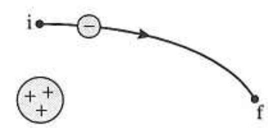
c. Is the electron's speed at f greater than, less than, or equal to its speed at i?

$V_f = V_i$

d. Are your answers to parts a and c consistent with each other?

Yes.

6. An electron moves along the trajectory from i to f.



a. Does the electric potential energy increase, decrease, or stay the same? Explain.

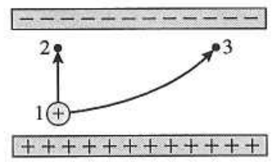
$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$

The potential energy varies as $\frac{1}{r}$, but because the two charges have opposite sign the potential energy actually increases as r increases.

b. Is the electron's speed at f greater than, less than, or equal to its speed at i? Explain.

Less than. Two opposite charges slow down as they move farther apart.

7. Inside a parallel-plate capacitor, two protons are launched with the same speed from point 1. One proton moves along the path from 1 to 2, the other from 1 to 3. Points 2 and 3 are the same distance from the positive plate.



a. Is $\Delta U_{1\to 2}$, the change in potential energy along the path $1 \to 2$, larger than, smaller than, or equal to $\Delta U_{1\to 3}$? Explain.

$\Delta U_{1\to 2} = \Delta U_{1\to 3}$

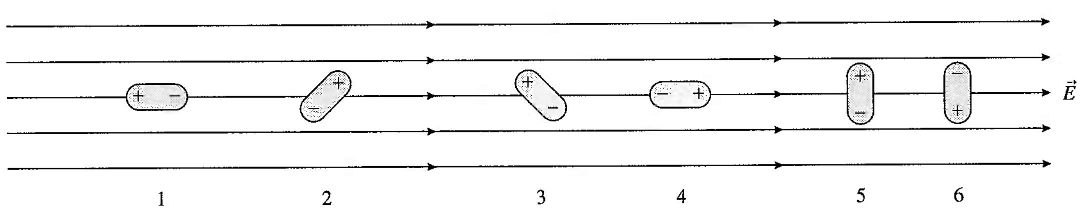
$\Delta U = qEs$ where s is the distance measured vertically from point 1 to point 2. Only the vertical displacement matters here because there is no horizontal component of E or of the force.

b. Is the proton's speed v_2 at point 2 larger than, smaller than, or equal to v_3? Explain.

$V_2 = V_3$ The change in kinetic energy $\Delta K_{1\to 2}$ is equal to $\Delta K_{1\to 3}$ because $\Delta U_{1\to 2} = \Delta U_{1\to 3}$ and the total energy is constant.

29.3 The Potential Energy of a Dipole

8. Rank in order, from most positive to most negative, the potential energies U_1 to U_6 of these six electric dipoles in a uniform electric field.



Order: $U_1 > U_3 > U_6 = U_5 > U_2 > U_4$

Explanation:

$U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$

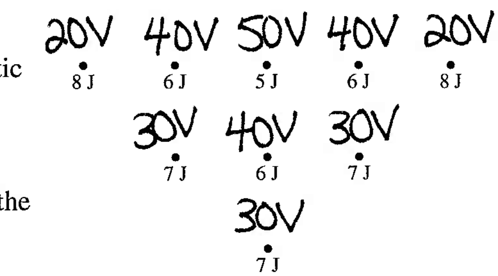
$U_1 = -pE\cos 180° = +pE$ $\quad U_4 = -pE\cos 0° = -pE$

$U_2 = -pE\cos 45° = -0.707pE$ $\quad U_5 = -pE\cos 90° = 0$

$U_3 = -pE\cos 135° = +0.707pE$ $\quad U_6 = -pE\cos(-90°) = 0$

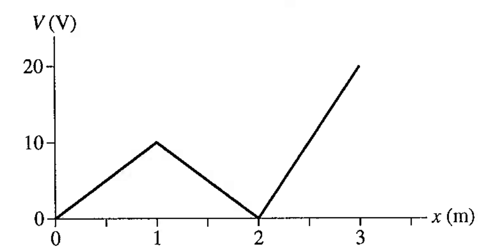
29.4 The Electric Potential

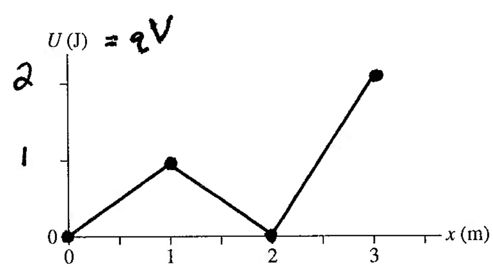
9. Charged particles with $q = +0.1$ C are fired with 10 J of kinetic energy toward a region of space in which there is an electric potential. The figure shows the kinetic energy of the charged particles as they arrive at nine different points in the region. Determine the electric potential at each of these points. Write the value of the potential above each of the dots. Assume that the particles start from a point where the electric potential is zero.



Ex.
- $K_2 = 7J$ $U_2 = 3J$
  $V_2 = \frac{U_2}{q} = \frac{3J}{0.1C} = 30V$
- $K_1 = 10J$ $U_1 = qV_1 = 0$ $V_1 = 0$

10. a. The graph on the left shows the electric potential along the x-axis. Use the axes on the right to draw a graph of the potential energy of a 0.1 C charged particle in this region of space. Provide a numerical scale on the energy axis.





b. If the charged particle is shot toward the right from $x = 1$ m with 1.0 J of kinetic energy, where is its turning point? Explain.

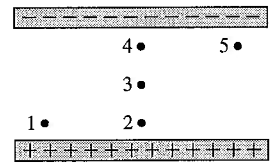
At $x = 1m$, the total energy $E = K_1 + U_1 = 1J + 1J = 2J$.
At the turning point $v = 0$ so $K_2 = 0$
$E = K_2 + U_2$ $2J = 0J + U_2$ $U_2 = 2J$ which occurs at $x = 3m$.

c. Will the charged particle of part b ever reach $x = 0$ m? If so, how much kinetic energy will it have at that point? If not, why not?

Yes it will reach $x = 0m$. It will have 2J of kinetic energy at that point.

29.5 The Electric Potential Inside a Parallel-Plate Capacitor

11. Rank in order, from largest to smallest, the electric potentials V_1 to V_5 at points 1 to 5.

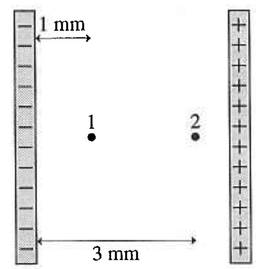


Order: $V_1 = V_2 > V_3 > V_4 = V_5$

Explanation:

$V = Es$ where s is measured from the negative plate.

12. The figure shows two points inside a capacitor. Let $V = 0$ V at the negative plate.



a. What is the ratio V_2/V_1 of the electric potential at these two points? Explain.

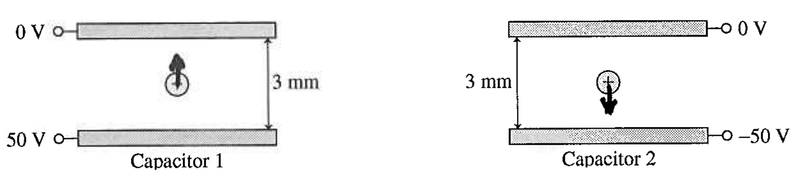
$\frac{V_2}{V_1} = \frac{Es_2}{Es_1} = \frac{s_2}{s_1} = \frac{3\text{mm}}{1\text{mm}} = 3$

b. What is the ratio E_2/E_1 of the electric field strength at these two points? Explain.

$\frac{E_2}{E_1} = 1$ The field inside a parallel-plate capacitor is a constant.

$E = \frac{\eta}{\epsilon_0}$

13. The figure shows two capacitors, each with a 3 mm separation. A proton is released from rest in the center of each capacitor.



a. Draw an arrow on each proton to show the direction it moves.

b. Which proton reaches a capacitor plate first? Or are they simultaneous? Explain.

Simultaneous. The electric field strength is the same in both capacitors.

14. A capacitor with plates separated by distance d is charged to a potential difference ΔV_C. All wires and batteries are disconnected, then the two plates are pulled apart (with insulated handles) to a new separation of distance $2d$.

a. Does the capacitor charge Q change as the separation increases? If so, by what factor? If not, why not?

The capacitor charge remains constant. No charge flows off the capacitor plates. So, by conservation of charge, $Q_1 = Q_2$.

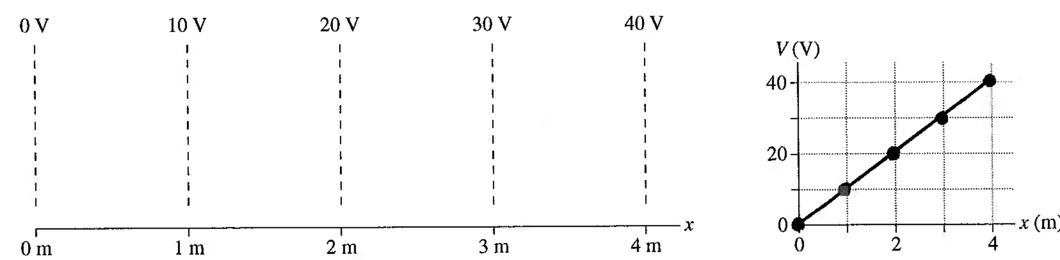
b. Does the electric field strength E change as the separation increases? If so, by what factor? If not, why not?

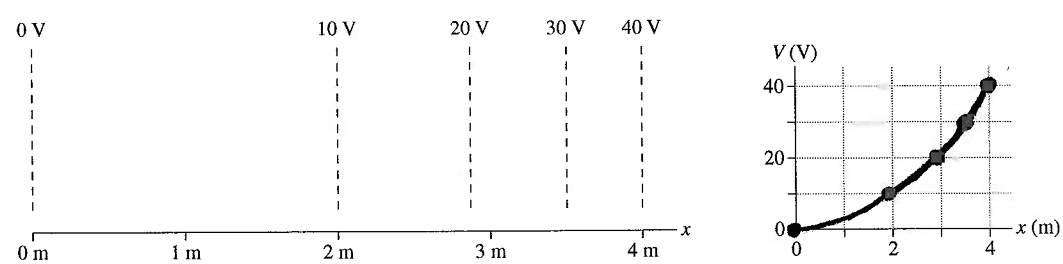
The electric field strength also remains constant. $E = \frac{\eta}{\epsilon_0} = \frac{Q/A}{\epsilon_0}$ Q, A and ϵ_0 are all constant.

c. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?

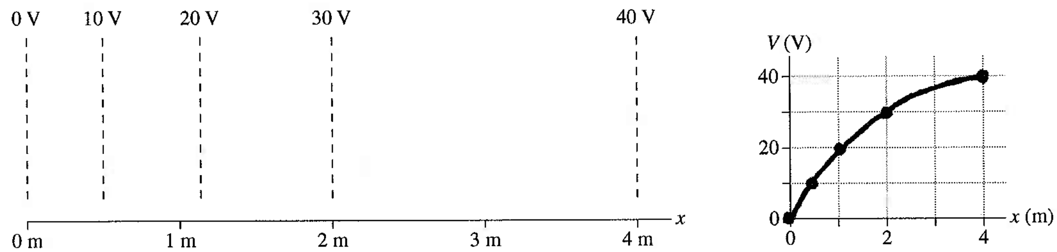
Yes. $\Delta V_C = Ed$ It increases by a factor of 2 because the separation increases by a factor of 2.

15. Each figure shows a contour map on the left and a set of graph axes on the right. Draw a graph of V versus x. Your graph should be a straight line or a smooth curve.

a.


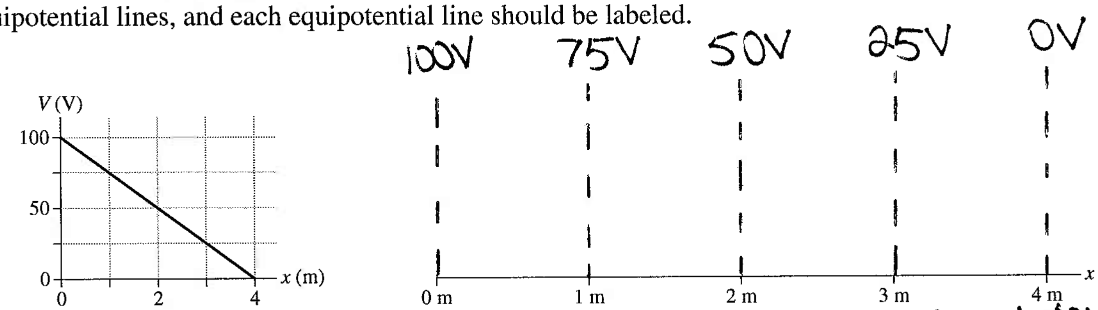
b.


c.

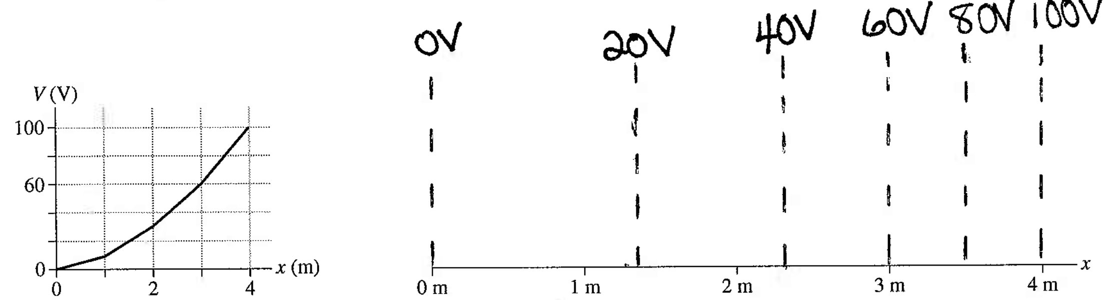


16. Each figure shows a V-versus-x graph on the left and an x-axis on the right. Assume that the potential varies with x but not with y. Draw a contour map of the electric potential. Your figures should look similar to the contour maps in Question 15. There should be a uniform difference between equipotential lines, and each equipotential line should be labeled.

a.

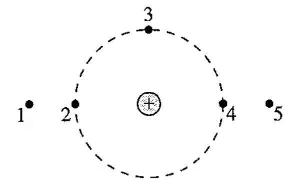


b.



29.6 The Electric Potential of a Point Charge

17. Rank in order, from largest to smallest, the electric potentials V_1 to V_5 at points 1 to 5.



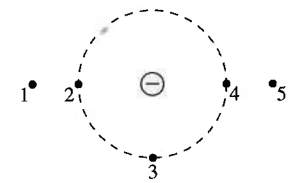
Order: $V_2 = V_3 = V_4 > V_1 = V_5$

Explanation:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

As r increases, V decreases.

18. Rank in order, from least negative to most negative, the electric potentials V_1 to V_5 at points 1 to 5.



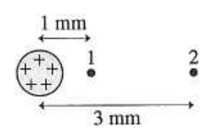
Order: $V_1 = V_5 > V_2 = V_3 = V_4$

Explanation:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

because q is a negative charge, as r increases V increases (becomes less negative).

19. The figure shows two points near a positive point charge.



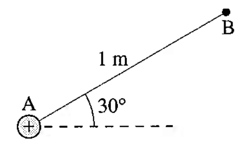
a. What is the ratio V_1/V_2 of the electric potentials at these two points? Explain.

$$\frac{V_1}{V_2} = \frac{\frac{1}{4\pi\epsilon_0}\frac{q}{r_1}}{\frac{1}{4\pi\epsilon_0}\frac{q}{r_2}} = \frac{r_2}{r_1} = \frac{3\text{mm}}{1\text{mm}} = 3$$

b. What is the ratio E_1/E_2 of the electric field strengths at these two points? Explain.

$$\frac{E_1}{E_2} = \frac{\frac{1}{4\pi\epsilon_0}\frac{q}{r_1^2}}{\frac{1}{4\pi\epsilon_0}\frac{q}{r_2^2}} = \frac{r_2^2}{r_1^2} = \frac{(3\text{mm})^2}{(1\text{mm})^2} = 9$$

20. A 1 nC positive point charge is located at point A. The electric potential at point B is



(a. 9 V) b. $9 \cdot \sin 30°$ V c. $9 \cdot \cos 30°$ V d. $9 \cdot \tan 30°$ V

Explain the reason for your choice.

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{(9.0\times10^9 \frac{Nm^2}{C^2})(1.0\times10^{-9}C)}{1m} = 9V$$

21. An inflatable metal balloon of radius R is charged to a potential of 1000 V. After all wires and batteries are disconnected, the balloon is inflated to a new radius $2R$.

a. Does the potential of the balloon change as it is inflated? If so, by what factor? If not, why not?

Yes. It decreases by a factor of 2.

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} \quad V_2 = \frac{1}{4\pi\epsilon_0}\frac{Q}{(2R)} \quad Q = \text{constant}$$

b. Does the potential at a point at distance $r = 4R$ change as the balloon is inflated? If so, by what factor? If not, why not?

No. Outside a sphere the potential is the same as that of a point charge Q located at the center of the sphere. The amount of charge does not change.

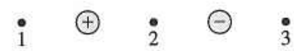
29.7 The Electric Potential of Many Charges

22. Each figure below shows three points in the vicinity of two point charges. The charges have equal magnitudes. Rank in order, from largest to smallest, the potentials V_1, V_2, and V_3.

a.

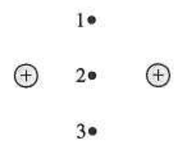
$V_2 > V_1 = V_3$

b.



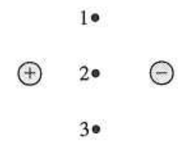
$V_1 > V_2 > V_3$

c.



$V_2 > V_1 = V_3$

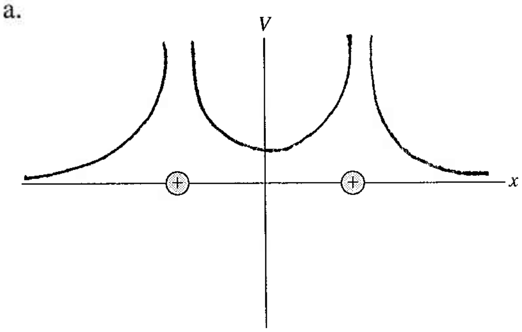
d.



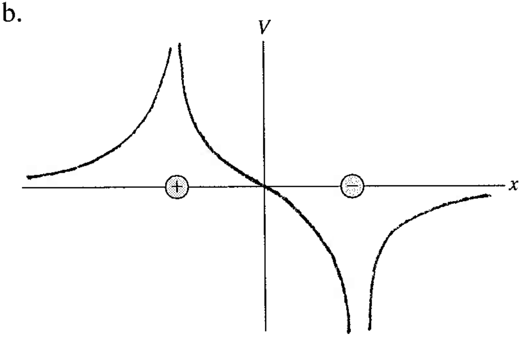
$V_1 = V_2 = V_3$

23. On the axes below, draw a graph of V versus x for the two point charges shown.

a.



b.

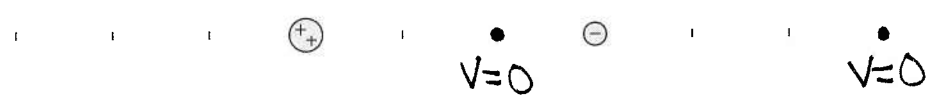


24. For each pair of charges below, are there any points (other than at infinity) at which the electric potential is zero? If so, identify them on the figure with a dot and a label. If not, why not?

a.

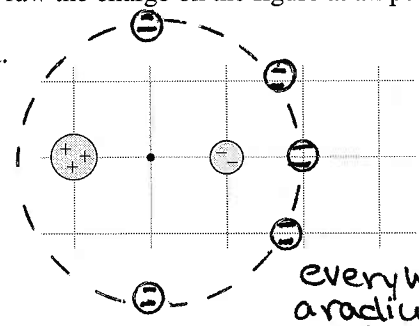
No. The potential is always positive.

b.



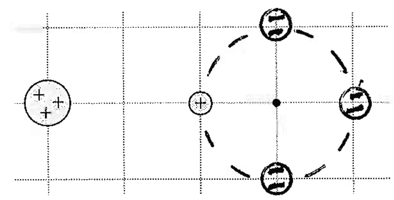
25. For each pair of charges below, at which grid point or points could a double-negative point charge ($q = -2$) be placed so that the potential at the dot is 0 V? There may be more than one possible point. Draw the charge on the figure at all points that work.

a.



everywhere on a radius of 2 circle centered at dot

b.

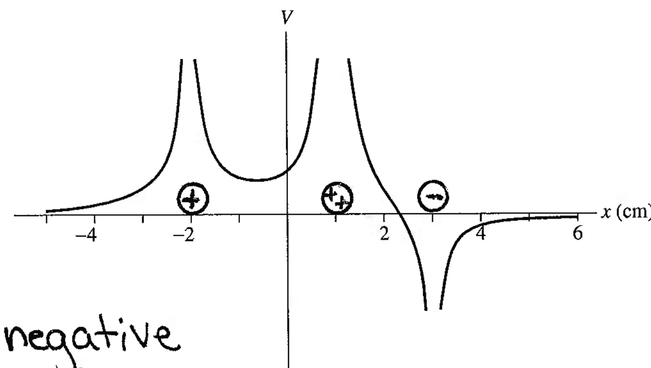


everywhere on a circle of radius 1 centered on the dot

26. The graph shows the electric potential along the x-axis due to point charges on the x-axis.

a. Draw the charges *on the axis of the figure*. Note that the charges may have different magnitudes.

b. An electron is placed at $x = 2$ cm. Is its potential energy positive, negative, or zero? Explain.



Its potential energy is negative
$U = qV$ q is negative and V is positive at $x = 2$ cm.

c. If the electron is released from rest at $x = 2$ cm, will it move right, move left, or remain at $x = 2$ cm? Base your explanation on energy concepts.

The electron will move left.
A charge will move in the direction of decreasing potential energy.

27. A ring has radius R and charge Q. The ring is shrunk to a new radius $\frac{1}{2}R$ with no change in its charge. By what factor does the on-axis potential at $z = R$ increase?

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2+R^2}}$$

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{(\frac{R}{2})^2+z^2}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{\frac{1}{4}R^2+R^2}}$$

$$\frac{V_2}{V_1} = \frac{\sqrt{2R^2}}{\sqrt{\frac{5}{4}R^2}} = \frac{\sqrt{2}}{\sqrt{\frac{5}{4}}} = 1.26$$

V increases by a factor of 1.26.

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Chapter 28 Exercises and Problems

#19

$\Phi_A = \frac{q_1 + q_3}{\varepsilon_0} = \frac{-q}{\varepsilon_0}$

$\Phi_B = \frac{q_2 + q_1}{\varepsilon_0} = \frac{3q}{\varepsilon_0}$

$\Phi_A - \Phi_B = \frac{q_3 - q_2}{\varepsilon_0} = \frac{-4q}{\varepsilon_0}$

$\Phi_C = \frac{q_2 + q_3}{\varepsilon_0} = \frac{-2q}{\varepsilon_0}$

$\Phi_A - \Phi_B + \Phi_C = \frac{2q_3}{\varepsilon_0} = \frac{-6q}{\varepsilon_0} \Rightarrow \boxed{q_3 = -3q.}$

$\Phi_C = \frac{q_2 - 3q}{\varepsilon_0} = \frac{-2q}{\varepsilon_0} \Rightarrow \boxed{q_2 = q}$

$\Phi_A = \frac{q_1 - 3q}{\varepsilon_0} = \frac{-q}{\varepsilon_0} \Rightarrow \boxed{q_1 = 2q}$

#38 a) $\rho = \frac{Q_{total}}{V} = \frac{Q_{total}}{\frac{4}{3}\pi r^3} = \frac{80 \times 10^{-9}\,C}{\frac{4}{3}\pi (0.2\,m)^3} = 2.39 \times 10^{-6}\,\frac{C}{m^3}$.

b) $q_{encl} = \rho V_{encl.} = \rho \frac{4}{3}\pi r_{encl}^3$

For $r = 5\,cm$, we get

$q_{encl} = \frac{4}{3}\pi (2.3873\ldots \times 10^{-6}\,\frac{C}{m^3})(0.05\,m)^3$

$= \boxed{1.25 \times 10^{-9}\,C.}$

Similarly, for $r = 10$ cm, we get

$q_{encl} = \boxed{1.00 \times 10^{-8}\,C}$.

For $r = 20$ cm, we just get our total charge $\boxed{80 \times 10^{-9}\,C}$.

c) Use Gauss' law for a sphere.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q_{encl}}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q_{encl}}{r^2}$$

For $r = 5$cm,

$$E = \frac{1}{4\pi(8.85 \times 10^{-12}\,C^2/N \cdot m^2)}\frac{1.25 \times 10^{-9}\,C}{(0.05\,m)^2} = \boxed{4.50 \times 10^{3}\,\frac{N}{C}}.$$

Similarly, for $r = 10$ cm,

$$\boxed{E = 8.99 \times 10^{3}\,\frac{N}{C}}.$$

Finally, for $r = 20$ cm,

$$\boxed{E = 1.80 \times 10^{4}\,\frac{N}{C}}.$$

Chapter 29 Exercises and Problems

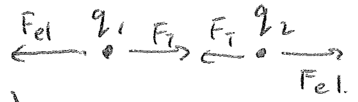
#40 a) Charged spheres act like point charges, so

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{1}{4\pi(8.85 \times 10^{-12}\,C^2/N \cdot m^2)}\frac{(2 \times 10^{-6}\,C)^2}{0.05\,m}$$

$$= \boxed{7.19 \times 10^{-1}\,J.}$$

b) $F_{el} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} = \frac{U}{r} = \frac{0.7193\ldots \text{ N}\cdot\text{m}}{0.05\text{ m}} = \boxed{1.44\text{ N}} = F_T.$

(in this case!)



c) There are no external forces, so the centre of mass is fixed.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

When the charges are far apart, they are moving at constant speed, so $v = \frac{\Delta x}{\Delta t}$. Now consider changes in centre of mass and set it to 0.

$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} = 0$$

Divide both sides by Δt (or take time derivative).

$$\frac{m_1 \frac{\Delta x_1}{\Delta t} + m_2 \frac{\Delta x_2}{\Delta t}}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$

$$\Rightarrow m_1 v_1 = -m_2 v_2$$

Actually, momentum conservation would have been <u>much</u> faster. Let $m_1 = 2g$, $m_2 = 4g$.

$$v_1 = -\frac{m_2}{m_1} v_2 = -\frac{4.0g}{2.0g} v_2 = -2v_2.$$

Conservation of energy / work-energy theorem:

$$\Delta U = -\Delta E_k = -\left(\tfrac{1}{2}m_1 v_{1f}^2 - \tfrac{1}{2}m_1 v_{1i}^2 + \tfrac{1}{2}m_2 v_{2f}^2 - \tfrac{1}{2}m_2 v_{2i}^2\right)$$

($v_{1i}^2 \to 0$, $v_{2i}^2 \to 0$)

$$= -\tfrac{1}{2}m_1 v_{1f}^2 - \tfrac{1}{2}m_2 v_{2f}^2 = -\tfrac{1}{2}m_1(-2v_{2f})^2 - \tfrac{1}{2}m_2 v_{2f}^2$$

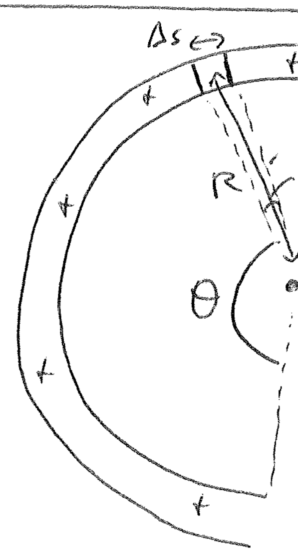
$$= -v_{2f}^2\left(\tfrac{1}{2}\right)(4m_1 + m_2) \Rightarrow v_{2f}^2 = \frac{-2\Delta U}{4m_1 + m_2}.$$

$$v_{2f} = \sqrt{\frac{-2\Delta U}{4m_1 + m_2}} = \sqrt{\frac{-2(-7.19 \times 10^{-1}\,J)}{4(0.002)\,kg + 0.004\,kg}}$$

$$= 1.09 \times 10^{1}\ \frac{m}{s}$$

$$|v_{1f}| = +2v_{2f} = 21.9\ \frac{m}{s}.$$

(#71) $V = \int \frac{dq}{4\pi\varepsilon_0 r}$ ← goal.



$$V = \sum_i \frac{\Delta q}{4\pi\varepsilon_0 r_i}$$

Chop up arc. Notice $r_i = R$ for all i. Use $\Delta q = \lambda \Delta s$.
$\Delta s = R\Delta\theta$.

$$V = \sum_i \frac{\lambda \Delta s}{4\pi\varepsilon_0 R} = \sum_i \frac{\lambda R \Delta\theta}{4\pi\varepsilon_0 R}$$

Convert to integral. $\sum_i \to \int_0^\pi$, $\Delta\theta \to d\theta$.

$$V = \int_0^\pi \frac{\lambda R\, d\theta}{4\pi\varepsilon_0 R} = \frac{\lambda\pi}{4\pi\varepsilon_0} = \frac{\lambda}{4\varepsilon_0} = \frac{Q/\pi R}{4\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 R}.$$

Could also have not used integral! $\sum \Delta q = Q$.