Tutorial 8 Problems

A selection of the following problems were done:

Workbook (2nd edition)
Chapter 30:
4, 12

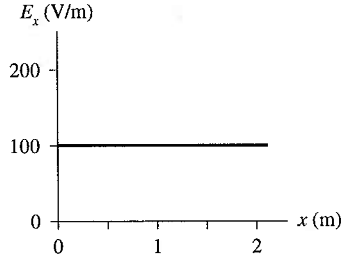
Textbook (2nd edition)
Chapter 30:
9, 11, 29, 44

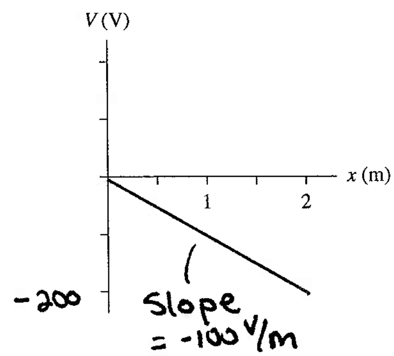
30 Potential and Field

30.1 Connecting Potential and Field

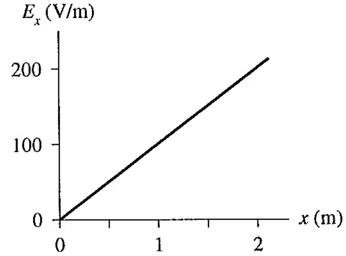
1. The top graph shows the x-component of $\vec{E}$ as a function of x. On the axes below the graph, draw the graph of V versus x in this same region of space. Let $V = 0$ V at $x = 0$ m. Include an appropriate vertical scale. (Hint: Integration is the area under the curve.)

a.

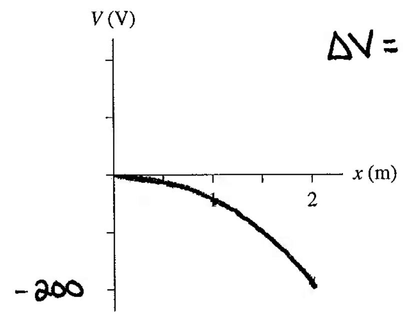




b.

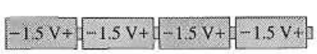


$$\Delta V = -\int_{0m}^{2m} E_x \, dx$$



30.2 Sources of Electric Potential

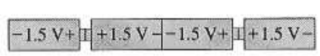
2. What is ΔV_{series} for each group of 1.5 V batteries?

a. 

$\Delta V_{series} =$ 6V

b. 

$\Delta V_{series} =$ 3V

c. 

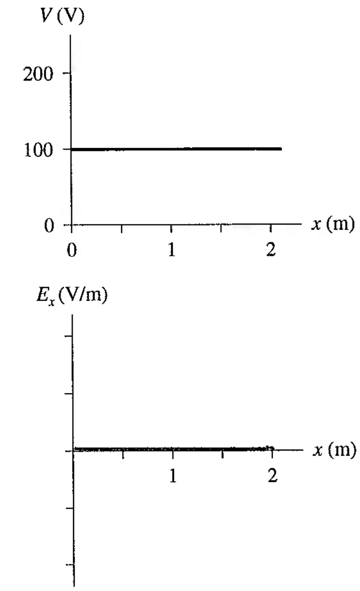
$\Delta V_{series} =$ 0V

30.3 Finding the Electric Field from the Potential

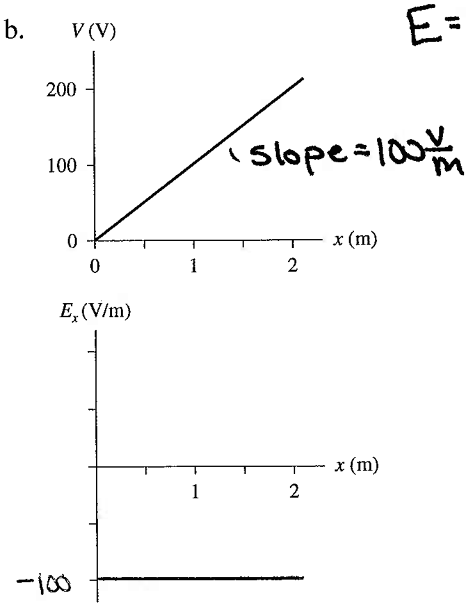
3. The top graph shows the electric potential as a function of x. On the axes below the graph, draw the graph of E_x versus x in this same region of space. Add an appropriate scale on the vertical axis.

$$E = -\frac{\partial V}{\partial x}$$

a.

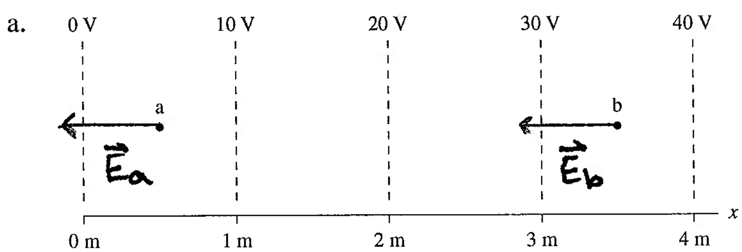


b.



4. For each contour map:
   i. Estimate the electric fields $\vec{E}_a$ and $\vec{E}_b$ at points a and b. Don't forget that $\vec{E}$ is a vector. Show how you made your estimate.
   ii. On the contour map, draw the electric field vectors at points a and b.

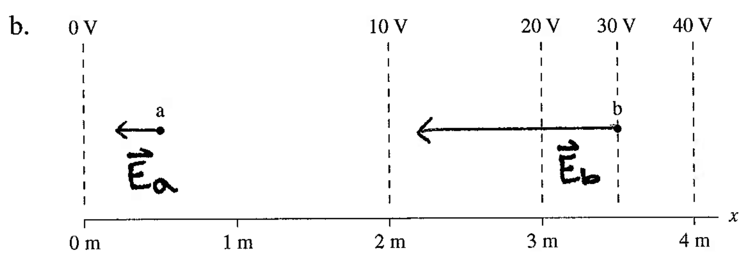
a.



$\vec{E}_a = -\frac{\Delta V}{\Delta S} = \frac{10V}{1m} = 10\frac{V}{m}$

$\vec{E}_b = 10\frac{V}{m}$

b.

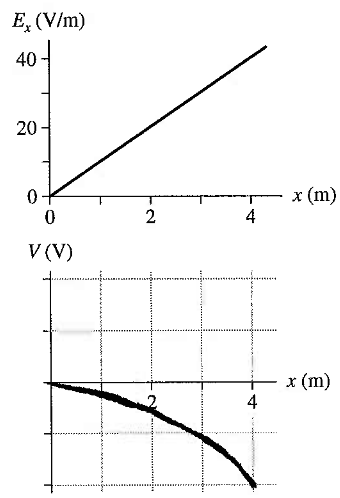


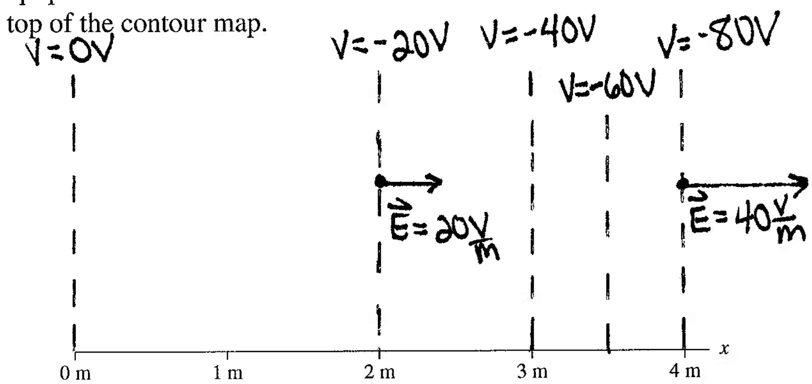
$\vec{E}_a = \frac{10V - 0V}{2m - 0m} = 5\frac{m}{m}$

$\vec{E}_b = \frac{40V - 20V}{4m - 3m} = 20\frac{V}{m}$

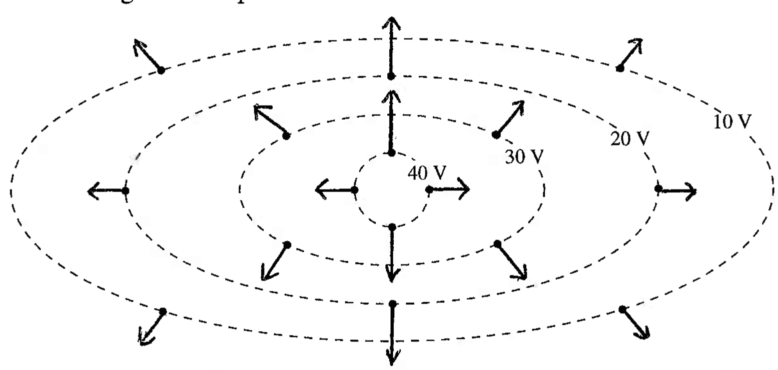
The minus sign in the equation means that the electric field vector points "downhill" on the contour map.

5. The top graph shows E_x versus x for an electric field that is parallel to the x-axis.
   a. Draw the graph of V versus x in this region of space. Let $V = 0$ V at $x = 0$ m. Add an appropriate scale on the vertical axis. (Hint: Integration is the area under the curve.)
   b. Use dashed lines to draw a contour map above the x-axis on the right. Space your equipotential lines every 20 volts and label each equipotential line.
   c. Draw electric field vectors on top of the contour map.

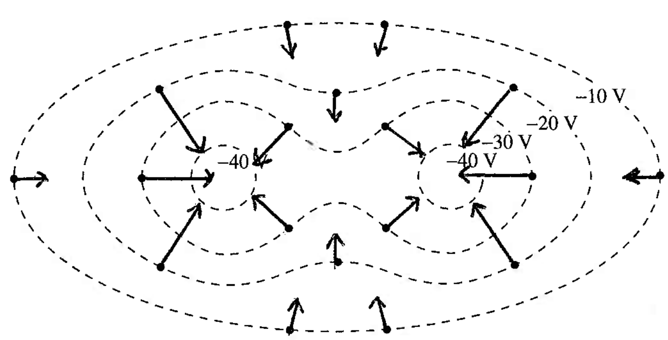




6. Draw the electric field vectors at the dots on this contour map. The length of each vector should be proportional to the field strength at that point.



7. Draw the electric field vectors at the dots on this contour map. The length of each vector should be proportional to the field strength at that point.



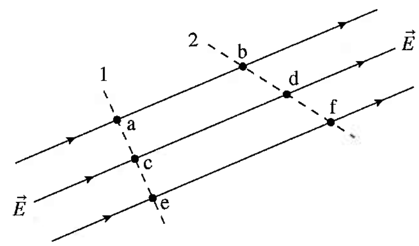
8. a. Suppose $\vec{E} = \vec{0}$ V/m throughout some region of space. Is $V = 0$ V in this region? Explain.

Not necessarily. The potential can have any value, but it cannot change its value in this region of space because $E = -\frac{\partial V}{\partial s}$
($\Delta V = 0V$)

b. Suppose $V = 0$ V throughout some region of space. Is $\vec{E} = \vec{0}$ V/m in this region? Explain.

Yes. If V has any constant value throughout a region of space, then the $\vec{E}$ field is 0 in that region.

9. The figure shows an electric field diagram. Dashed lines 1 and 2 are two surfaces in space, not physical objects.



a. Is the electric potential at point a higher than, lower than, or equal to the electric potential at point b? Explain.

The electric field vector points toward decreasing potential. Therefore $V_a > V_b$.

b. Rank in order, from largest to smallest, the potential differences ΔV_{ab}, ΔV_{cd}, and ΔV_{ef}.

Order: $\Delta V_{ab} > \Delta V_{cd} > \Delta V_{ef}$

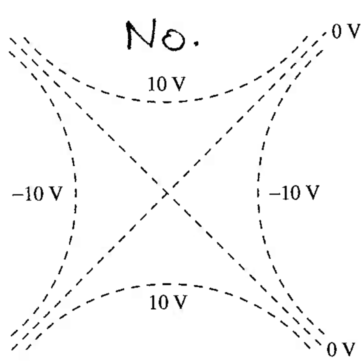
Explanation:
$\Delta V = -E\Delta S$
$\Delta S_{ab} < \Delta S_{cd} < \Delta S_{ef}$
The minus sign reverses the inequality direction.

c. Is surface 1 an equipotential surface? What about surface 2? Explain why or why not.

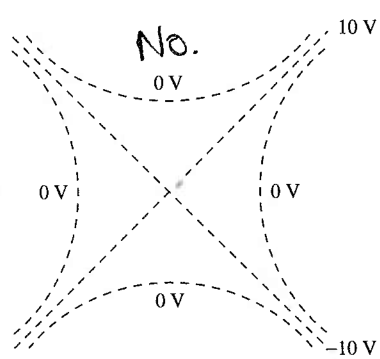
Yes. Surface 1 is an equipotential surface because it is perpendicular to the electric field vectors. Surface 2 is not perpendicular so it is not an equipotential surface.

10. For each of the figures below, is this a physically possible potential map if there are no free charges in this region of space? If so, draw an electric field line diagram on top of the potential map. If not, why not?

a.



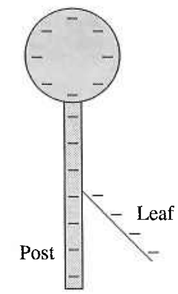
b.



Potential lines will never cross because no single point can have two different potential values.

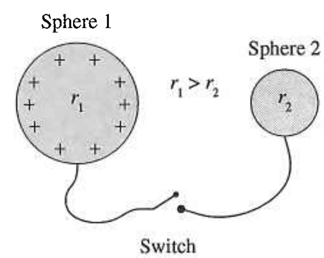
30.4 A Conductor in Electrostatic Equilibrium

11. The figure shows a negatively charged electroscope. The gold leaf stands away from the rigid metal post. Is the electric potential of the leaf higher than, lower than, or equal to the potential of the post? Explain.



Equal. When a conductor is in electrostatic equilibrium, the entire conductor is at the same potential.

12. Two metal spheres are connected by a metal wire that has a switch in the middle. Initially the switch is open. Sphere 1, with the larger radius, is given a positive charge. Sphere 2, with the smaller radius, is neutral. Then the switch is closed. Afterward, sphere 1 has charge Q_1, is at potential V_1, and the electric field strength at its surface is E_1. The values for sphere 2 are Q_2, V_2, and E_2.



a. Is V_1 larger than, smaller than, or equal to V_2? Explain.

$V_1 = V_2$ Both spheres and the wire become one conductor all at the same potential.

b. Is Q_1 larger than, smaller than, or equal to Q_2? Explain.

$Q_1 > Q_2$, $V_1 = V_2$

$$\frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_1} = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2} \quad \frac{Q_1}{r_1} = \frac{Q_2}{r_2} \quad r_1 > r_2 \Rightarrow Q_1 > Q_2$$

c. Is E_1 larger than, smaller than, or equal to E_2? Explain.

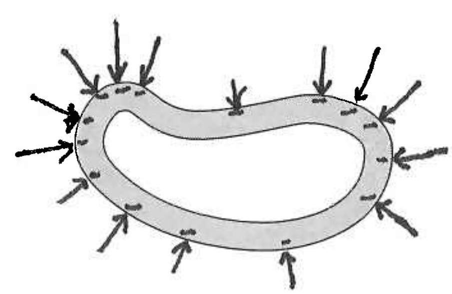
$E_1 < E_2$

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r_1^2} \quad E_2 = \frac{1}{4\pi\epsilon_0}\frac{Q_2}{r_2^2}$$

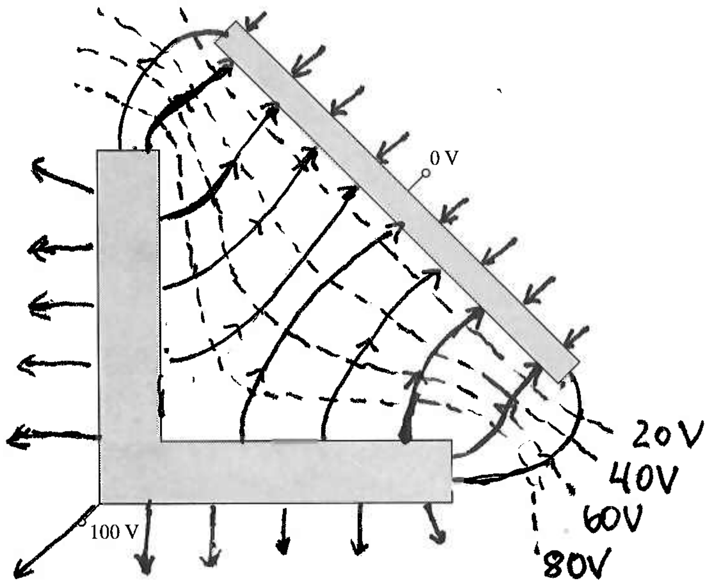
13. The figure shows a hollow metal shell. A negatively charged rod touches the top of the sphere, transferring charge to the shell. Then the rod is removed.

a. Show on the figure the equilibrium distribution of charge.

b. Draw the electric field diagram.

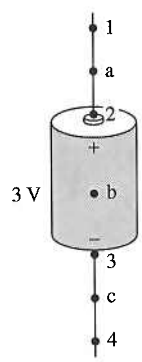


14. The figure shows two flat metal electrodes that are held at potentials of 100 V and 0 V.
   a. Used dashed lines to sketch a reasonable approximation of the 20 V, 40 V, 60 V, and 80 V equipotential lines.
   b. Draw enough electric field lines to indicate the shape of the electric field. Use solid lines with arrowheads.



15. The figure shows two 3 V batteries with metal wires attached to each end. Points a and c are *inside* the wire. Point b is inside the battery. For each figure:
   - What are the potential differences ΔV_{12}, ΔV_{23}, ΔV_{34}, and ΔV_{14}?
   - Does the electric field at a, b, and c point left, right, up, or down? Or is $\vec{E} = \vec{0}$?

a.

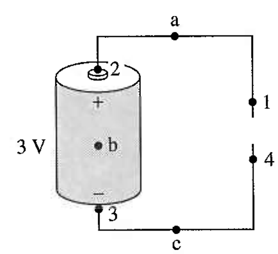


$\Delta V_{12} =$ 0V $\Delta V_{23} =$ 3V

$\Delta V_{34} =$ 0V $\Delta V_{14} =$ 3V

$\vec{E}_a$ Zero $\vec{E}_b$ Down $\vec{E}_c$ Zero

b.



$\Delta V_{12} =$ 0V $\Delta V_{23} =$ 3V

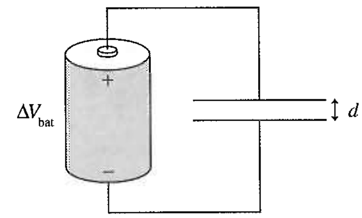
$\Delta V_{34} =$ 0V $\Delta V_{14} =$ 3V

$\vec{E}_a$ Zero $\vec{E}_b$ Down $\vec{E}_c$ Zero

30.5 Capacitance and Capacitors

30.6 The Energy Stored in a Capacitor

16. A parallel-plate capacitor with plate separation d is connected to a battery that has potential difference ΔV_{bat}. Without breaking any of the connections, insulating handles are used to increase the plate separation to $2d$.



a. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?

No, it does not change because the upper plate is still connected to the positive electrode of the battery and the bottom plate to the negative electrode.

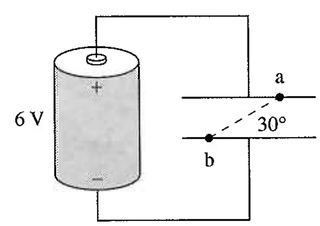
b. Does the capacitance change? If so, by what factor? If not, why not?

Yes. $C = \frac{\epsilon_0 A}{d}$ So when the plate separation is doubled the capacitance decreases by a factor of 2.

c. Does the capacitor charge Q change? If so, by what factor? If not, why not?

Yes. It also decreases by a factor of 2. $C = \frac{Q}{\Delta V_C}$ and ΔV_C is unchanged.

17. For the capacitor shown, the potential difference ΔV_{ab} between points a and b is



a. 6 V	b. 6 · sin 30° V	c. 6/sin 30° V
d. 6 · tan 30° V	e. 6 · cos 30° V	f. 6/cos 30° V

Explain your choice.

The entire top plate is at the same potential as the positive electrode of the battery (6V) and the bottom plate is at the same potential as the negative electrode (0V). Each capacitor plate is an equipotential surface.

Knight, 2nd edition

Chapter 30 Exercises and Problems

#9

Imagine a positive test charge is at the dot, not moving. It will want to move towards lower potential to decrease its potential energy. $\vec{E}$ points in the direction of acceleration if there are no other forces, so $\vec{E}$ points to the left.

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

V only changes in the $\hat{i}$ direction, so

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} \approx -\frac{\Delta V}{\Delta x}\hat{i} = -\frac{(100\text{ V})}{(1\text{cm})}\hat{i} = -\frac{100}{0.01}\frac{\text{V}}{\text{m}}\hat{i} = -10000\frac{\text{V}}{\text{m}}\hat{i}.$$

#11

$$E_x = -\frac{\partial V}{\partial x}$$

In other words, find the slope of the V vs. x graph.

For $0 < x < 10\text{ cm}$,

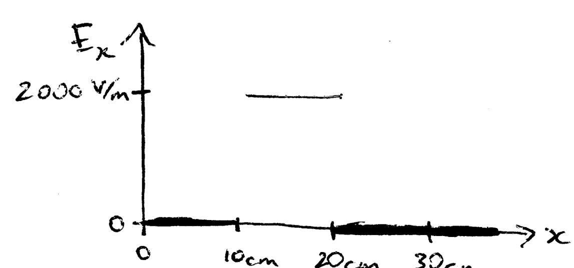
$$E_x = -\frac{\partial V}{\partial x} = -\frac{(0\text{V})}{10\text{cm}} = 0\frac{\text{V}}{\text{m}}.$$

For $10\text{cm} < x < 20\text{cm}$,

$$E_x = -\frac{(-100\text{ V} - 100\text{ V})}{(20\text{ cm} - 10\text{cm})} = \frac{200\text{ V}}{10\text{cm}} = \frac{200}{0.10}\frac{\text{V}}{\text{m}} = 2000\frac{\text{V}}{\text{m}}.$$

For $x > 20\text{cm}$,

$$E_x = -\frac{(0\text{V})}{10\text{cm}} = 0\frac{\text{V}}{\text{m}}$$



#29

$$U_c = \frac{Q^2}{2C}.$$

Note that Q rises linearly to 300 μC in 3s. That is,

$Q = \alpha t, \quad 0 \le t \le 3s, \quad \text{where } \alpha = \dfrac{200\mu C}{3s}.$

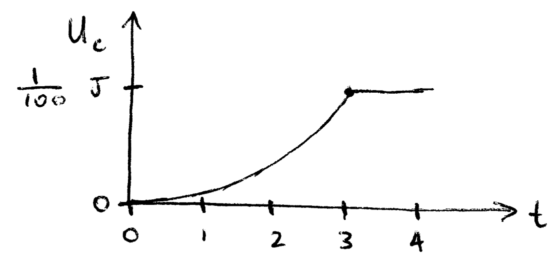
$Q = 200\mu C, \quad t \ge 3s.$

Plug this into our equation for U_c:

$$U_c = \begin{cases} \alpha^2 t^2/2C, & 0 \le t \le 3s \\ (200\mu C)^2/2C, & 3s \le t. \end{cases}$$

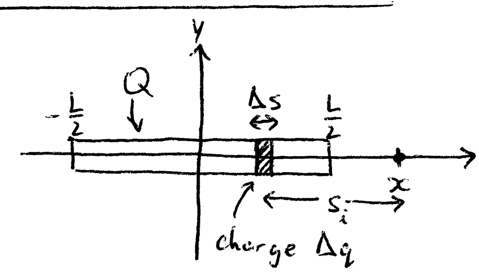
$$= \begin{cases} \left(\dfrac{200\times10^{-6}C}{3s}\right)^2 \left(\dfrac{1/2}{2\times10^{-6}F}\right) t^2, & 0 \le t \le 3s \\ \dfrac{(200\times10^{-6}C)^2}{2(2\times10^{-6}F)}, & t \ge 3s. \end{cases}$$

$$= \begin{cases} \dfrac{t^2}{900}\dfrac{J}{s^2}, & 0 \le t \le 3s \\ \dfrac{1}{100} J, & t \ge 3s. \end{cases}$$



#44

a) Split the rod into a bunch of small pieces that look like point charges and sum up the potentials due to all the pieces.



$$V_i = \frac{\Delta q}{4\pi\varepsilon_0 s_i}$$

$$V_{total} = \sum_i V_i = \sum_i \frac{\Delta q}{4\pi\varepsilon_0 s_i}$$

$\dfrac{\Delta q}{\Delta s} = \dfrac{Q}{L}$ since the charge density of the rod is assumed uniform.

$$\Rightarrow \Delta q = \frac{Q}{L}\Delta s$$

$$V_{total} = \sum_i \frac{Q\,\Delta s}{4\pi\varepsilon_0 L s_i} \xrightarrow[s_i \to s,\ \Delta s \to ds]{\Sigma \to \int} \int_{x-\frac{1}{2}L}^{x+\frac{1}{2}L} \frac{Q\,ds}{4\pi\varepsilon_0 L s}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\left[\ln\left(x+\tfrac{1}{2}L\right) - \ln\left(x-\tfrac{1}{2}L\right)\right] = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x+\frac{1}{2}L}{x-\frac{1}{2}L}\right).$$

b) $$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} = -\frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{x+\frac{1}{2}L} - \frac{1}{x-\frac{1}{2}L}\right]\hat{i}$$

$$= -\frac{Q}{4\pi\varepsilon_0 L}\left[\frac{(x-\frac{1}{2}L)-(x+\frac{1}{2}L)}{(x+\frac{1}{2}L)(x-\frac{1}{2}L)}\right]\hat{i} = -\frac{Q}{4\pi\varepsilon_0 L}\left[\frac{-L}{(x+\frac{1}{2}L)(x-\frac{1}{2}L)}\right]\hat{i}$$

$$= \frac{Q}{4\pi\varepsilon_0}\frac{1}{(x+\frac{1}{2}L)(x-\frac{1}{2}L)}\hat{i}.$$

The nice thing about this approach to finding the electric field is that you don't have to deal with vectors. The benefits would be more obvious in a 2D problem.