Tutorial 9 Problems

A selection of the following problems were done:

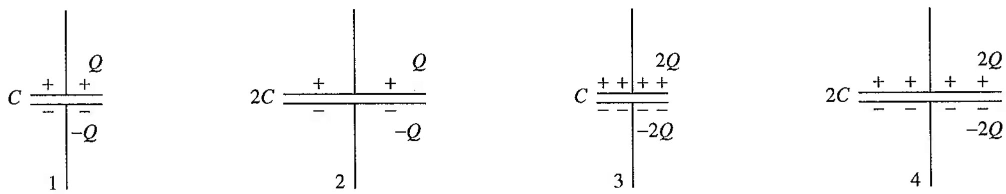
Workbook (2nd edition)
Chapter 30:
19

Chapter 31:
6

Textbook (2nd edition)
Chapter 30:
67

Chapter 31:
42, 58

18. Rank in order, from largest to smallest, the potential differences $(\Delta V_C)_1$ to $(\Delta V_C)_4$ of these four capacitors.



Order: $(\Delta V_c)_3 > (\Delta V_c)_1 = (\Delta V_c)_4 > (\Delta V_c)_2$

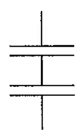
Explanation:

$(\Delta V_c)_1 = \frac{Q}{C}$ $(\Delta V_c)_2 = \frac{Q}{2C} = \frac{1}{2}(\Delta V_c)_1$ $(\Delta V_c)_3 = \frac{2Q}{C} = 2(\Delta V_c)_1$

$(\Delta V_c)_4 = \frac{2Q}{2C} = (\Delta V_c)_1$

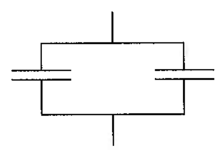
19. Each capacitor in the circuits below has capacitance C. What is the equivalent capacitance of the group of capacitors?

a.



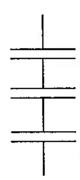
$C_{eq} = \frac{C}{2}$

b.



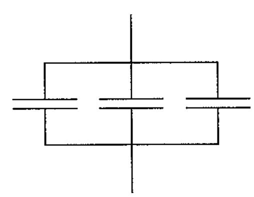
$C_{eq} = 2C$

c.



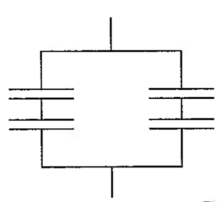
$C_{eq} = \frac{C}{3}$

d.



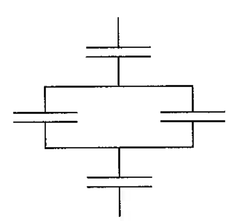
$C_{eq} = 3C$

e.



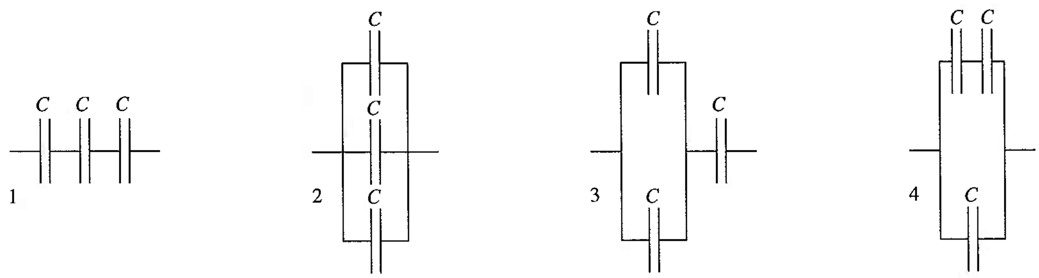
$C_{eq} = C$

f.



$C_{eq} = \frac{2C}{5}$

20. Rank in order, from largest to smallest, the equivalent capacitances $(C_{eq})_1$ to $(C_{eq})_4$ of these four groups of capacitors.



Order: $(C_{eq})_2 > (C_{eq})_4 > (C_{eq})_3 > (C_{eq})_1$

Explanation:

$(C_{eq})_1 = \left(\frac{1}{C}+\frac{1}{C}+\frac{1}{C}\right)^{-1} = \frac{1}{3}C$; $(C_{eq})_2 = C+C+C = 3C$;

$(C_{eq})_3 = \left(\frac{1}{2C}+\frac{1}{C}\right)^{-1} = \frac{2}{3}C$; $(C_{eq})_4 = \frac{C}{2}+C = \frac{3}{2}C$

21. Rank in order, from largest to smallest, the energies $(U_C)_1$ to $(U_C)_4$ stored in each of these capacitors.

Order: $(U_c)_2 > (U_c)_1 > (U_c)_3 > (U_c)_4$

Explanation:

$U_1 = \frac{1}{2}C(\Delta V_c)^2$

$U_2 = \frac{1}{2}\left(\frac{1}{2}C\right)(2\Delta V_c)^2 = 2U_1$

$U_3 = \frac{1}{2}(2C)\left(\frac{1}{2}\Delta V_c\right)^2 = \frac{1}{2}U_1$

$U_4 = \frac{1}{2}(4C)\left(\frac{1}{3}\Delta V\right)^2 = \frac{4}{9}U_1$

30.7 Dielectrics

22. An air-insulated capacitor is charged until the electric field strength inside is 10,000 V/m, then disconnected from the battery. When a dielectric is inserted between the capacitor plates, the electric field strength is reduced to 2000 V/m.

    a. Does the amount of charge on the capacitor plates increase, decrease, or stay the same when the dielectric is inserted? If it increases or decreases, by what factor?

    The amount of charge stays the same because charge is conserved and it is disconnected from the battery so it has no where to go.

    b. Does the potential difference between the capacitor plates increase, decrease, or stay the same when the dielectric is inserted? If it increases or decreases, by what factor?

    The potential difference decreases by a factor of 5. The plates are still the same distance apart, but the field is smaller.

23. The plates of an air-insulated capacitor are charged to ±100 nC, then left connected to the battery. When a dielectric is inserted between the plates, the charge on the plates increases to ±500 nC.

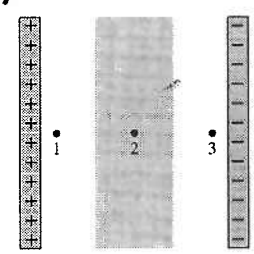
    a. Does the potential difference across the capacitor increase, decrease, or stay the same when the dielectric is inserted? If it increases, by what factor?

    The potential difference stays the same because it is fixed by the connection to the battery.

    b. Does the electric field strength inside the capacitor increase, decrease, or stay the same when the dielectric is inserted? If it increases, by what factor?

    The electric field strength stays the same because it is fixed by the connection to the battery also. $E = V/d$

24. The gap between two capacitor plates is *partially* filled with a dielectric. Rank in order, from largest to smallest, the electric field strengths E_1, E_2, and E_3 at points 1, 2, and 3.



Order: $E_1 = E_3 > E_2$

Explanation:

The field is reduced in the dielectric due to the induced charge separation in the dielectric.

31 Current and Resistance

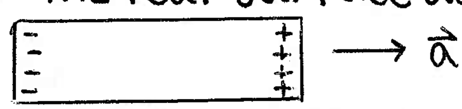
31.1 The Electron Current

1. A lightbulb is connected with wires to a battery, and the bulb is glowing. Are simple observations and measurements you can make on this circuit able to distinguish a current composed of positive charge carriers from a current composed of negative charge carriers? If so, describe how you can tell which it is. If not, why not?

No, either flow could explain the observations we can make. A flow of positive charges in one direction through the circuit would "look the same" as a flow of negative charges in the opposite direction in terms of the opposite we can make.

2. Describe experimental evidence to support the claim that the charge carriers in metals are electrons. Use both pictures and words.

In 1916, Tolman and Steward caused a metal rod to accelerate very quickly and found a negative charge on the rear surface. This is an indication that the charge carriers are negative and they were thrown to the rear surface as the rod was accelerated.



3. Are the charge carriers always electrons? If so, why is this the case? If not, describe a situation in which a current is due to some other charge carrier. No.

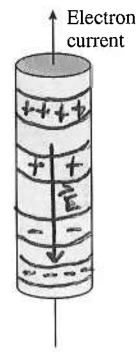
Electrons are the charge carriers in metals. But other conductors, like ionic solutions or semiconductors have charge carriers that are not electrons.

31.2 Creating a Current

4. The electron drift speed in a wire is exceedingly slow—typically only a fraction of a millimeter per second. Yet when you turn on a light switch, a light bulb several meters away seems to come on instantly. Explain how to resolve this apparent paradox.

Because the wire between the switch and the bulb is already full of electrons, a flow of electrons from the switch into the wire immediately causes electrons to flow from the other end of the wire into the bulb.

5. The figure shows a segment of a current-carrying metal wire.



a. Is there an electric field inside the wire? If so, draw and label an arrow on the figure to show its direction. If not, why not?

See figure.

b. If there is an electric field, draw on the figure a possible arrangement of charges that could be the source charges causing the field. See figure.

6. a. If the electrons in a current-carrying wire collide with the positive ions *more* frequently, does their drift speed increase or decrease? Explain.

If the collisions occur more frequently that means that the mean time between collisions, τ, decreases. Then the drift speed also decreases. $v_d = \frac{e\tau}{m}E$

b. Does an increase in the collision frequency make the wire a better conductor or a worse conductor? Explain.

A worse conductor. $i = nAv_d$
A smaller drift speed means a smaller current which means fewer electrons going past each second.

c. Would you expect a metal to be a better conductor at high temperature or at low temperature? Explain.

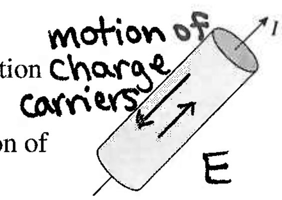
Metals are better conductors at low temperatures because there are more collisions at higher temperatures due to increased thermal motion.

31.3 Current and Current Density

7. What is the difference between current and current density?

Current is the rate at which charge moves through a wire. Current density, J, is the current per square meter of cross section. $J = \frac{I}{A}$

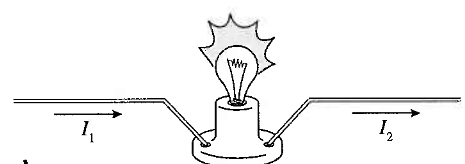
8. The figure shows a segment of a current-carrying metal wire.



a. Draw an arrow on the figure, using a **black** pen or pencil, to show the direction of motion of the charge carriers.

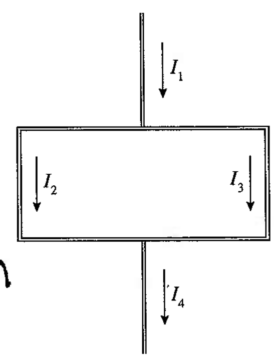
b. Draw an arrow on the figure, using a **red** pen or pencil, to show the direction of the electric field.

9. Is I_2 greater than, less than, or equal to I_1? Explain.



$I_2 = I_1$ The law of conservation of current states that the current is the same at all points in a current-carrying wire. The current in, I_1, equals the current out, I_2 of the bulb.

10. All wires in this figure are made of the same material and have the same diameter. Rank in order, from largest to smallest, the currents I_1 to I_4.

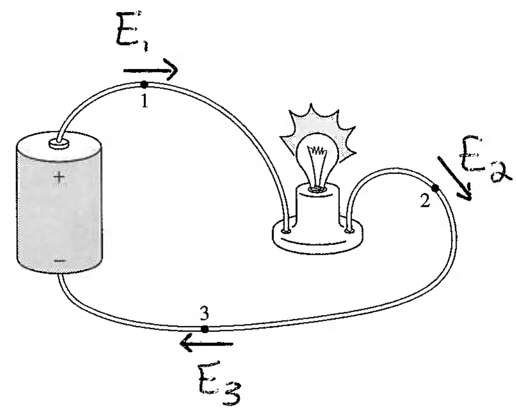


Order: $I_1 = I_4 > I_2 = I_3$

Explanation:

Conservation of current insures that the current into the top junction point, I_1, must equal the current out of that point $I_2 + I_3$. Similarly at the bottom junction point $I_2 + I_3 = I_4$.

11. A lightbulb is connected to a battery with 1-mm-diameter wires. The bulb is glowing.



a. Draw arrows at points 1, 2, and 3 to show the direction of the electric field at those points. (The points are *inside* the wire.)

b. Rank in order, from largest to smallest, the field strengths E_1, E_2, and E_3.

Order: $E_1 = E_2 = E_3$

Explanation:

The current is the same at every point in the wire and $I \propto E$.

12. A wire carries a 4 A current. What is the current in a second wire that delivers twice as much charge in half the time?

$I_1 = \frac{\Delta Q_1}{\Delta t_1} = 4A$; $I_2 = \frac{2\Delta Q_1}{\frac{1}{2}\Delta t_1} = 4\frac{\Delta Q_1}{\Delta t_1} = 4(4A) = 16A$

13. The current density in a wire is 1000 A/m^2. What will the current density be if the current is doubled and the wire's diameter is halved?

$J = I/A$ If the diameter is halved, the area is reduced to 1/4. So $J' = 2I/A/4 = 8J$.
The current density will be 8000 A/m^2.

31.4 Conductivity and Resistivity

14. Metal 1 and metal 2 are each formed into 1-mm-diameter wires. The electric field needed to cause a 1 A current in metal 1 is larger than the electric field needed to cause a 1 A current in metal 2. Which metal has the larger conductivity? Explain.

Metal 2.

$E_1 = \frac{I}{\sigma_1 A} \quad E_2 = \frac{I}{\sigma_2 A} \quad \frac{E_1}{E_2} = \frac{\sigma_2}{\sigma_1}$

15. If a metal is heated, does its conductivity increase, decrease, or stay the same? Explain.

As the temperature increases the conductivity decreases. As the temperature increases, the thermal vibrations of the lattice atoms increase so the collisions become more frequent which decreases τ, the mean time of collisions, and thereby decreases σ.

16. Wire 1 and wire 2 are made from the same metal. Wire 1 has twice the diameter and half the electric field of wire 2. What is the ratio I_1/I_2?

$I_1 = E_1 \sigma A_1 = E_1 \sigma (\pi r_1^2) \quad I_2 = E_2 \sigma (\pi r_2^2); E_1 = \frac{E_2}{2}$

$I_1 = \left(\frac{E_2}{2}\right) \sigma \pi (2r_2)^2 \quad r_1 = 2r_2$

$I_1 = 2(E_2 \sigma \pi r_2^2) = 2I_2 \quad \frac{I_1}{I_2} = 2$

17. Wire 1 and wire 2 are made from the same metal. Wire 2 has a larger diameter than wire 1. The electric field strengths E_1 and E_2 are equal.

a. Compare the values of the two current densities. Is J_1 greater than, less than, or equal to J_2? Explain.

$J_1 = J_2$

Same metal means $\sigma_1 = \sigma_2$

also $E_1 = E_2$ and $J = \sigma E$

b. Compare the values of the currents I_1 and I_2.

$I_1 = \sigma_1 E_1 A_1 \quad \sigma_1 = \sigma_2$

$I_2 = \sigma_2 E_2 A_2 \quad E_1 = E_2$

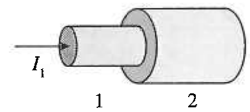
$A_2 > A_1 \Rightarrow I_2 > I_1$

c. Compare the values of the electron drift speeds $(v_d)_1$ and $(v_d)_2$.

$J = n e v_d$ since $J_1 = J_2$

then $(v_d)_1 = (v_d)_2$

18. A wire consists of two segments of different diameters but made from the same metal. The current in segment 1 is I_1.



a. Compare the values of the currents in the two segments. Is I_2 greater than, less than, or equal to I_1? Explain.

$I_1 = I_2$ Due to conservation of current, the current everywhere in the wire is the same. The number of charges passing per unit must be the same in wires 1 and 2.

b. Compare the values of the current densities J_1 and J_2.

$J_1 > J_2$

$J_1 = \frac{I}{A_1}$ $J_2 = \frac{I}{A_2}$ $A_1 < A_2$

c. Compare the strengths of the electric fields E_1 and E_2 in the two segments.

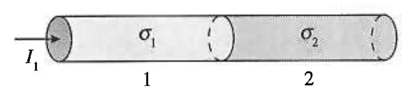
$\sigma_1 = \sigma_2 = \sigma$

$E_1 = \frac{J_1}{\sigma}$ $E_2 = \frac{J_2}{\sigma}$ $E_1 > E_2$

d. Compare the values of the electron drift speeds $(v_d)_1$ and $(v_d)_2$.

$J = neV_d$ since $J_1 > J_2$ then $(V_d)_1 > (V_d)_2$

19. A wire consists of two equal-diameter segments. Their conductivities and electron densities differ, with $\sigma_2 > \sigma_1$ and $n_2 > n_1$. The current in segment 1 is I_1.



a. Compare the values of the currents in the two segments. Is I_2 greater than, less than, or equal to I_1? Explain.

Due to conservation of current I_1 must be equal to I_2.

b. Compare the strengths of the current densities J_1 and J_2.

$J_1 = \frac{I_1}{A_1}$; $J_2 = \frac{I_2}{A_2}$ Since $I_1 = I_2$ and $A_1 = A_2$, then $J_1 = J_2$.

c. Compare the strengths of the electric fields E_1 and E_2 in the two segments.

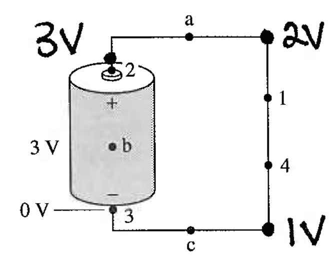
$E_1 = \frac{J_1}{\sigma_1}$; $E_2 = \frac{J_2}{\sigma_2}$ $J_1 = J_2$ and $\sigma_2 > \sigma_1$ therefore $E_1 > E_2$.

d. Compare the values of the electron drift speeds $(v_d)_1$ and $(v_d)_2$.

$J = neV_d$ If $n_2 > n_1$, then $J_2 = J_1$ means $(V_d)_1 > (V_d)_2$.

31.5 Resistance and Ohm's Law

20. A continuous metal wire connects the two ends of a 3 V battery with a rectangular loop. The negative terminal of the battery has been chosen as the point where $V = 0$ V.



a. Locate and label the approximate points along the wire where $V = 3$ V, $V = 2$ V, and $V = 1$ V.

b. Points a and c are *inside* the wire. Point b is inside the battery. Does the electric field at a, b, and c point left, right, up, or down? Or is $\vec{E} = \vec{0}$?

$\vec{E}_a$ Right $\vec{E}_b$ Down $\vec{E}_c$ Left

c. In moving through the *wire* from point 2 to point 3, does the potential increase, decrease, or not change? If the potential changes, by how much does it change?

The potential decreases and the decrease is linear with the distance traveled around the wire. $\Delta V_{23} = -3V$

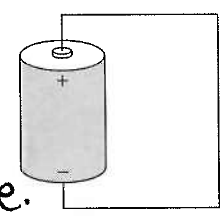
d. In moving through the *battery* from point 2 to point 3, does the potential increase, decrease, or not change? If the potential changes, by how much does it change?

The potential decreases by 3V.

e. In moving all the way around the loop in a clockwise direction, starting from point 2 and ending at point 2, is the net change in the potential positive, negative, or zero?

Zero. $\Delta V_{23} = -3V$ and $\Delta V_{32} = +3V$

21. a. Which direction—clockwise or counterclockwise—does an electron travel through the wire? Explain.



Counterclockwise. The electron will be repelled by the negative electrode of the battery and attracted to the positive electrode.

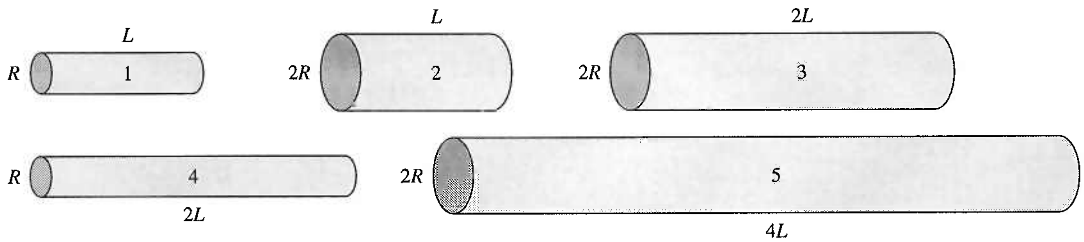
b. Does an electron's electric potential energy increase, decrease, or stay the same as it moves through the wire? Explain. Decreases.

A charge always moves in the direction of decreasing potential energy as it gains kinetic energy.

c. If you answered "decrease" in part b, where does the energy go? If you answered "increase" in part b, where does the energy come from?

The energy goes into the increase in the kinetic energy of the electron and therefore into an increase in its speed.

22. The wires below are all made of the same material. Rank in order, from largest to smallest, the resistances R_1 to R_5 of these wires.



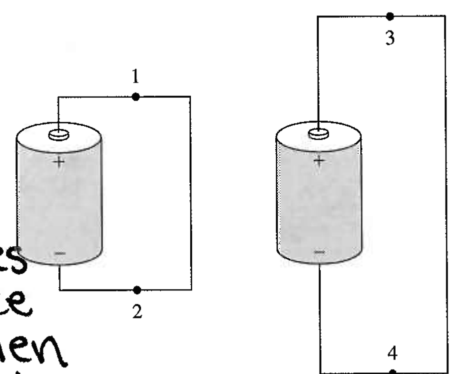
Order: $R_4 > R_1 = R_5 > R_3 > R_2$

Explanation:

$R = \frac{\rho L}{A}$ $R_1 = \frac{\rho L}{\pi R^2}$ $R_2 = \frac{\rho L}{(\pi)(2R)^2} = \frac{1}{4} R_1$ $R_3 = \frac{\rho (2L)}{(\pi)(2R)^2} = \frac{1}{2} R_1$

$R_4 = \frac{\rho(2L)}{\pi R^2} = 2R_1$ $R_5 = \frac{\rho(4L)}{(\pi)(2R)^2} = R_1$

23. The two circuits use identical batteries and wires of equal diameters. Rank in order, from largest to smallest, the currents I_1, I_2, I_3, and I_4 at points 1 to 4.

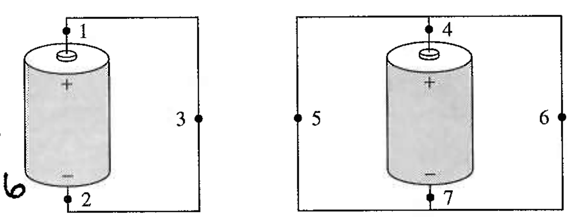


Order: $I_1 = I_2 > I_3 = I_4$

Explanation:

Conservation of current requires $I_1 = I_2$ and $I_3 = I_4$. However, since the wire on the right is longer, then its resistance is greater ($R = \frac{\rho L}{A}$) and therefore the current is smaller ($I = V/R$).

24. The two circuits use identical batteries and wires of equal diameters. Rank in order, from largest to smallest, the currents I_1 to I_7 at points 1 to 7.

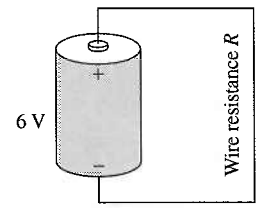


Order: $I_4 = I_7 > I_1 = I_2 = I_3 = I_5 = I_6$

Explanation:

Conservation of current requires $I_1 = I_2 = I_3$. The current in each wire is $I = \Delta V/R$. The wires are identical and have the same resistance. The batteries have the same potential difference. Therefore $I_5 = I_6 = I_1$. From conservation of current, the current at point 4 must go partially to point 5 and partially to point 6. Therefore $I_4 > I_5$.

25. A wire is connected to the terminals of a 6 V battery. What is the potential difference ΔV_{wire} between the ends of the wire, and what is the current I through the wire, if the wire has the following resistances:



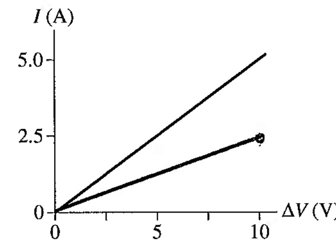
a. $R = 1\ \Omega$ $\Delta V_{wire} =$ 6V $I =$ 6A

b. $R = 2\ \Omega$ $\Delta V_{wire} =$ 6V $I =$ 3A

c. $R = 3\ \Omega$ $\Delta V_{wire} =$ 6V $I =$ 2A

d. $R = 6\ \Omega$ $\Delta V_{wire} =$ 6V $I =$ 1A

26. The graph shows the current-versus-potential-difference relationship for a resistor R.

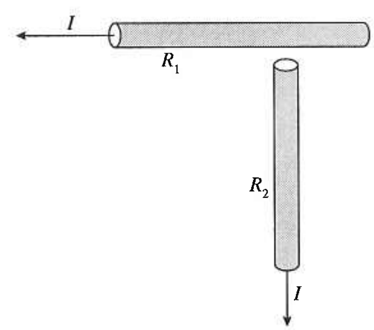


a. What is the numerical value of R?

$R = \frac{1}{\text{slope}} = \frac{10V}{5A} = 2\Omega$

b. Suppose the length of the resistor is doubled. On the figure, draw the current-versus-potential-difference graph for the longer resistor.

27. For resistors R_1 and R_2:



a. Which end (left, right, top, or bottom) is more positive?

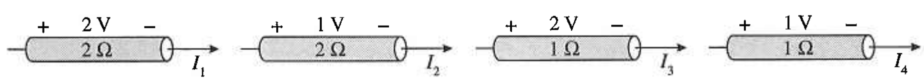
R_1 right R_2 top

b. In which direction (such as left to right or top to bottom) does the potential decrease?

R_1 right to left

R_2 top to bottom

28. Rank in order, from largest to smallest, the currents I_1 to I_4 through these four resistors.



Order: $I_3 > I_1 = I_4 > I_2$

Explanation:

$I = \frac{\Delta V}{R}$ $I_1 = \frac{2V}{2\Omega} = 1A$ $I_2 = \frac{1V}{2\Omega} = 0.5A$

$I_3 = \frac{2V}{1\Omega} = 2A$ $I_4 = \frac{1V}{1\Omega} = 1A$

29. Which, if any, of these statements are true? (More than one may be true.)
   i. A battery supplies the energy to a circuit.
   ii. A battery is a source of potential difference. The potential difference between the terminals of the battery is always the same.
   iii. A battery is a source of current. The current leaving the battery is always the same.

   Explain your choice or choices.

i. True. The chemical reactions in the electrolytes separate the positive and negative charges. This creates a potential difference. The charges flowing in the circuit have energy due to this potential difference.

ii. It is true that a battery is a source of potential difference. But the potential difference is always the same ONLY for an ideal battery. In a real battery there are energy losses so the terminal voltage is not always the same.

iii. False. The current leaving the battery depends upon the resistance in the circuit.

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Chapter 30 Exercises and Problems

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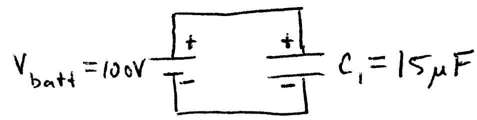
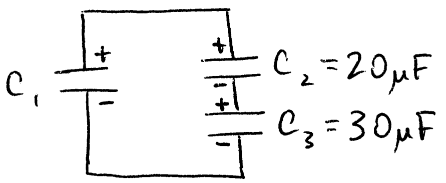
At first, the circuit is like this:



After waiting a long time, the capacitor C_1 charges up so that

$$Q_1^i = V_{batt} C_1.$$

Now flick the switch the other way. The circuit is now this:



C_1 will start with charge Q_1^i, but will lose charge until it gets to some charge Q_1. C_2 and C_3 will gain charges Q_2 and Q_3. Now apply Kirchhoff's loop rule. For a positively charged particle, V increases as you go towards the positive plate of a capacitor and decreases as you move towards the negative plate of a capacitor. Applying Kirchhoff's loop rule clockwise, starting below C_1, yields

$$+\frac{Q_1}{C_1} - \frac{Q_2}{C_2} - \frac{Q_3}{C_3} = 0.$$

Charge is conserved, so the amount of positive charge on C_1 and C_2 must equal the initial charge Q_1^i. The same thing happens with C_1 and C_3's negative charges, so we get

$$\left.\begin{aligned} Q_1 + Q_3 &= Q_1^i \\ Q_1 + Q_2 &= Q_1^i \end{aligned}\right\} \Rightarrow Q_2 = Q_3.$$

Now substitute these equations into our loop rule equation.

$$0 = \frac{Q_1}{C_1} - Q_2\left(\frac{1}{C_2} + \frac{1}{C_3}\right) = \frac{Q_1}{C_1} - (Q_1^i - Q_1)\left(\frac{1}{C_2} + \frac{1}{C_3}\right).$$

$$Q_1 = Q_1^i\left(\frac{1}{C_2} + \frac{1}{C_3}\right)\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = C_1 V_{batt}\left(\frac{1}{C_2} + \frac{1}{C_3}\right)\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

$$= (15\times10^{-6}F)(100V)\left(\frac{1}{20\times10^{-6}F} + \frac{1}{30\times10^{-6}F}\right)\left(\frac{1}{15\times10^{-6}F} + \frac{1}{20\times10^{-6}F} + \frac{1}{30\times10^{-6}F}\right)^{-1}$$

$$= \frac{1}{1200}\,C \approx 8.3\times10^{-4}\,C.$$

Now find Q_2:

$$Q_2 = Q_1^i - Q_1 = C_1 V_{batt} - Q_1 = (100\text{V})(15\times10^{-6}\text{F}) - \frac{1}{1200}\text{C}$$

$$= \frac{1}{1500}\text{C} \approx 6.7\times10^{-4}\text{C}.$$

We saw before that $Q_2 = Q_3$, so

$$Q_3 = \frac{1}{1500}\text{C} \approx 6.7\times10^{-4}\text{C}.$$

The potential differences across each capacitor are

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{\frac{1}{1200}\text{C}}{15\times10^{-6}\text{F}} = 55.5\ldots\text{V} \approx 56\text{V}$$

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{\frac{1}{1500}\text{C}}{20\times10^{-6}\text{F}} = 33.3\ldots\text{V} \approx 33\text{V}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{\frac{1}{1500}\text{C}}{30\times10^{-6}\text{F}} = 22.2\ldots\text{V} \approx 22\text{V}$$

Notice that $\Delta V_1 - \Delta V_2 - \Delta V_3 = 0$, as required by Kirchhoff's loop rule. Good.

Also notice that if you have the algebraic expressions for what you want to solve in a problem, you can take limits to check whether your answer is right or not. For instance, consider the equation we derived for Q_1. If we take the limit where $C_1 \gg C_2$ and $C_1 \gg C_3$, then that means C_1 can store much more charge than C_2 and C_3, given the same potential difference between each of them. If C_1 is bigger than the other capacitances by a large margin, then C_1 should not lose much charge when it is connected to C_2 and C_3. Indeed, when we take the limit that $C_1 \to \infty$, we find that

$$Q_1 \longrightarrow Q_1^i \left(\frac{1}{C_2} + \frac{1}{C_3}\right)\left(\frac{1}{\infty} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1} = Q_1^i.$$

That is, Q_1 doesn't change from its initial value after being charged by the battery.

Chapter 31 Exercises and Problems

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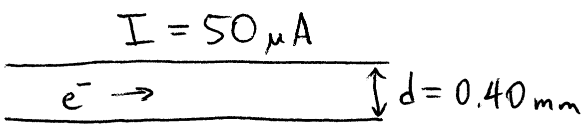


a) The current tells us how much charge per second hits the screen, but in Coulombs/s. We just need to convert this to the number of electrons per second.

$$\frac{\Delta N}{\Delta t} = \frac{I}{e} = \frac{50 \times 10^{-6}\,A}{1.60 \times 10^{-19}\,C} = 3.125 \times 10^{14}\,\frac{\text{electrons}}{s}$$

$$\approx 3.1 \times 10^{14}\,\frac{\text{electrons}}{s}.$$

b) $$J = \frac{I}{A} = \frac{I}{\pi(\frac{1}{2}d)^2} = \frac{50 \times 10^{-6}\,A}{\pi(\frac{1}{4})(0.4 \times 10^{-3}\,m)^2} = 397.887\ldots\,\frac{A}{m^2}$$

$$\approx 4.0 \times 10^2\,\frac{A}{m^2}.$$

c) There are at least two ways to approach this. You can use kinematics or you can use energy. Let's try energy. Recall for a constant electric field,

$$-\Delta U = \vec{F} \cdot \Delta\vec{x} = \Delta E_k$$

$$= q_e E \Delta x = \frac{1}{2} m_e v^2$$

$$\Rightarrow E = \frac{\frac{1}{2} m_e v^2}{q_e \Delta x} = \frac{\frac{1}{2}(9.11 \times 10^{-31}\,kg)(4 \times 10^7\,\frac{m}{s})^2}{(1.60 \times 10^{-19}\,C)(5 \times 10^{-3}\,m)} = 9.11 \times 10^5\,\frac{V}{m}$$

$$\approx 9.1 \times 10^5\,\frac{V}{m}.$$

Notice we neglected the sign of q_e since we only need to find electric field strength, which is not a vector.

d) Each electron has energy

$$E_k = \frac{1}{2} m_e v^2.$$

ΔN of these electrons have energy

$$\Delta N E_k = \frac{1}{2} \Delta N m_e v^2.$$

If ΔN electrons strike the screen in time Δt, we get an average power of

$$P = \frac{\Delta N}{\Delta t} E_k = \left(\frac{\Delta N}{\Delta t}\right)\left(\tfrac{1}{2} m_e v^2\right) = \underbrace{\left(3.125\times10^{14}\ \tfrac{1}{s}\right)}_{\text{from part a)}}\left(\tfrac{1}{2}\right)\left(9.11\times10^{-31}\ kg\right)\left(4\times10^{7}\ \tfrac{m}{s}\right)^2$$

$$= 0.22775\ W \approx 0.23\ W.$$

#58

By conservation of charge and current, we know

$$I_{left} = I_{right}.$$

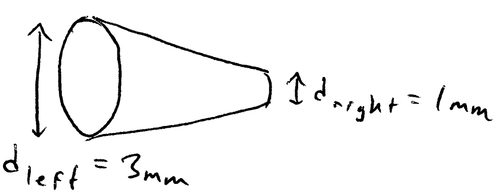
The current in a wire is given by

$$I = n_e e v_d A \quad \text{(equation 31.12 of the textbook).}$$

Assuming the wire is made of the same material through the entire thing, n_e shouldn't change. Thus, we have

$$I_{left} = n_e e v_{d_{left}} A_{left} = I_{right} = n_e e v_{d_{right}} A_{right}$$

$$v_{d_{left}} = \frac{v_{d_{right}} A_{right}}{A_{left}} = v_{d_{right}} \frac{(\pi)\left(\frac{1}{2} d_{right}\right)^2}{\pi\left(\frac{1}{2} d_{left}\right)^2}$$



$$= \frac{\left(0.5\times10^{-4}\ \frac{m}{s}\right)(\pi)\left(\frac{1}{4}\right)(0.001\ m)^2}{(\pi)\left(\frac{1}{4}\right)(0.003\ m)^2}$$

$$= 5.6\times10^{-6}\ \frac{m}{s}.$$

Notice that $v_{d_{left}} < v_{d_{right}}$. This is expected. Imagine a garden hose with water flowing through it. If you squeeze the nozzle end of the hose, water comes out faster. Constricting a wire is like squeezing a water hose, assuming I stays constant.