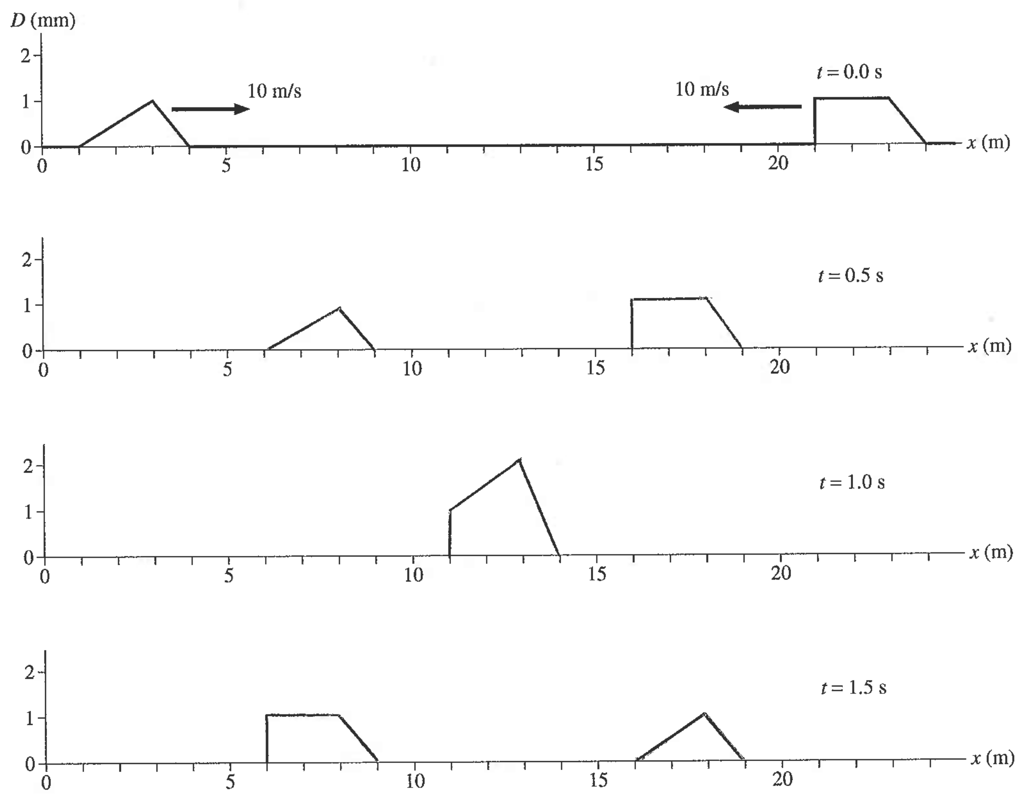
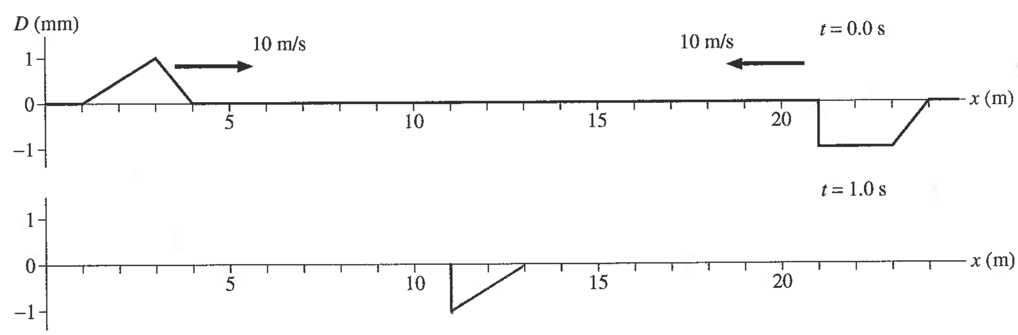
21 Superposition

21.1 The Principle of Superposition

1. Two pulses on a string, both traveling at 10 m/s, are approaching each other. Draw snapshot graphs of the string at the three times indicated.



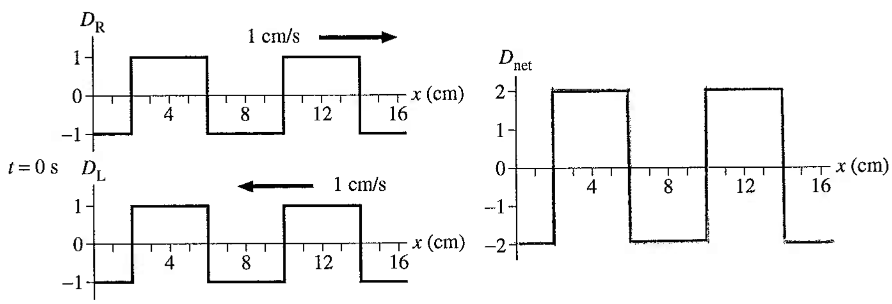
2. Two pulses on a string, both traveling at 10 m/s, are approaching each other. Draw a snapshot graph of the string at $t = 1$ s.

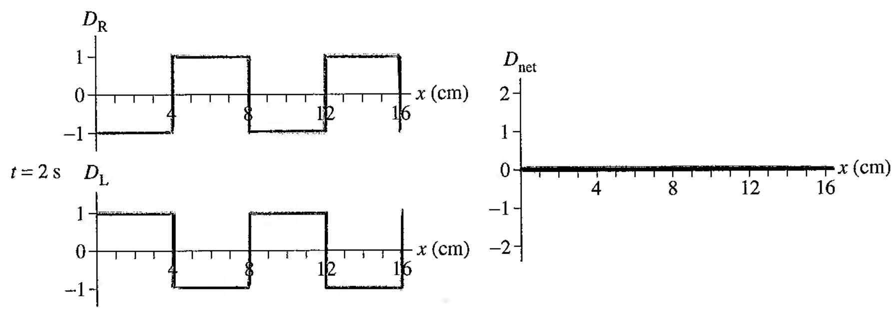


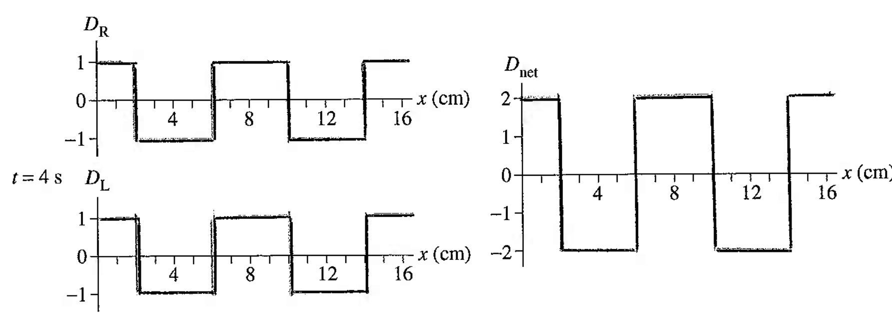
21.2 Standing Waves

21.3 Transverse Standing Waves

3. Two waves are traveling in opposite directions along a string. Each has a speed of 1 cm/s and an amplitude of 1 cm. The first set of graphs below shows each wave at $t = 0$ s.
   a. On the axes at the right, draw the superposition of these two waves at $t = 0$ s.
   b. On the axes at the left, draw each of the two displacements every 2 s until $t = 8$ s. The waves extend beyond the graph edges, so new pieces of the wave will move in.
   c. On the axes at the right, draw the superposition of the two waves at the same instant.







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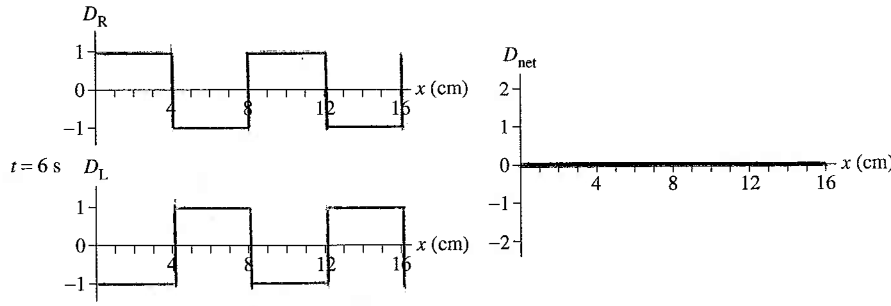
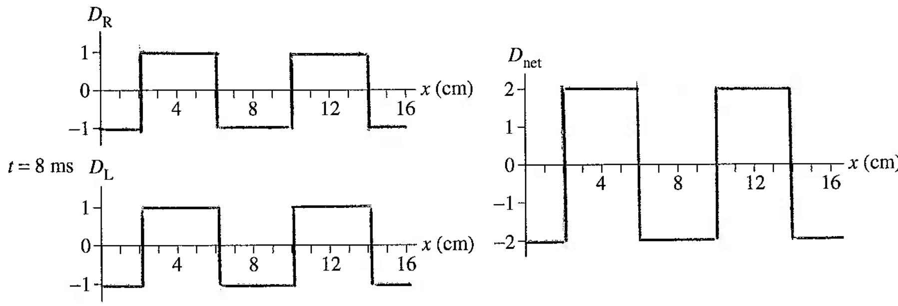
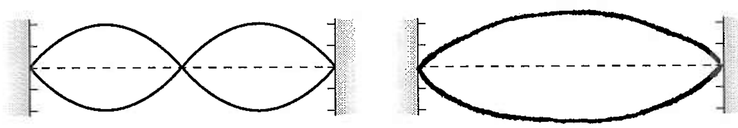




4. The figure shows a standing wave on a string.

a. Draw the standing wave if the tension is quadrupled while the frequency is held constant.

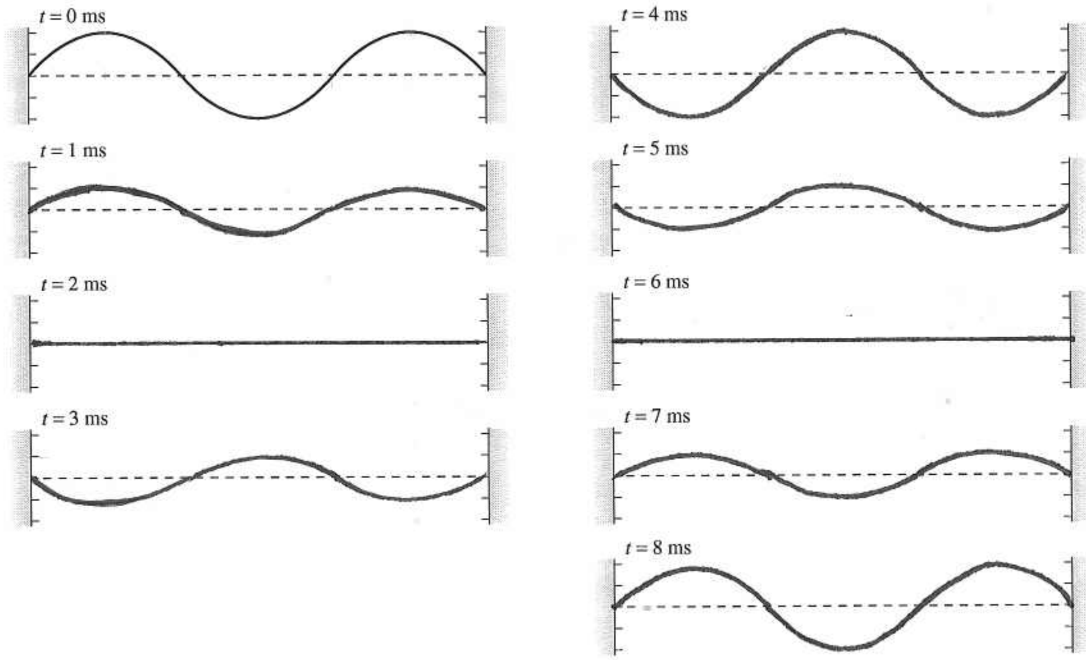


Original wave, tension T Tension $4T$

b. Suppose the tension is merely doubled while the frequency remains constant. Will there be a standing wave? If so, how many antinodes will it have? If not, why not?

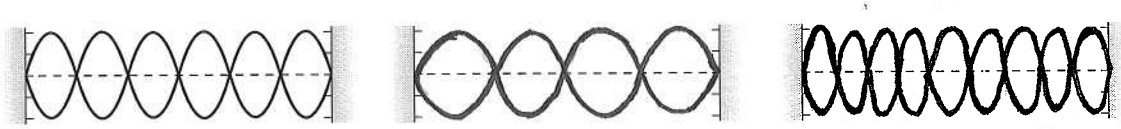
There will be no standing wave because $v \to \sqrt{2}v$ and so $\lambda \to \sqrt{2}\lambda$, but this will not have nodes at the boundaries.

5. This standing wave has a period of 8 ms. Draw snapshot graphs of the string every 1 ms from $t = 1$ ms to $t = 8$ ms. Think carefully about the proper amplitude at each instant.



6. The figure shows a standing wave on a string. It has frequency f.

a. Draw the standing wave if the frequency is changed to $\frac{2}{3}f$ and to $\frac{3}{2}f$.

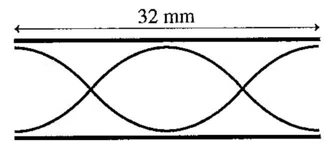


Original wave, frequency f Frequency $\frac{2}{3}f$ Frequency $\frac{3}{2}f$

b. Is there a standing wave if the frequency is changed to $\frac{1}{4}f$? If so, how many antinodes does it have? If not, why not?

There is no standing wave at $f/4$.
At f, $\lambda = \frac{2L}{6}$. Therefore at $f/4$, $\lambda = 4\left(\frac{2L}{6}\right) = \frac{4}{3}L$,
but this wavelength cannot meet the boundary conditions for a standing wave.

21.4 Standing Sound Waves and Musical Acoustics



7. The picture shows a displacement standing sound wave in a 32-mm-long tube of air that is open at both ends.
   a. Which mode (value of m) standing wave is this? 2
   b. Are the air molecules vibrating vertically or horizontally? Explain.

   Horizontally. Sound waves are longitudinal.

   c. At what distances from the left end of the tube do the molecules oscillate with maximum amplitude?

   0 mm, 16 mm, and 32 mm.

8. The purpose of this exercise is to visualize the motion of the air molecules for the standing wave of Exercise 7. On the next page are nine graphs, every one-eighth of a period from $t = 0$ to $t = T$. Each graph represents the displacements at that instant of time of the molecules in a 32-mm-long tube. Positive values are displacements to the right, negative values are displacements to the left.
   a. Consider nine air molecules that, in equilibrium, are 4 mm apart and lie along the axis of the tube. The top picture on the right shows these molecules in their equilibrium positions. The dotted lines down the page—spaced 4 mm apart—are reference lines showing the equilibrium positions. Read each graph carefully, then draw nine dots to show the positions of the nine air molecules at each instant of time. The first one, for $t = 0$, has already been done to illustrate the procedure.

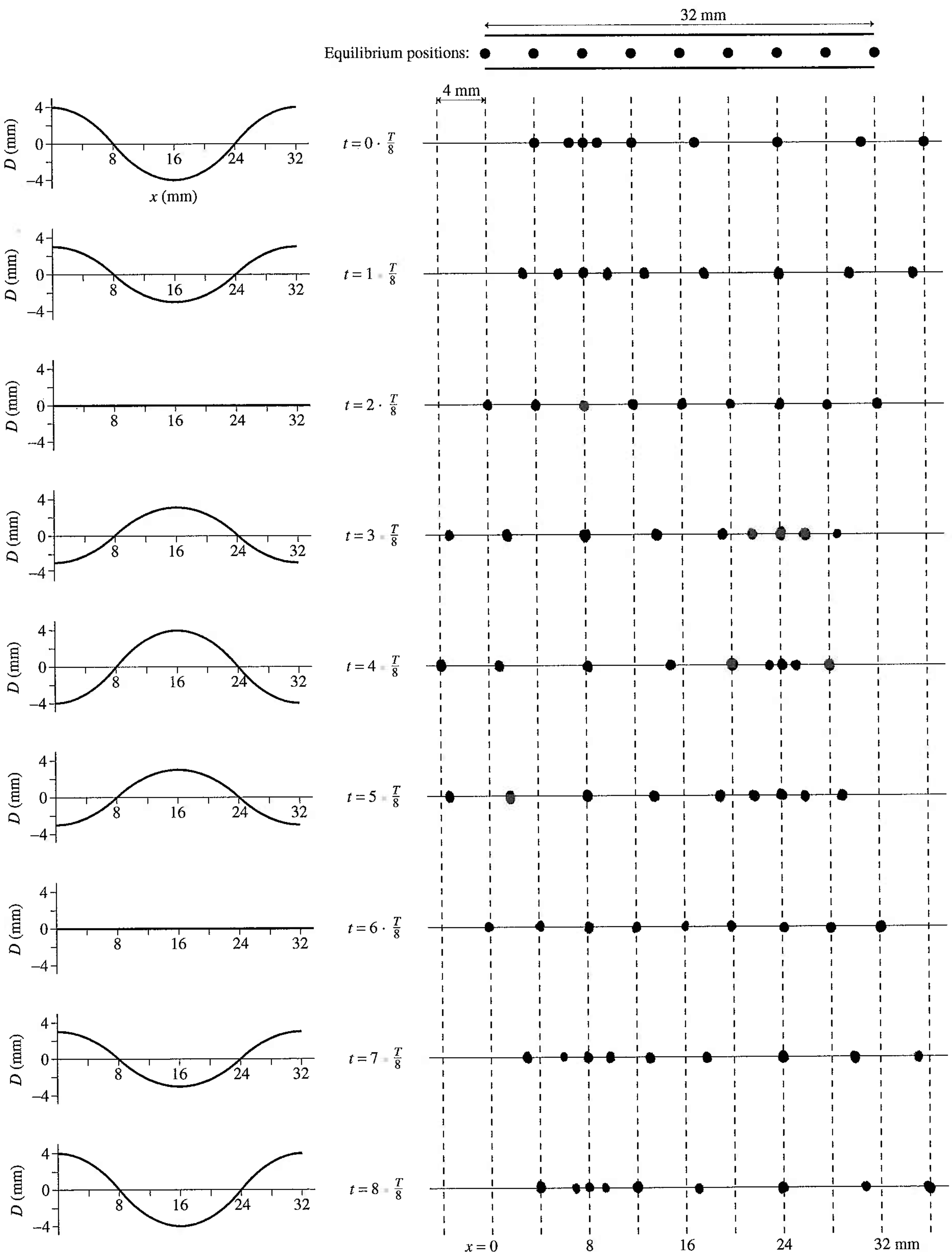
   Note: It's a good approximation to assume that the left dot moves in the pattern 4, 3, 0, −3, −4, −3, 0, 3, 4 mm; the second dot in the pattern 3, 2 , 0, −2, −3, −2, 0, 2, 3 mm; and so on.

   b. At what times does the air reach maximum compression, and where does it occur?

Max compression at time	Max compression at position
0	8 mm
T/2	24 mm
T	8 mm

   c. What is the relationship between the positions of maximum compression and the nodes of the standing wave?

   The points of maximum compression are nodes.



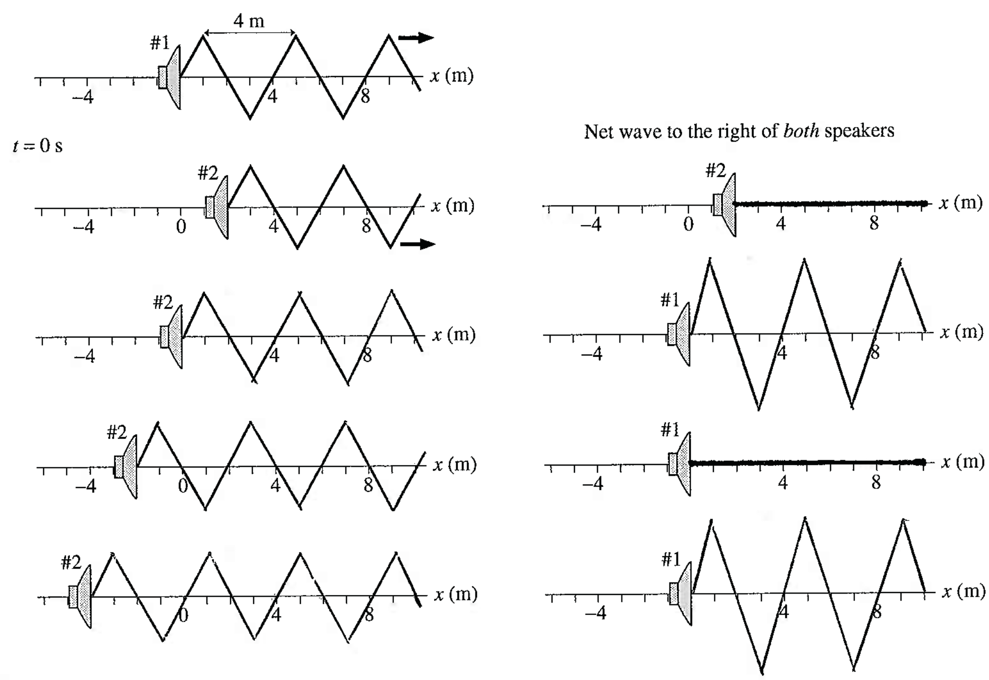
32 mm
Equilibrium positions:
4 mm
$t = 0 \cdot \frac{T}{8}$
$t = 1 \cdot \frac{T}{8}$
$t = 2 \cdot \frac{T}{8}$
$t = 3 \cdot \frac{T}{8}$
$t = 4 \cdot \frac{T}{8}$
$t = 5 \cdot \frac{T}{8}$
$t = 6 \cdot \frac{T}{8}$
$t = 7 \cdot \frac{T}{8}$
$t = 8 \cdot \frac{T}{8}$
D (mm)
x (mm)
$x = 0$
8
16
24
32 mm

21.5 Interference in One Dimension

21.6 The Mathematics of Interference

9. The figure shows a snapshot graph at $t = 0$ s of loudspeakers emitting triangular-shaped sound waves. Speaker 2 can be moved forward or backward along the axis. Both speakers vibrate in phase at the same frequency. The second speaker is drawn below the first, so that the figure is clear, but you want to think of the two waves as overlapped as they travel along the x-axis.

   a. On the left set of axes, draw the $t = 0$ s snapshot graph of the second wave if speaker 2 is placed at each of the positions shown. The first graph, with $x_{speaker} = 2$ m, is already drawn.



   b. On the right set of axes, draw the superposition $D_{net} = D_1 + D_2$ of the waves from the two speakers. D_{net} exists only to the right of *both* speakers. It is the net wave traveling to the right.

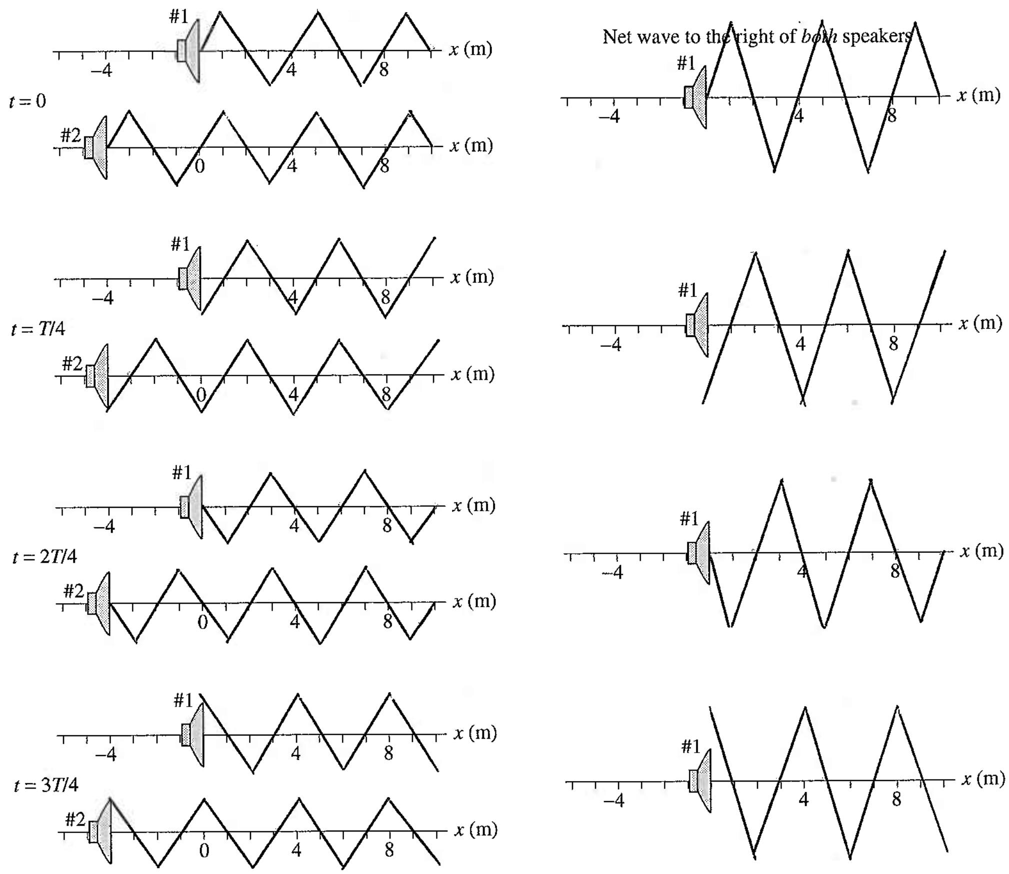
   c. What separations between the speakers give constructive interference? 0, 4 mm

   d. What are the $\Delta x/\lambda$ ratios at the points of constructive interference? 0, 1

   e. What separations between the speakers give destructive interference? -2m, +2m

   f. What are the $\Delta x/\lambda$ ratios at the points of destructive interference? $\pm 1/2$

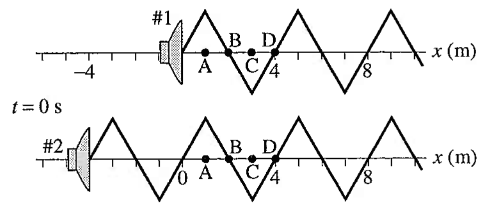
10. Consider the two loudspeakers of Exercise 9.
    a. Copy the speaker 1 and 2 graphs from Exercise 9 onto the first set of axes below for the situation in which speaker 2 is 4 m behind speaker 1. Then draw their superposition on the axes at the right. This simply repeats your last set of graphs from Exercise 9.
    b. On the axes on the left, draw snapshot graphs of the two waves at times $t = \frac{1}{4}T, \frac{2}{4}T$, and $\frac{3}{4}T$, where T is the wave's period.
    c. On the right axes, draw the superposition of the two waves.



d. Is the net wave a traveling wave or a standing wave? Use your *observations* to explain.

It is a traveling wave.
The wave does not maintain nodes.

11. Two loudspeakers are shown at $t = 0$ s. Speaker 2 is 4 m behind speaker 1.



a. What is the wavelength λ? 4m

b. Is the interference constructive or destructive?

Constructive

c. What is the phase constant ϕ_{10} for wave 1? 0

What is the phase constant ϕ_{20} for wave 2? 2π

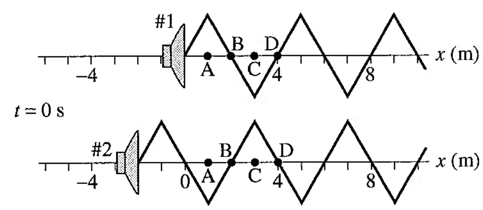
d. At points A, B, C, and D on the x-axis, what are:

- The distances x_1 and x_2 to the two speakers?
- The path length difference $\Delta x = x_2 - x_1$?
- The phases ϕ_1 and ϕ_2 of the two waves at the point (not the phase constant)?
- The phase difference $\Delta\phi = \phi_2 - \phi_1$?

Point A is already filled in to illustrate.

	x_1	x_2	Δx	ϕ_1	ϕ_2	$\Delta\phi$
Point A	1 m	5 m	4 m	0.5π rad	2.5π rad	2π rad
Point B	2m	6m	4m	π rad	3π rad	2π rad
Point C	3m	7m	4m	1.5π rad	3.5π rad	2π rad
Point D	4m	8m	4m	2π rad	4π rad	2π rad

e. Now speaker 2 is placed only 2 m behind speaker 1. Is the interference constructive or destructive?



Destructive

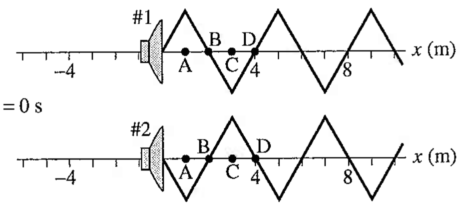
f. Repeat step c for the points A, B, C, and D.

	x_1	x_2	Δx	ϕ_1	ϕ_2	$\Delta\phi$
Point A	1 m	3 m	2 m	0.5π rad	1.5π rad	π rad
Point B	2m	4m	2m	π rad	2π rad	π rad
Point C	3m	5m	2m	1.5π rad	2.5π rad	π rad
Point D	4m	6m	2m	2π rad	3π rad	π rad

g. When the interference is constructive, what is $\Delta x/\lambda$? 1 What is $\Delta\phi$? 2π

h. When the interference is destructive, what is $\Delta x/\lambda$? 1/2 What is $\Delta\phi$? π

12. Two speakers are placed side-by-side at $x = 0$ m. The waves are shown at $t = 0$ s.



a. Is the interference constructive or destructive?

b. What is the phase constant ϕ_{10} for wave 1? 0

What is the phase constant ϕ_{10} for wave 2? π

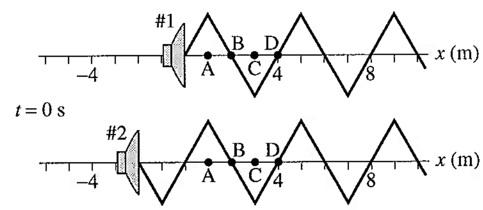
c. At points A, B, C, and D on the x-axis, what are:

- The distances x_1 and x_2 to the two speakers?
- The path length difference $\Delta x = x_2 - x_1$?
- The phases ϕ_1 and ϕ_2 of the two waves at the point (not the phase constant)?
- The phase difference $\Delta\phi = \phi_2 - \phi_1$?

	x_1	x_2	Δx	ϕ_1	ϕ_2	$\Delta\phi$
Point A	1 m	1 m	0	0.5π	1.5π	π
Point B	2 m	2 m	0	π	2π	π
Point C	3 m	3 m	0	1.5π	2.5π	π
Point D	4 m	4 m	0	2π	3π	π

d. Speaker 2 is moved back 2 m. Does this change its phase constant ϕ_0?

No



e. Is the interference constructive or destructive?

Constructive

f. Repeat step c for the points A, B, C, and D.

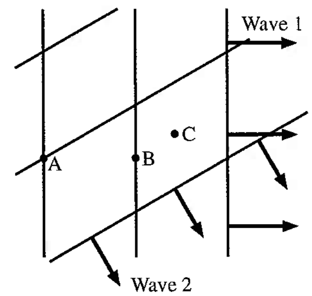
	x_1	x_2	Δx	ϕ_1	ϕ_2	$\Delta\phi$
Point A	1 m	3 m	2 m	0.5π	2.5π	2π
Point B	2 m	4 m	2 m	π	3π	2π
Point C	3 m	5 m	2 m	1.5π	3.5π	2π
Point D	4 m	6 m	2m	2π	4π	2π

13. Review your answers to the Exercises 11 and 12. Is it the separation path length difference Δx or the phase difference $\Delta\phi$ between the waves that determines whether the interference is constructive or destructive? Explain.

The phase difference determines whether the interference is constructive or destructive. If the phase difference is an even multiple of π, the interference will be constructive, if an odd multiple of π, then destructive. Path length differences can lead to phase differences, however.

21.7 Interference in Two and Three Dimensions

14. This is a snapshot graph of two plane waves passing through a region of space. Each has a 2 mm amplitude. At each lettered point, what are the displacements of each wave and the net displacement?

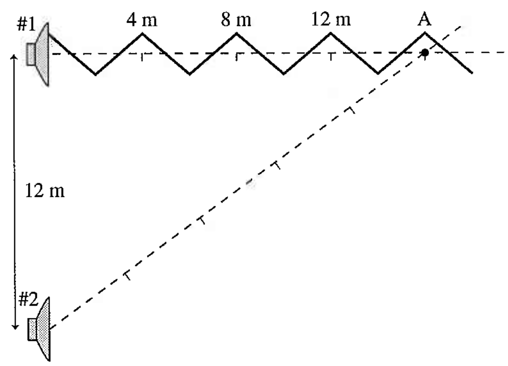


a. Point A: $D_1 =$ 2mm $D_2 =$ 2mm $D_{net} =$ 4mm

b. Point B: $D_1 =$ 2mm $D_2 =$ -2mm $D_{net} =$ 0

c. Point C: $D_1 =$ -2mm $D_2 =$ -2mm $D_{net} =$ -4mm

15. Speakers 1 and 2 are 12 m apart. Both emit identical triangular sound waves with $\lambda = 4$ m and $\phi_0 = \pi/2$ rad. Point A is $r_1 = 16$ m from speaker 1.



a. What is distance r_2 from speaker 2 to A?

$\sqrt{(12m)^2 + (16m)^2} = 20m$

b. Draw the wave from speaker 2 along the dashed line to just past point A.

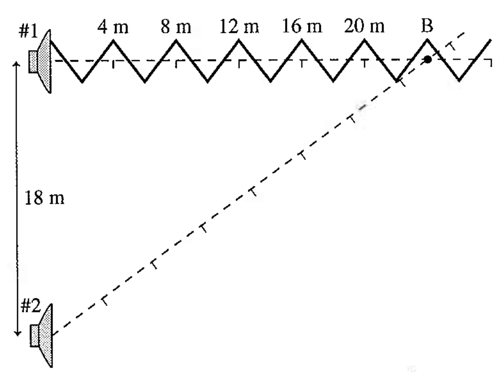
c. At A, is wave 1 a crest, trough, or zero? crest

At A, is wave 2 a crest, trough, or zero? crest

d. What is the path length difference $\Delta r = r_2 - r_1$? 4m What is the ratio $\Delta r/\lambda$? 1

e. Is the interference at point A constructive, destructive, or in between? Constructive

16. Speakers 1 and 2 are 18 m apart. Both emit identical triangular sound waves with $\lambda = 4$ m and $\phi_0 = \pi/2$ rad. Point B is $r_1 = 24$ m from speaker 1.



a. What is distance r_2 from speaker 2 to B?

$\sqrt{(18m)^2 + (24m)^2} = 30m$

b. Draw the wave from speaker 2 along the dashed line to just past point A.

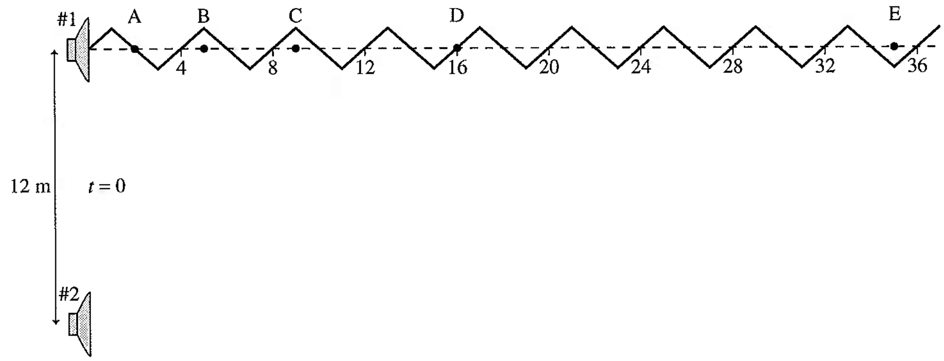
c. At B, is wave 1 a crest, trough, or zero? crest

At B, is wave 2 a crest, trough, or zero? trough

d. What is the path length difference $\Delta r = r_2 - r_1$? 6m What is the ratio $\Delta r/\lambda$? 1.5

e. Is the interference at point B constructive, destructive, or in between? destructive

17. Two speakers 12 m apart emit identical triangular sound waves with $\lambda = 4$ m and $\phi_0 = 0$ rad. The distances r_1 to points A, B, C, D, and E are shown in the table below.



a. For each point, fill in the table and determine whether the interference is constructive (C) or destructive (D).

Point	r_1	r_2	Δr	$\Delta r/\lambda$	C or D
A	2.2 m	12.2m	10m	2.5	D
B	5.0 m	13m	8m	2	C
C	9.0 m	15m	6m	1.5	D
D	16 m	20m	4m	1	C
E	35 m	37m	2m	0.5	D

b. Are there any points to the right of E, on the line straight out from speaker 1, for which the interference is either exactly constructive or exactly destructive? If so, where? If not, why not?

No, for all points to the right of E, the path difference will be less than 2m, but can never be 0m. Therefore neither exact constructive nor destructive interference is possible.

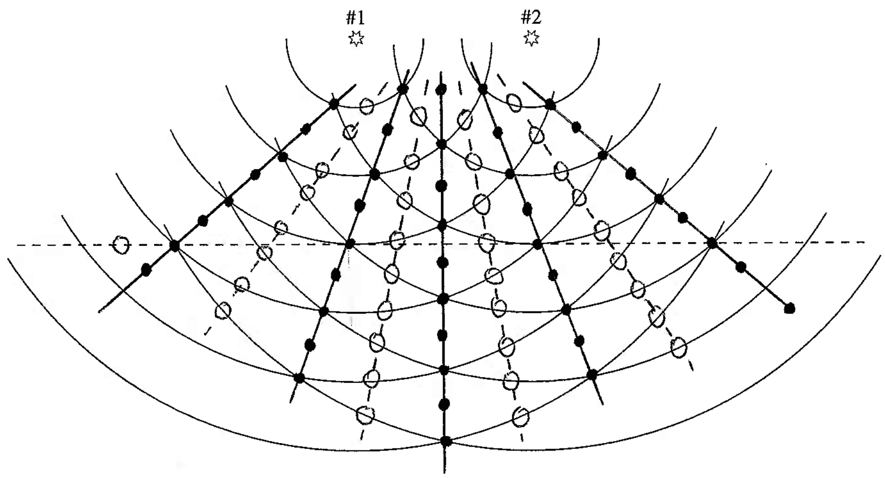
c. Suppose you start at speaker 1 and walk straight away from the speaker for 50 m. Describe what you will hear as you walk.

The sound will get louder and softer as you walk through points of maximum constructive and destructive interference.

18. The figure shows the wave-front pattern emitted by two loudspeakers.

   a. Draw a dot • at points where there is constructive interference. These will be points where two crests overlap *or* two troughs overlap.

   b. Draw an open circle ○ at points where there is destructive interference. These will be points where a crest overlaps a trough.



   c. Use a **black** line to draw each "ray" of constructive interference. Use a **red** line to draw each "ray" of destructive interference. ——— Black - - - Red

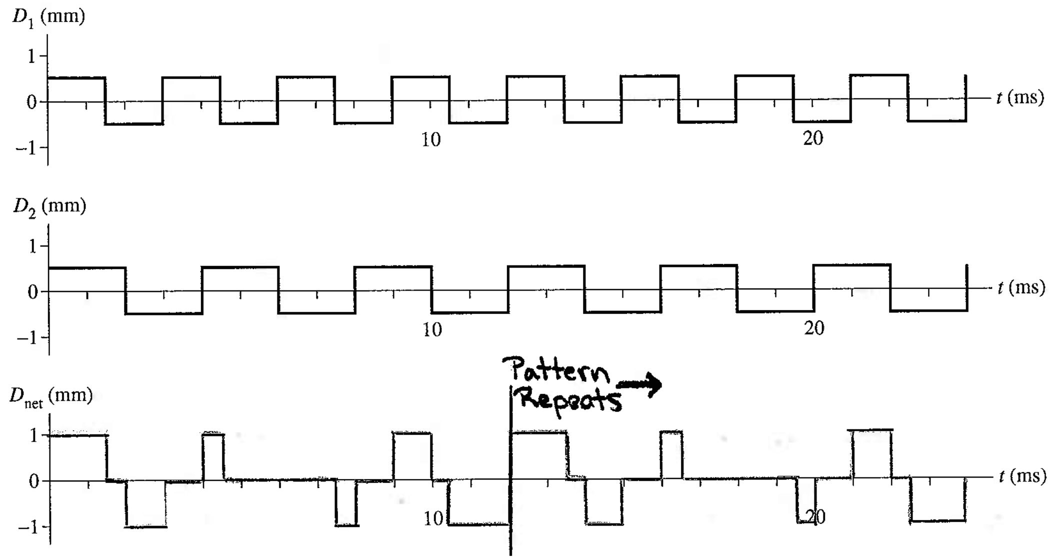
   d. Draw a graph on the axes above of the sound intensity you would hear if you walked along the horizontal dashed line. Use the same horizontal scale as the figure so that your graph lines up with the figure above it.

   e. Suppose the phase constant of speaker 2 is increased by π rad. Describe what will happen to the interference pattern.

The "rays" of constructive and destructive interference will exchange places with each other.

21.8 Beats

19. The two waves arrive simultaneously at a point in space from two different sources.



a. Period of wave 1? 3 ms Frequency of wave 1? 333 Hz

b. Period of wave 2? 4 ms Frequency of wave 2? 250 Hz

c. Draw the graph of the net wave at this point on the third set of axes. Be accurate, use a ruler!

d. Period of the net wave? 12 ms Frequency of the net wave? 83.3 Hz

e. Is the frequency of the superposition what you would expect as a beat frequency? Explain.

Yes, the superposition has a frequency equal to the difference in frequency of the two waves.