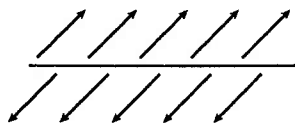


28 Gauss's Law

28.1 Symmetry

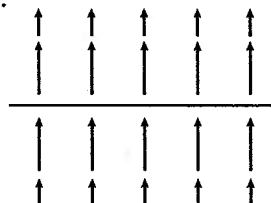
1. An infinite plane of charge is seen edge on. The sign of the charge is not given. Do the electric fields shown below have the same symmetry as the charge? If not, why not?

a.



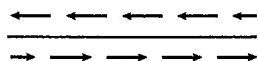
No. The field is not reflected in a plane coming out of the page.

b.



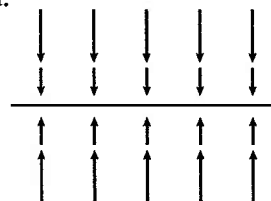
No. The field is not symmetric under a reflection in a plane coming out of the page.

c.



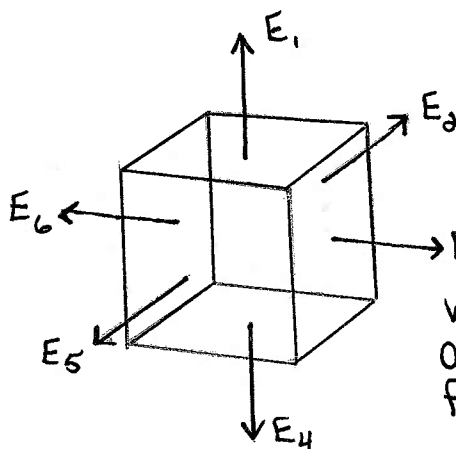
No. The field is not reflected up and down.

d.

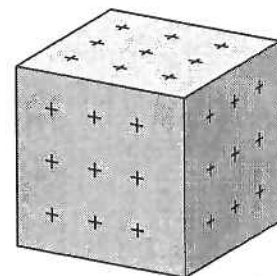


Yes. This field has the same symmetry as the charge.

2. Suppose you had a uniformly charged cube. Can you use symmetry alone to deduce the shape of the cube's electric field? If so, sketch and describe the field shape. If not, why not?



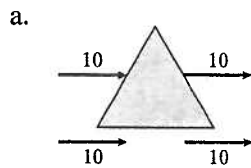
Choose a Gaussian surface in the shape of a cube. The electric field at each face will have the same magnitude and be perpendicular to that face.



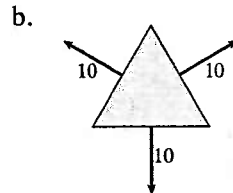
28.2 The Concept of Flux

3. The figures shown below are cross sections of three-dimensional closed surfaces. They have a flat top and bottom surface above and below the plane of the page. However, the electric field is everywhere parallel to the page, so there is no flux through the top or bottom surface. The electric field is uniform over each face of the surface. The field strength, in N/C, is shown.

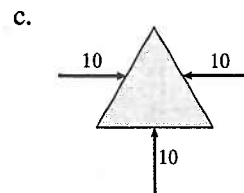
For each, does the surface enclose a net positive charge, a net negative charge, or no net charge?



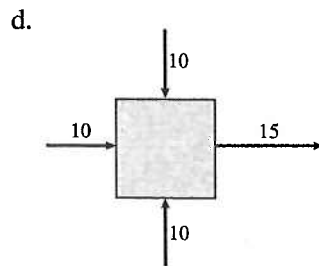
$$Q_{\text{net}} = 0$$



$$Q_{\text{net}} = +$$



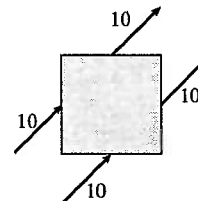
$$Q_{\text{net}} = -$$



$$Q_{\text{net}} = -$$



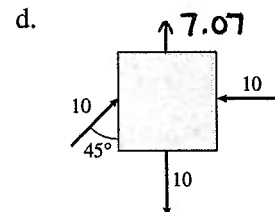
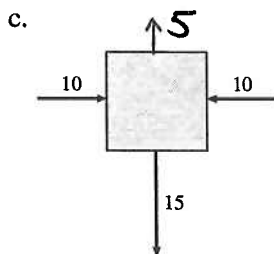
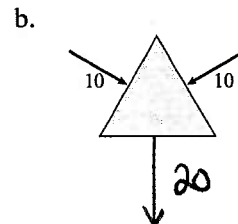
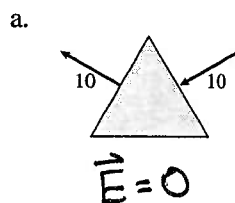
$$Q_{\text{net}} = +$$



$$Q_{\text{net}} = 0$$

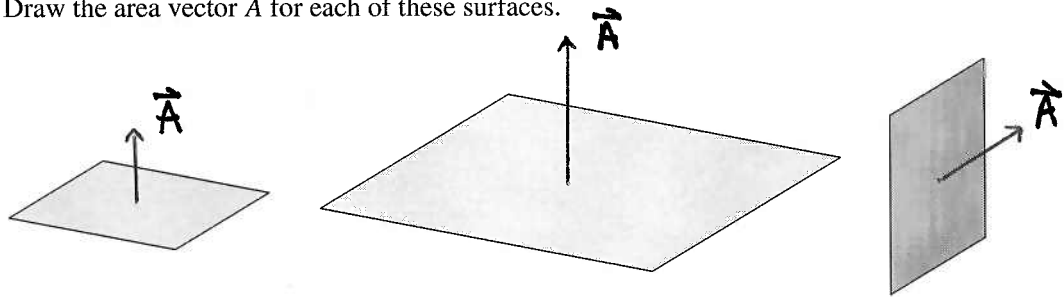
4. The figures shown below are cross sections of three-dimensional closed surfaces. They have a flat top and bottom surface above and below the plane of the page, but there is no flux through the top or bottom surface. The electric field is uniform over each face of the surface. The field strength, in N/C, is shown.

Each surface contains no net charge. Draw the missing electric field vector (or write $\vec{E} = \vec{0}$) in the proper direction. Write the field strength beside it.



28.3 Calculating Electric Flux

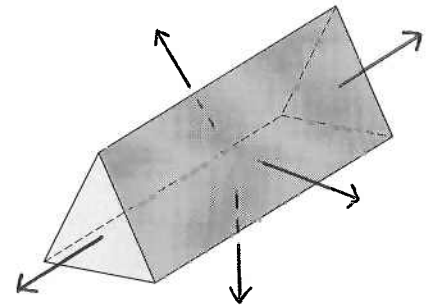
5. Draw the area vector \vec{A} for each of these surfaces.



6. How many area vectors are needed to characterize this closed surface?

5

Draw them.

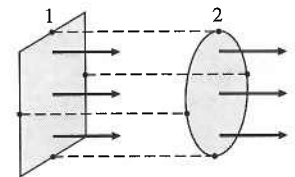


7. The diameter of the circle equals the edge length of the square. Is the electric flux Φ_1 through the square larger than, smaller than, or equal to the electric flux Φ_2 through the circle? Explain.

Because $A_1 > A_2$ and $E_1 = E_2$

$$\Phi_1 > \Phi_2$$

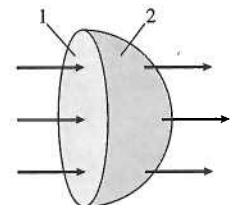
$$\Phi_1 = E_1 A_1, \Phi_2 = E_2 A_2$$



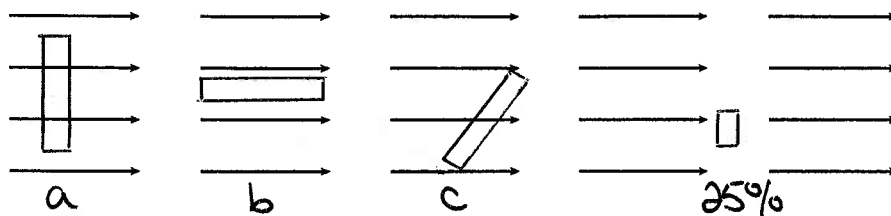
8. Is the electric flux Φ_1 through the circle larger than, smaller than, or equal to the electric flux Φ_2 through the hemisphere? Explain.

Any flux into surface 1 must come out of surface 2.

$$\Phi_1 = \Phi_2$$



9. A uniform electric field is shown below.



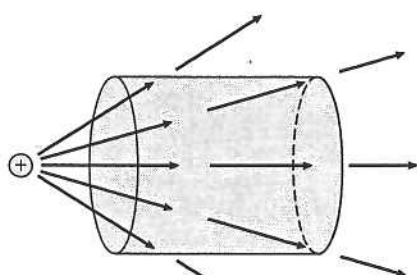
Draw and label an *edge view* of three square surfaces, all the *same size*, for which

- The flux is maximum.
- The flux is minimum.
- The flux has half the value of the flux through square 1.

Give the tilt angle of any squares not perpendicular to the field lines.

10. Is the net electric flux through each of the closed surfaces below positive (+), negative (-), or zero (0)?

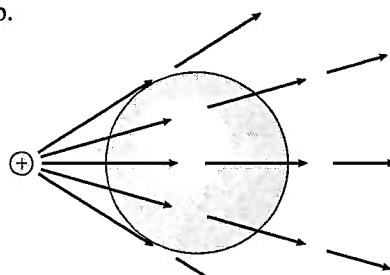
a.



$$\Phi = 0$$

All flux lines that flow in also flow out.

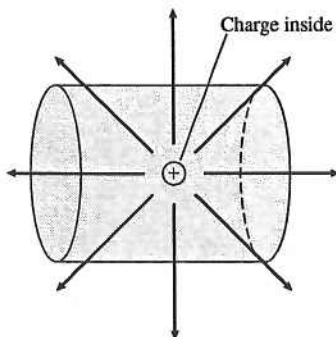
b.



$$\Phi = 0$$

All flux lines that flow into the surface also flow out.

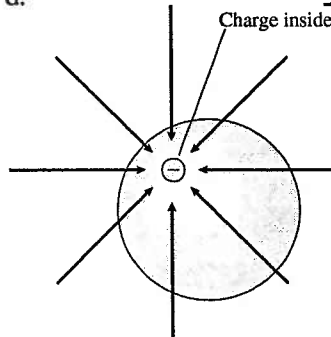
c.



$$\Phi = +$$

Flux only flows out.

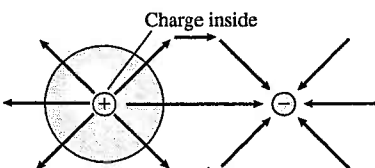
d.



$$\Phi = -$$

Flux only flows in.

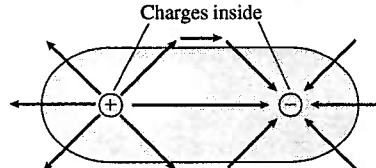
e.



$$\Phi = +$$

Flux only flows out.

f.



$$\Phi = 0$$

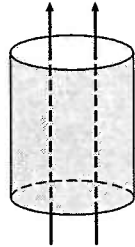
The amount of flux into the closed surface is equal to the amount of flux out.

28.4 Gauss's Law

28.5 Using Gauss's Law

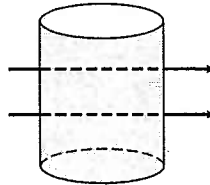
11. For each of the closed cylinders shown below, are the electric fluxes through the top, the wall, and the bottom positive (+), negative (-), or zero (0)? Is the net flux positive, negative, or zero?

a.



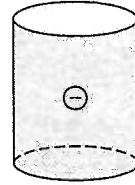
$$\begin{aligned}\Phi_{\text{top}} &= + \\ \Phi_{\text{wall}} &= 0 \\ \Phi_{\text{bot}} &= - \\ \Phi_{\text{net}} &= 0\end{aligned}$$

b.



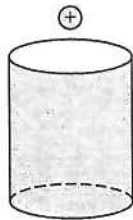
$$\begin{aligned}\Phi_{\text{top}} &= 0 \\ \Phi_{\text{wall}} &= 0 \\ \Phi_{\text{bot}} &= 0 \\ \Phi_{\text{net}} &= 0\end{aligned}$$

c.



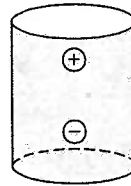
$$\begin{aligned}\Phi_{\text{top}} &= - \\ \Phi_{\text{wall}} &= - \\ \Phi_{\text{bot}} &= - \\ \Phi_{\text{net}} &= -\end{aligned}$$

d.



$$\begin{aligned}\Phi_{\text{top}} &= - \\ \Phi_{\text{wall}} &= + \\ \Phi_{\text{bot}} &= + \\ \Phi_{\text{net}} &= 0\end{aligned}$$

e.



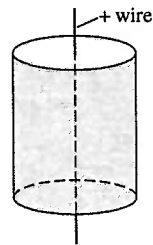
$$\begin{aligned}\Phi_{\text{top}} &= + \\ \Phi_{\text{wall}} &= 0 \\ \Phi_{\text{bot}} &= - \\ \Phi_{\text{net}} &= 0\end{aligned}$$

f.



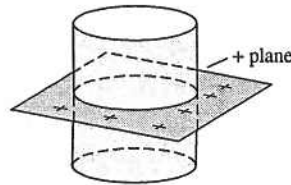
$$\begin{aligned}\Phi_{\text{top}} &= - \\ \Phi_{\text{wall}} &= 0 \\ \Phi_{\text{bot}} &= + \\ \Phi_{\text{net}} &= 0\end{aligned}$$

g.



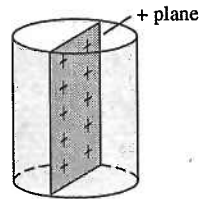
$$\begin{aligned}\Phi_{\text{top}} &= 0 \\ \Phi_{\text{wall}} &= + \\ \Phi_{\text{bot}} &= 0 \\ \Phi_{\text{net}} &= +\end{aligned}$$

h.



$$\begin{aligned}\Phi_{\text{top}} &= + \\ \Phi_{\text{wall}} &= 0 \\ \Phi_{\text{bot}} &= + \\ \Phi_{\text{net}} &= +\end{aligned}$$

i.



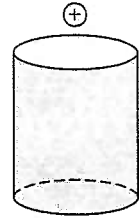
$$\begin{aligned}\Phi_{\text{top}} &= 0 \\ \Phi_{\text{wall}} &= + \\ \Phi_{\text{bot}} &= 0 \\ \Phi_{\text{net}} &= +\end{aligned}$$

12. For this closed cylinder, $\Phi_{\text{top}} = -15 \text{ Nm}^2/\text{C}$ and $\Phi_{\text{bot}} = 5 \text{ Nm}^2/\text{C}$.

What is Φ_{wall} ?

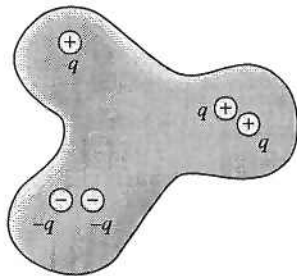
$$\Phi_{\text{wall}} = 10 \text{ Nm}^2/\text{C}$$

$$\Phi_{\text{top}} + \Phi_{\text{bot}} + \Phi_{\text{wall}} = 0$$



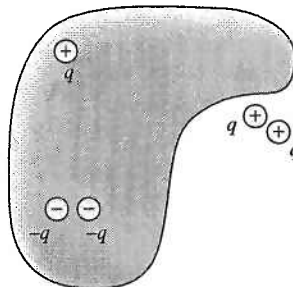
13. What is the electric flux through each of these surfaces? Give your answers as multiples of q/ϵ_0 .

a.



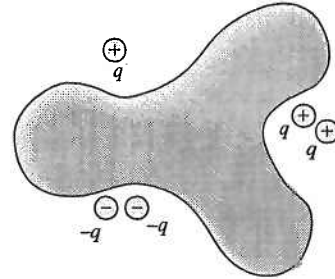
$$\Phi_e = +2/\epsilon_0$$

b.



$$\Phi_e = -2/\epsilon_0$$

c.



$$\Phi_e = 0$$

14. What is the electric flux through each of these surfaces?

Give your answers as multiples of q/ϵ_0 .

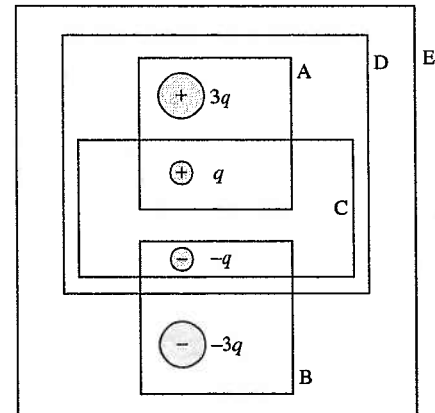
$$\Phi_A = +4q/\epsilon_0$$

$$\Phi_B = -4q/\epsilon_0$$

$$\Phi_C = 0$$

$$\Phi_D = +3q/\epsilon_0$$

$$\Phi_E = 0$$

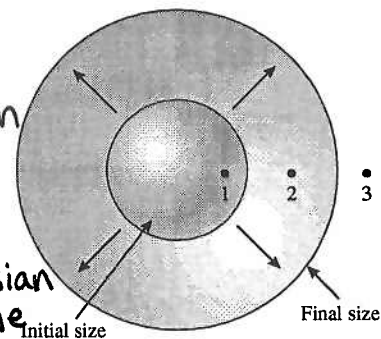


15. A charged balloon expands as it is blown up, increasing in size from the initial to final diameters shown. Do the electric fields at points 1, 2, and 3 increase, decrease, or stay the same? Explain your reasoning for each.

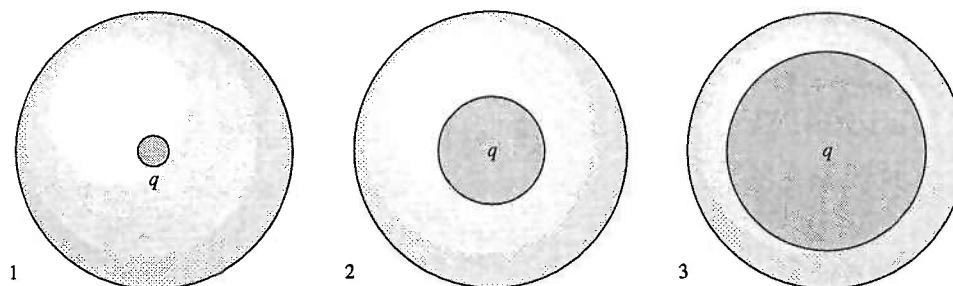
Point 1: Stays the same. A spherical Gaussian surface through 1 never encloses any charge, so the field at 1 is always zero.

Point 2: Decreases. As the balloon expands past point 2, the charge enclosed by a Gaussian surface through 2 decreases to zero, so the

Point 3: Stays the same. A field decreases to zero. Spherical Gaussian surface through 3 always encloses all the charge on the balloon. The field at 3 is always as if the entire charge were located at the center of the balloon.



16. Three charges, all the same charge q , are surrounded by three spheres of equal radii.



- a. Rank in order, from largest to smallest, the fluxes Φ_1 , Φ_2 , and Φ_3 through the spheres.

Order: $\Phi_1 = \Phi_2 = \Phi_3$

Explanation:

The flux is equal to the total charge enclosed by the surface divided by ϵ_0 . Each surface encloses the same amount of charge.

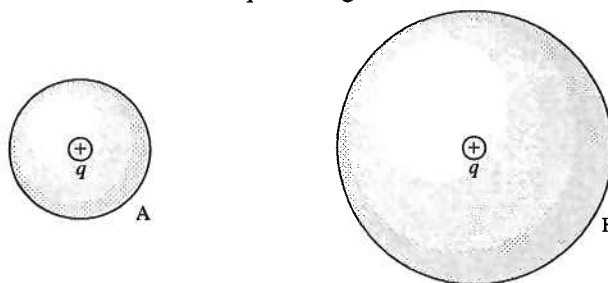
- b. Rank in order, from largest to smallest, the electric field strengths E_1 , E_2 , and E_3 on the surfaces of the spheres.

Order: $E_1 = E_2 = E_3$

Explanation:

The electric field outside a sphere of total charge q is the same as the field of a point charge q at the center.

17. Two spheres of different diameters surround equal charges. Three students are discussing the situation.



Student 1: The flux through spheres A and B are equal because they enclose equal charges.

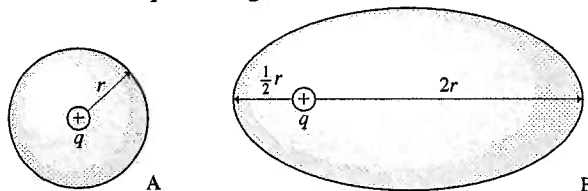
Student 2: But the electric field on sphere B is weaker than the electric field on sphere A. The flux depends on the electric field strength, so the flux through A is larger than the flux through B.

Student 3: I thought we learned that flux was about surface area. Sphere B is larger than sphere A, so I think the flux through B is larger than the flux through A.

Which of these students, if any, do you agree with? Explain.

Student 1. The area increases as r^2 and the electric field strength decreases as $1/r^2$ so the flux is the same through spheres A and B.

18. A sphere and an ellipsoid surround equal charges. Four students are discussing the situation.



Student 1: The fluxes through A and B are equal because the average radius is the same.

Student 2: I agree that the fluxes are equal, but it's because they enclose equal charges.

Student 3: The electric field is not perpendicular to the surface for B, and that makes the flux through B less than the flux through A.

Student 4: I don't think that Gauss's law even applies to a situation like B, so we can't compare the fluxes through A and B.

Which of these students, if any, do you agree with? Explain.

Student 2. As the area increases, the electric field decreases by the same factor so that the fluxes are equal.

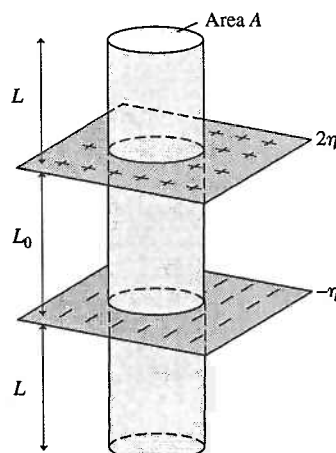
19. Two parallel, infinite planes of charge have charge densities 2η and $-\eta$. A Gaussian cylinder with cross section A extends distance L to either side.

a. Is \vec{E} perpendicular or parallel to the surface at the:

Top \perp Bottom \perp Wall \parallel

b. Is the electric field E_{top} emerging from the top surface stronger than, weaker than, or equal in strength to the field E_{bot} emerging from the bottom? Explain.

Equal. $E_{\text{top}} = \frac{2\eta}{\epsilon_0} - \frac{\eta}{\epsilon_0} = \frac{\eta}{\epsilon_0}$ up
 $E_{\text{bottom}} = \frac{2\eta}{\epsilon_0} - \frac{\eta}{\epsilon_0} = \frac{\eta}{\epsilon_0}$ down



c. By inspection, write the electric fluxes through the three surfaces in terms of E_{top} , E_{bot} , E_{wall} , L, L_0 , and A. (You may not need all of these.)

$\Phi_{\text{top}} = E_{\text{top}} A$ $\Phi_{\text{bot}} = E_{\text{bot}} A$ $\Phi_{\text{wall}} = 0$

d. How much charge is enclosed within the cylinder? Write Q_{in} in terms of η , L, L_0 , and A.

$Q_{\text{in}} = (2\eta - \eta)A = \eta A$

e. By combining your answers from parts b, c, and d, use Gauss's law to determine the electric field strength above the top plane. Show your work.

$\Phi_{\text{top}} = E_{\text{top}} A = \frac{Q_{\text{in}}}{\epsilon_0} = \frac{\eta A}{\epsilon_0}$

$E_{\text{top}} = \frac{\eta}{\epsilon_0}$

28.6 Conductors in Electrostatic Equilibrium

20. A small metal sphere hangs by a thread within a larger, hollow conducting sphere. A charged rod is used to transfer positive charge to the outer surface of the hollow sphere.

a. Suppose the thread is an insulator. After the charged rod touches the outer sphere and is removed, are the following surfaces positive, negative, or not charged?

The small sphere: not charged

The inner surface of the hollow sphere: not charged

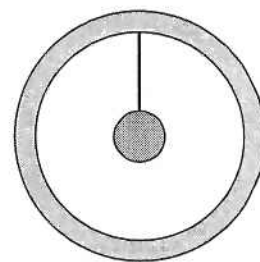
The outer surface of the hollow sphere: positive

b. Suppose the thread is a conductor. After the charged rod touches the outer sphere and is removed, are the following surfaces positive, negative, or not charged?

The small sphere: not charged

The inner surface of the hollow sphere: not charged

The outer surface of the hollow sphere: positive

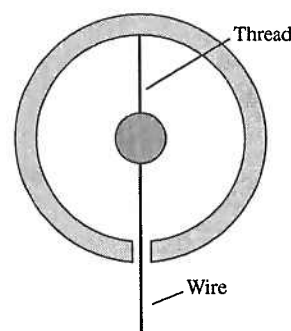


21. A small metal sphere hangs by an insulating thread within a larger, hollow conducting sphere. A conducting wire extends from the small sphere through, but not touching, a small hole in the hollow sphere. A charged rod is used to transfer positive charge to the wire. After the charged rod has touched the wire and been removed, are the following surfaces positive, negative, or not charged?

The small sphere: positive

The inner surface of the hollow sphere: negative

The outer surface of the hollow sphere: positive



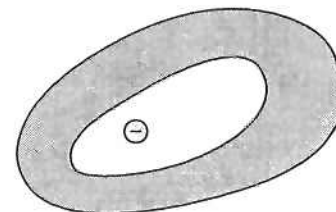
22. A -10 nC point charge is inside a hole in a conductor. The conductor has no net charge.

a. What is the total charge on the inside surface of the conductor?

$+10 \text{ nC}$

b. What is the total charge on the outside surface of the conductor?

-10 nC



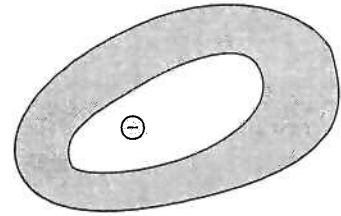
23. A -10 nC point charge is inside a hole in a conductor. The conductor has a net charge of $+10 \text{ nC}$.

a. What is the total charge on the inside surface of the conductor?

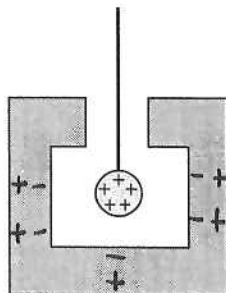
$+10 \text{ nC}$

b. What is the total charge on the outside surface of the conductor?

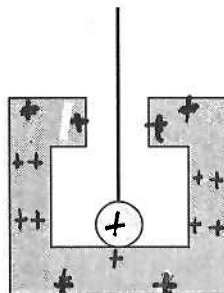
0



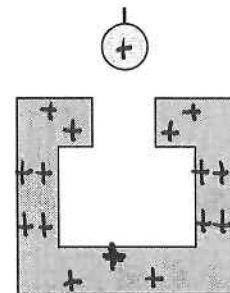
24. An insulating thread is used to lower a positively charged metal ball into a metal container. Initially, the container has no net charge. Use plus and minus signs to show the charge distribution on the ball at the times shown in the figure. (The ball's charge is already shown in the first frame.)



Ball hasn't touched



Ball has touched



Ball has been withdrawn