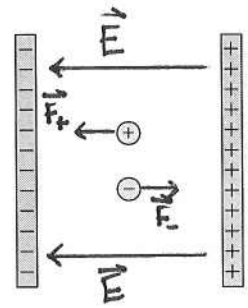
29 The Electric Potential

29.1 Electric Potential Energy

29.2 The Potential Energy of Point Charges



1. A positive point charge and a negative point charge are inside a parallel-plate capacitor. The point charges interact only with the capacitor, not with each other. Let the negative capacitor plate be the zero of potential energy for both charges.

   a. Use a **black** pen or pencil to draw the electric field vectors inside the capacitor.

   b. Use a **red** pen or pencil to draw the forces acting on the two charges.

   c. Is the potential energy of the *positive* point charge positive, negative, or zero? Explain.

   Positive. U=qEs with q, E and s all positive. The positive charge will move in the direction of decreasing potential when released from rest.

   d. In which direction (right, left, up, or down) does the potential energy of the positive charge decrease? Explain.

   Left. When released from rest the positive charge moves toward the negative plate. Its kinetic energy increases and its potential energy decreases.

   e. In which direction will the positive charge move if released from rest? Use the concept of energy to explain your answer.

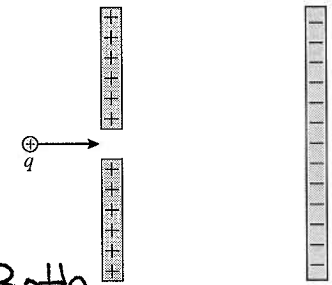
   Left. Kinetic energy increases as it accelerates toward the negative plate and potential energy decreases.

   f. Does your answer to part e agree with the force vector you drew in part b? Yes

   g. Repeat steps c to f for the *negative* point charge.

   c. negative. U=qEs with q negative
   d. right. The potential energy becomes more negative as the charge moves to the right.
   e. right. It moves toward the direction of decreasing potential energy as its kinetic energy increases.
   f. yes.

2. A positive charge q is fired through a small hole in the positive plate of a capacitor. Does q speed up or slow down inside the capacitor? Answer this question twice:



a. First using the concept of force.

It speeds up. A positive charge inside the capacitor experiences an attractive force due to the negative plate and a repulsive force due to the positive plate. Both forces point toward the right.

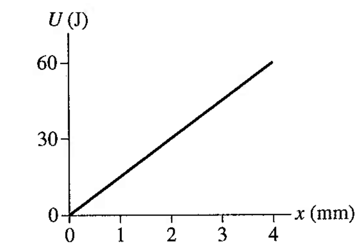
b. Second using the concept of energy.

The positive charge speeds up because it gains kinetic energy as it moves toward the negative plate in the direction of decreasing potential energy.

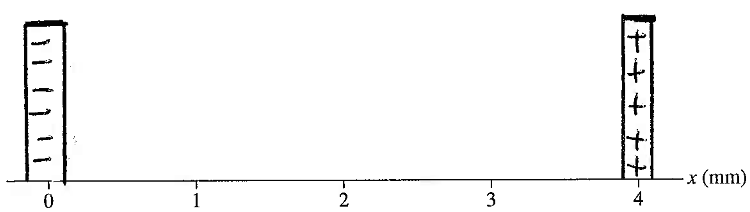
3. Charge $q_1 = 3$ nC is distance r from a positive point charge Q. Charge $q_2 = 1$ nC is distance $2r$ from Q. What is the ratio U_1/U_2 of their potential energies due to their interactions with Q?

$$\frac{U_1}{U_2} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_1 Q}{r}}{\frac{1}{4\pi\varepsilon_0}\frac{q_2 Q}{2r}} = \frac{q_1(2r)}{q_2(r)} = \frac{3\text{nC}(2r)}{1\text{nC}(r)} = 6$$

4. The figure shows the potential energy of a positively charged particle in a region of space.



a. What possible arrangement of source charges is responsible for this potential energy? Draw the source charges above the axis below.



b. With what kinetic energy should the charged particle be launched from $x = 0$ mm to have a turning point at $x = 3$ mm? Explain.

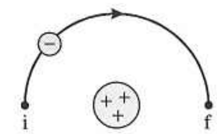
$-\Delta K = \Delta U$

$K_i - K_f = qEd \quad K_i = qE(0.003\text{m}) = 45\text{J}$ (from the graph)

c. How much kinetic energy does this charged particle of part b have as it passes $x = 2$ mm?

For each 1mm of travel toward the right there is a gain of 15J of potential energy and a loss of 15J of kinetic energy. As it passes x=2mm it has 15J of kinetic energy.

5. An electron ($q = -e$) completes half of a circular orbit of radius r around a nucleus with $Q = +3e$.



a. How much work is done on the electron as it moves from i to f? Give either a numerical value or an expression from which you could calculate the value if you knew the radius. Justify your answer.

Zero. The motion is always perpendicular to the force.

b. By how much does the electric potential energy change as the electron moves from i to f?

$\Delta U = U_f - U_i = 0$

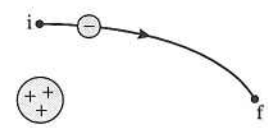
c. Is the electron's speed at f greater than, less than, or equal to its speed at i?

$v_f = v_i$

d. Are your answers to parts a and c consistent with each other?

Yes.

6. An electron moves along the trajectory from i to f.



a. Does the electric potential energy increase, decrease, or stay the same? Explain.

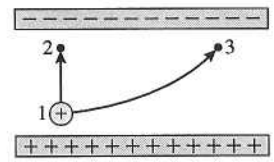
$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$

The potential energy varies as $\frac{1}{r}$, but because the two charges have opposite sign the potential energy actually increases as r increases.

b. Is the electron's speed at f greater than, less than, or equal to its speed at i? Explain.

Less than. Two opposite charges slow down as they move farther apart.

7. Inside a parallel-plate capacitor, two protons are launched with the same speed from point 1. One proton moves along the path from 1 to 2, the other from 1 to 3. Points 2 and 3 are the same distance from the positive plate.



a. Is $\Delta U_{1\to 2}$, the change in potential energy along the path $1 \to 2$, larger than, smaller than, or equal to $\Delta U_{1\to 3}$? Explain.

$\Delta U_{1\to 2} = \Delta U_{1\to 3}$

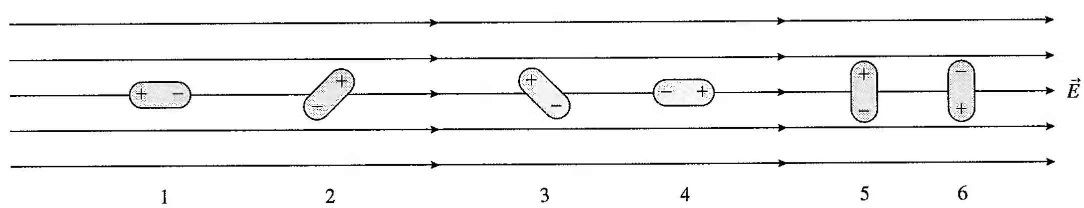
$\Delta U = qEs$ where s is the distance measured vertically from point 1 to point 2. Only the vertical displacement matters here because there is no horizontal component of E or of the force.

b. Is the proton's speed v_2 at point 2 larger than, smaller than, or equal to v_3? Explain.

$V_2 = V_3$ The change in kinetic energy $\Delta K_{1\to 2}$ is equal to $\Delta K_{1\to 3}$ because $\Delta U_{1\to 2} = \Delta U_{1\to 3}$ and the total energy is constant.

29.3 The Potential Energy of a Dipole

8. Rank in order, from most positive to most negative, the potential energies U_1 to U_6 of these six electric dipoles in a uniform electric field.



Order: $U_1 > U_3 > U_6 = U_5 > U_2 > U_4$

Explanation:

$U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$

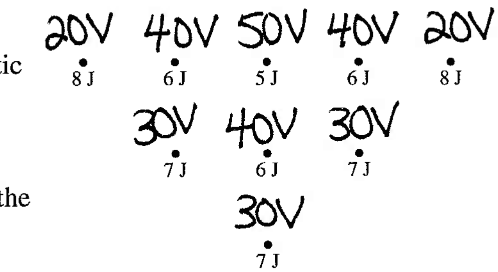
$U_1 = -pE\cos 180° = +pE$ $\quad U_4 = -pE\cos 0° = -pE$

$U_2 = -pE\cos 45° = -0.707pE$ $\quad U_5 = -pE\cos 90° = 0$

$U_3 = -pE\cos 135° = +0.707pE$ $\quad U_6 = -pE\cos(-90°) = 0$

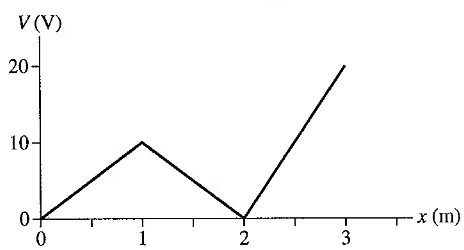
29.4 The Electric Potential

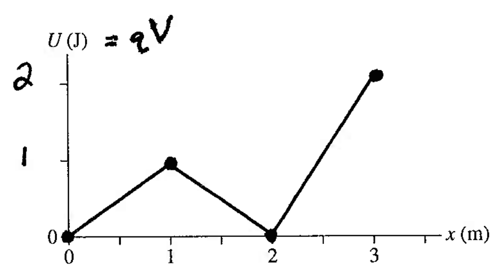
9. Charged particles with $q = +0.1$ C are fired with 10 J of kinetic energy toward a region of space in which there is an electric potential. The figure shows the kinetic energy of the charged particles as they arrive at nine different points in the region. Determine the electric potential at each of these points. Write the value of the potential above each of the dots. Assume that the particles start from a point where the electric potential is zero.



Ex.
• $K_2 = 7J$ $U_2 = 3J$
$V_2 = \frac{U_2}{q} = \frac{3J}{0.1C} = 30V$
• $K_1 = 10J$ $U_1 = qV_1 = 0$ $V_1 = 0$

10. a. The graph on the left shows the electric potential along the x-axis. Use the axes on the right to draw a graph of the potential energy of a 0.1 C charged particle in this region of space. Provide a numerical scale on the energy axis.





b. If the charged particle is shot toward the right from $x = 1$ m with 1.0 J of kinetic energy, where is its turning point? Explain.

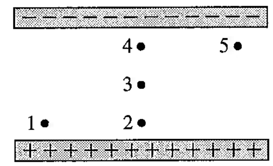
At $x = 1m$, the total energy $E = K_1 + U_1 = 1J + 1J = 2J$.
At the turning point $v = 0$ so $K_2 = 0$
$E = K_2 + U_2$ $2J = 0J + U_2$ $U_2 = 2J$ which occurs at $x = 3m$.

c. Will the charged particle of part b ever reach $x = 0$ m? If so, how much kinetic energy will it have at that point? If not, why not?

Yes it will reach $x = 0m$. It will have 2J of kinetic energy at that point.

29.5 The Electric Potential Inside a Parallel-Plate Capacitor

11. Rank in order, from largest to smallest, the electric potentials V_1 to V_5 at points 1 to 5.

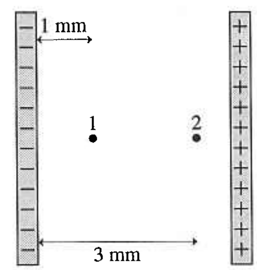


Order: $V_1 = V_2 > V_3 > V_4 = V_5$

Explanation:

$V = Es$ where s is measured from the negative plate.

12. The figure shows two points inside a capacitor. Let $V = 0$ V at the negative plate.



a. What is the ratio V_2/V_1 of the electric potential at these two points? Explain.

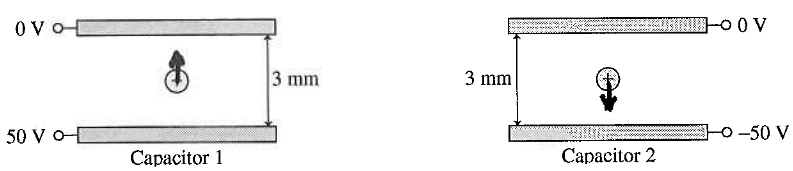
$\frac{V_2}{V_1} = \frac{Es_2}{Es_1} = \frac{s_2}{s_1} = \frac{3\text{mm}}{1\text{mm}} = 3$

b. What is the ratio E_2/E_1 of the electric field strength at these two points? Explain.

$\frac{E_2}{E_1} = 1$ The field inside a parallel-plate capacitor is a constant.

$E = \frac{\eta}{\epsilon_0}$

13. The figure shows two capacitors, each with a 3 mm separation. A proton is released from rest in the center of each capacitor.



a. Draw an arrow on each proton to show the direction it moves.

b. Which proton reaches a capacitor plate first? Or are they simultaneous? Explain.

Simultaneous. The electric field strength is the same in both capacitors.

14. A capacitor with plates separated by distance d is charged to a potential difference ΔV_C. All wires and batteries are disconnected, then the two plates are pulled apart (with insulated handles) to a new separation of distance $2d$.

a. Does the capacitor charge Q change as the separation increases? If so, by what factor? If not, why not?

The capacitor charge remains constant. No charge flows off the capacitor plates So, by conservation of charge, $Q_1 = Q_2$.

b. Does the electric field strength E change as the separation increases? If so, by what factor? If not, why not?

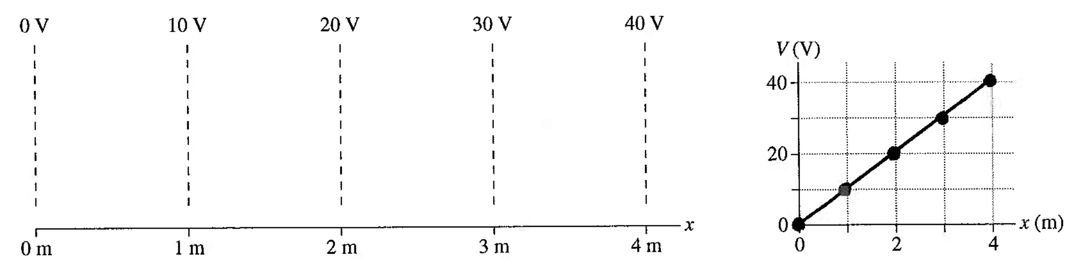
The electric field strength also remains constant. $E = \frac{\eta}{\epsilon_0} = \frac{Q/A}{\epsilon_0}$ Q, A and ϵ_0 are all constant.

c. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?

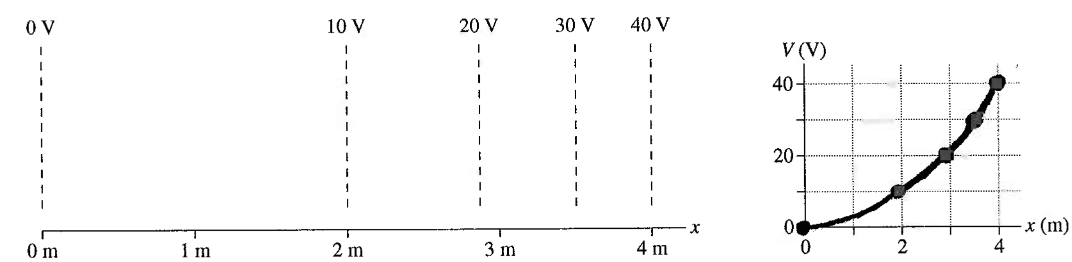
Yes. $\Delta V_C = Ed$ It increases by a factor of 2 because the separation increases by a factor of 2.

15. Each figure shows a contour map on the left and a set of graph axes on the right. Draw a graph of V versus x. Your graph should be a straight line or a smooth curve.

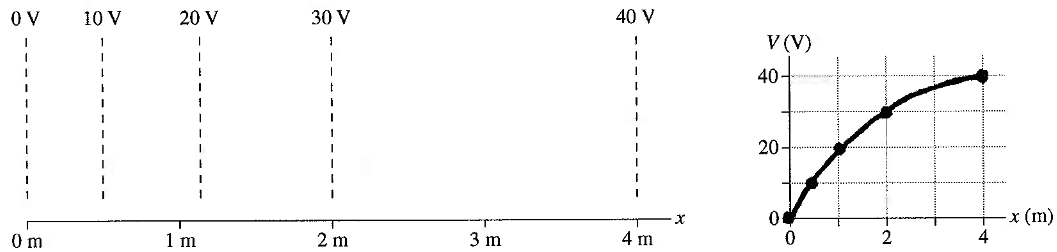
a.



b.

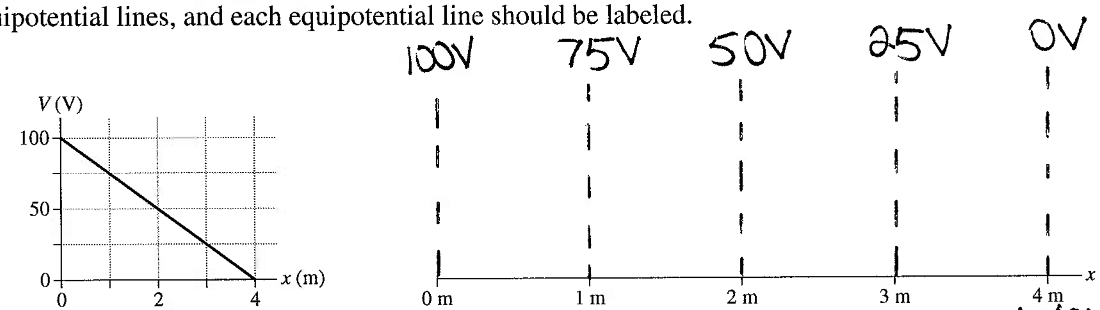


c.

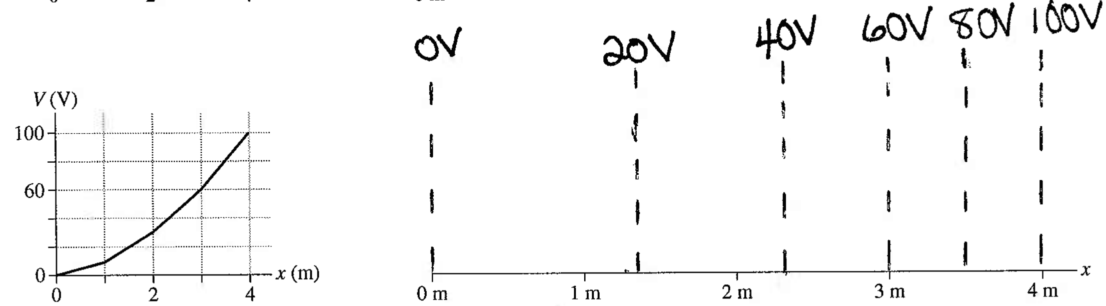


16. Each figure shows a V-versus-x graph on the left and an x-axis on the right. Assume that the potential varies with x but not with y. Draw a contour map of the electric potential. Your figures should look similar to the contour maps in Question 15. There should be a uniform difference between equipotential lines, and each equipotential line should be labeled.

a.

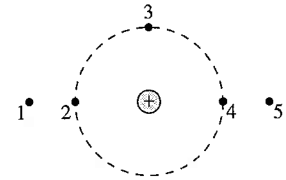


b.



29.6 The Electric Potential of a Point Charge

17. Rank in order, from largest to smallest, the electric potentials V_1 to V_5 at points 1 to 5.



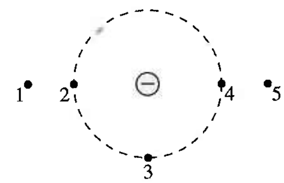
Order: $V_2 = V_3 = V_4 > V_1 = V_5$

Explanation:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

As r increases, V decreases.

18. Rank in order, from least negative to most negative, the electric potentials V_1 to V_5 at points 1 to 5.



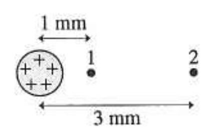
Order: $V_1 = V_5 > V_2 = V_3 = V_4$

Explanation:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

because q is a negative charge, as r increases V increases (becomes less negative).

19. The figure shows two points near a positive point charge.



a. What is the ratio V_1/V_2 of the electric potentials at these two points? Explain.

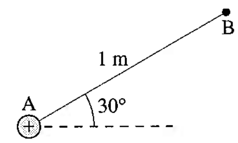
$$\frac{V_1}{V_2} = \frac{\frac{1}{4\pi\epsilon_0}\frac{q}{r_1}}{\frac{1}{4\pi\epsilon_0}\frac{q}{r_2}} = \frac{r_2}{r_1} = \frac{3\text{mm}}{1\text{ mm}} = 3$$

b. What is the ratio E_1/E_2 of the electric field strengths at these two points? Explain.

$$\frac{E_1}{E_2} = \frac{\frac{1}{4\pi\epsilon_0}\frac{q}{r_1^2}}{\frac{1}{4\pi\epsilon_0}\frac{q}{r_2^2}} = \frac{r_2^2}{r_1^2} = \frac{(3\text{mm})^2}{(1\text{mm})^2} = 9$$

20. A 1 nC positive point charge is located at point A. The electric potential at point B is

(a. 9 V) b. $9 \cdot \sin 30°$ V c. $9 \cdot \cos 30°$ V d. $9 \cdot \tan 30°$ V



Explain the reason for your choice.

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{(9.0\times10^9\ \frac{Nm^2}{C^2})(1.0\times10^{-9}C)}{1m} = 9V$$

21. An inflatable metal balloon of radius R is charged to a potential of 1000 V. After all wires and batteries are disconnected, the balloon is inflated to a new radius $2R$.

a. Does the potential of the balloon change as it is inflated? If so, by what factor? If not, why not?

Yes. It decreases by a factor of 2.

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} \quad V_2 = \frac{1}{4\pi\epsilon_0}\frac{Q}{(2R)} \quad Q = \text{constant}$$

b. Does the potential at a point at distance $r = 4R$ change as the balloon is inflated? If so, by what factor? If not, why not?

No. Outside a sphere the potential is the same as that of a point charge Q located at the center of the sphere. The amount of charge does not change.

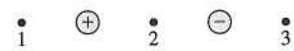
29.7 The Electric Potential of Many Charges

22. Each figure below shows three points in the vicinity of two point charges. The charges have equal magnitudes. Rank in order, from largest to smallest, the potentials V_1, V_2, and V_3.

a.

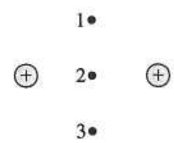
$V_2 > V_1 = V_3$

b.



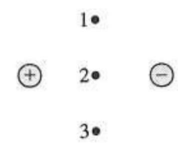
$V_1 > V_2 > V_3$

c.



$V_2 > V_1 = V_3$

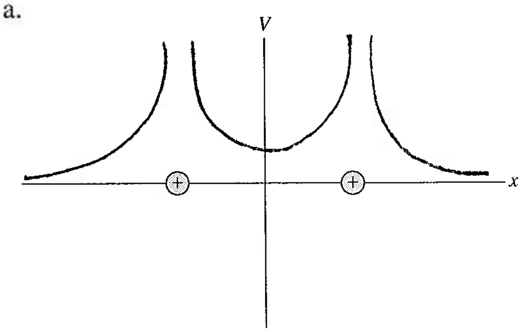
d.



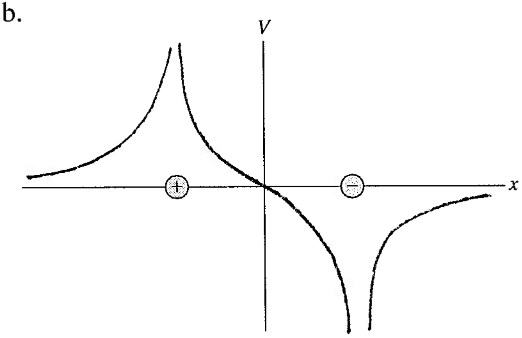
$V_1 = V_2 = V_3$

23. On the axes below, draw a graph of V versus x for the two point charges shown.

a.



b.



24. For each pair of charges below, are there any points (other than at infinity) at which the electric potential is zero? If so, identify them on the figure with a dot and a label. If not, why not?

a.

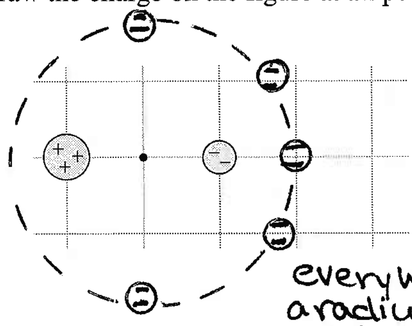
No. The potential is always positive.

b.

V=0 V=0

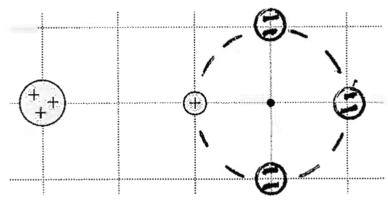
25. For each pair of charges below, at which grid point or points could a double-negative point charge ($q = -2$) be placed so that the potential at the dot is 0 V? There may be more than one possible point. Draw the charge on the figure at all points that work.

a.



everywhere on a radius of 2 circle centered at dot

b.

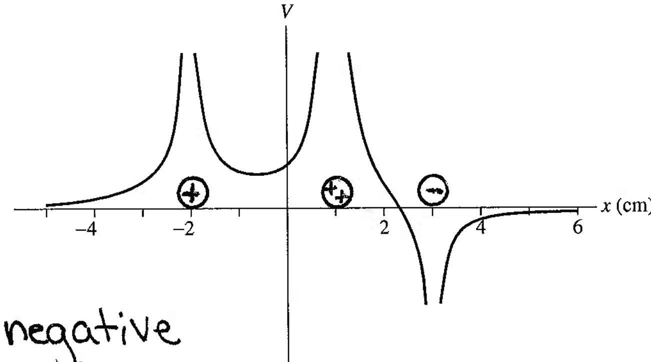


everywhere on a circle of radius 1 centered on the dot

26. The graph shows the electric potential along the x-axis due to point charges on the x-axis.

a. Draw the charges *on the axis of the figure*. Note that the charges may have different magnitudes.

b. An electron is placed at $x = 2$ cm. Is its potential energy positive, negative, or zero? Explain.



Its potential energy is negative
$U = qV$ q is negative and V is positive at $x = 2$ cm.

c. If the electron is released from rest at $x = 2$ cm, will it move right, move left, or remain at $x = 2$ cm? Base your explanation on energy concepts.

The electron will move left.
A charge will move in the direction of decreasing potential energy.

27. A ring has radius R and charge Q. The ring is shrunk to a new radius $\frac{1}{2}R$ with no change in its charge. By what factor does the on-axis potential at $z = R$ increase?

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2+R^2}}$$

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{(\frac{R}{2})^2+z^2}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{\frac{1}{4}R^2+R^2}}$$

$$\frac{V_2}{V_1} = \frac{\sqrt{2R^2}}{\sqrt{\frac{5}{4}R^2}} = \frac{\sqrt{2}}{\sqrt{\frac{5}{4}}} = 1.26$$

V increases by a factor of 1.26.