Dr. S. Johnson Physics 141 - Studio Physics II- Exam 2 July 9, 2007

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Test Taking Strategies

1) Read each problem carefully.

2) To get partial credit, **show all of your work.** Use extra paper if necessary.

3) **Put a box around your answer. Don't forget units on physical quantities!**

4) If you get stuck on a problem, or part of a problem, move on to the next part.

5) At the least, write down the formula(e) you would use to solve the problem, even if you can't finish it.

6) Draw a sketch of the problem and label the initial and final values.

7) Write down which quantities are known and which are unknown.

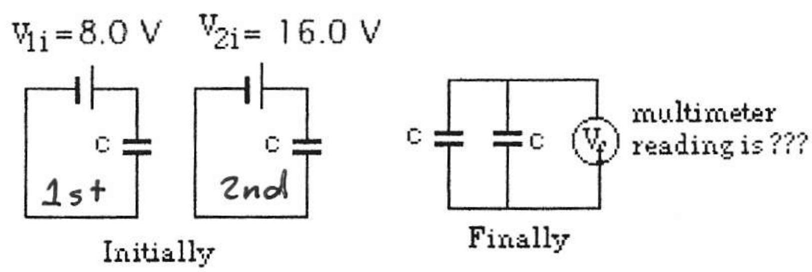
8) Check to see if your answer makes sense physically. In other words, is it much too large or too small for the situation you're given.

Mean 68.6 ± 14 B –

A	84 – 100		|	1
B	68 – 83		~~||||~~ ||	7
C	52 – 67		||||	4
D	36 – 51		||	2
F		<36	0	0

Conceptual/Short Exercises – 40 pts total

1) You examined a capacitance "puzzler" like the following in Unit 24.



a) What is the final multimeter reading in this situation? (2 pts)

$$V_f = \frac{8.0V + 16.0V}{2} = \boxed{12.0V}$$

b) Explain qualitatively in words how the charge on the two capacitors redistributes itself, i.e.which plates lose charge and which gain, when the two capacitors are connected together. (3 pts)

(Note: a gain in (+) charge means a loss of electrons.)

$q_{1i} = CV_{1i}$ $\quad q_{2i} = CV_{2i}$

The second capacitor has twice as much charge on it as the first. When the two capacitors are connected together the top plate of the second capacitor will lose (+) charge and the bottom plate will lose (–) charge. The (+) charge will go to the top plat of the firs cap, and the (–) will go to its bottom plate

c) If C = 45.0 μF, determine the magnitude of the charge on each of the two capacitors before and after they are connected together. (5 pts)

$$q_{1i} = (45.0\times10^{-6}F)(8.0V) = \boxed{3.6\times10^{-4}C}$$
$$q_{2i} = (45.0\times10^{-6}F)(16.0V) = \boxed{7.2\times10^{-4}C}$$
before

$$q_f = (45.0\times10^{-6}F)(12.0V) = \boxed{5.4\times10^{-4}C}$$
after on both

or

$$q_f = \frac{q_{1i} + q_{2i}}{2} = \frac{10.8\times10^{-4}C}{2} = 5.4\times10^{-4}C$$

2) a) An electron has an initial velocity of 15.0 km/s $\hat{\mathbf{j}}$ + 18.0 km/s $\hat{\mathbf{k}}$ and a constant acceleration of 2.50 X 10^{12} m/s^2 $\hat{\mathbf{i}}$ in a region in which uniform electric and magnetic fields are present. If $\vec{\mathbf{B}}$ = 500 μT $\hat{\mathbf{i}}$, find the electric field $\vec{\mathbf{E}}$. (7 pts)

$$\vec{v}_0 = 15.0\ \text{km/s}\ \hat{j} + 18.0\ \text{km/s}\ \hat{k} \qquad \vec{a} = 2.50\times10^{12}\ \frac{m}{s^2}\ \hat{i}$$

$$\vec{F} = \vec{F}_e + \vec{F}_m = -e\vec{E} + (-e)(\vec{v}\times\vec{B})$$

$$\vec{E} = \frac{\vec{F} + e(\vec{v}\times\vec{B})}{-e} = -\frac{m\vec{a}}{e} - (\vec{v}\times\vec{B})$$

$$= -\frac{(9.11\times10^{-31}\ kg)}{1.6\times10^{-19}\ C}(2.50\times10^{12}\ \frac{m}{s^2})\hat{i} - (15.0\times10^{3}\ \frac{m}{s}\hat{j} + 18.0\times10^{3}\ \frac{m}{s}\hat{k})\times(500\times10^{-6}\ T)\hat{i}$$

$$= -14.23\ \frac{N}{C}\hat{i} - 7.50\frac{N}{C}(-\hat{k}) - 9.00\frac{N}{C}\hat{j}$$

$$\boxed{\vec{E} = -14.2\ \frac{N}{C}\hat{i} - 9.00\ \frac{N}{C}\hat{j} + 7.50\ \frac{N}{C}\hat{k}}$$

b) Explain in words how your answer to a) would differ if the particle were a proton instead of an electron. (3 pts)

If the particle were a proton then $q = +e$

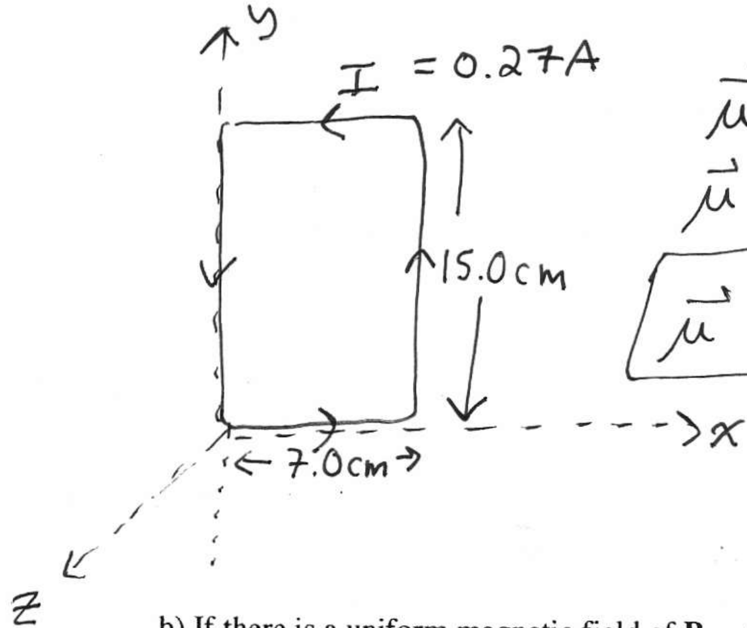
so: $\vec{F} = e\vec{E} + e(\vec{v}\times\vec{B})$ so:

(first term changes sign)

$\vec{E} = \frac{\vec{F}}{e} - (\vec{v}\times\vec{B})$ and $|\vec{F}|$ would be larger because $m_p >> m_e$. $\vec{E}$ would have a different magnitude and direction.

3) A rectangular 40-turn coil of wire of dimensions 15.0 cm by 7.0 cm carries a current of 0.27 A. The coil is hinged on the y-axis along one of the longer sides and lays in the x-y plane.

a) Draw a sketch of this coil and find its magnetic dipole moment. (5 pts)



$\vec{\mu} = NI\vec{A}$

$\vec{\mu} = (40)\ (0.27A)(0.07m \times 0.15m)\hat{k}$

$\boxed{\vec{\mu} = 0.11\, Am^2\, \hat{k}}$

b) If there is a uniform magnetic field of $\mathbf{B} = 0.25\ \mathrm{T}\ \mathbf{i} + 0.45\ \mathrm{T}\ \mathbf{j}$, find the magnitude and direction of the torque acting on the coil about the hinge line. (5 pts)

$\vec{\tau} = \vec{\mu} \times \vec{B}$

Total: $\vec{\tau} = (0.11\, Am^2)\hat{k} \times (0.25\, T\hat{\imath} + 0.45\, T\hat{\jmath})$

$\vec{\tau} = 0.0275\, Nm\,\hat{\jmath} \quad - 0.0495\, Nm\,\hat{\imath}$

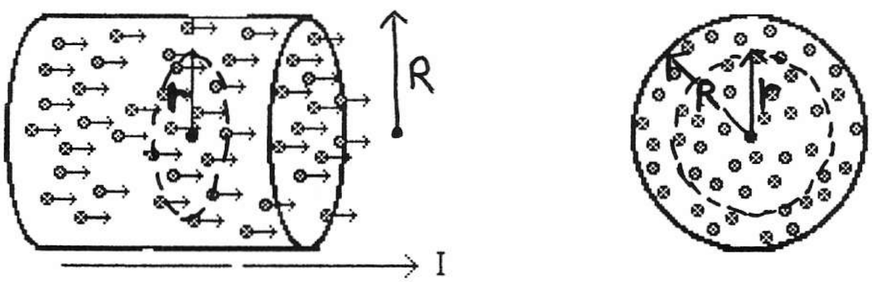
torque about the hinge line is along $\hat{\jmath}$:

$\vec{\tau}_y = 0.0275\, Nm\,\hat{\jmath}$

$\boxed{|\vec{\tau}_y| = 0.0275\, Nm}$

$\vec{\tau}_x = -0.0495\, Nm\,\hat{\imath}$ would cause loop to rotate about the x-axis if it could.

4) A cylindrical wire of radius *R* carries a current *I* uniformly distributed in it as shown.



SIDE VIEW FRONT VIEW

a) To find the magnetic field inside the wire one must draw a circular Amperian loop of radius $r < R$ as shown. What fraction of the total current *I* would be enclosed in such a loop? (4 pts)

$$I_{enc} = \frac{\pi r^2}{\pi R^2} I = \boxed{I\left(\frac{r^2}{R^2}\right)}$$

or

$$\boxed{\frac{I_{enc}}{I} = \frac{r^2}{R^2}}$$

b) Using Ampere's Law, determine the magnitude of the magnetic field at a distance *r* ($r < R$) from the centre of the wire. (6 pts)

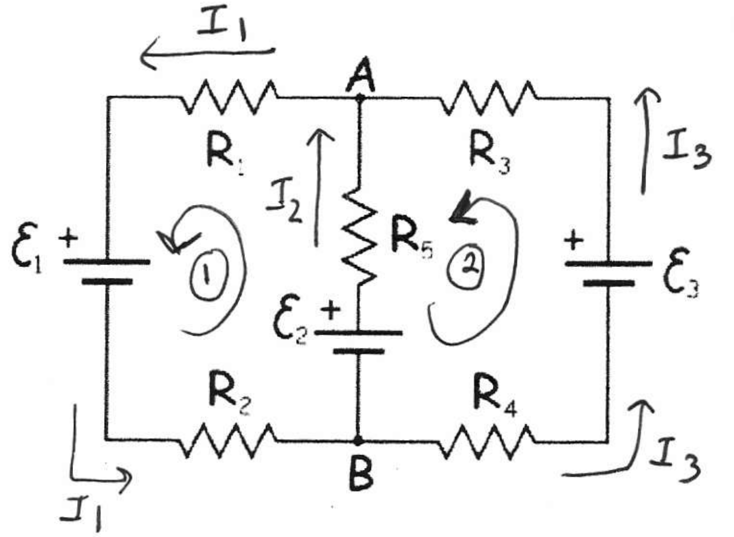
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

$$B\, 2\pi r = \mu_0 I \left(\frac{r^2}{R^2}\right) \qquad \boxed{B = \frac{\mu_0}{2\pi} I \left(\frac{r}{R^2}\right)}$$

Longer Problems – 60 pts total – 20 pts each

5) Consider the circuit shown below. Assume that $R_1 = R_2 = 5.0\ \Omega$, $R_3 = R_4 = 3.0\ \Omega$, $R_5 = 4.0\ \Omega$, $\mathcal{E}_1 = 6.0$ V, $\mathcal{E}_2 = 9.0$ V, and $\mathcal{E}_3 = 12.0$ V.

(a) Determine the current flowing through R_2. (5 pts)
(b) Determine the current flowing through R_5. (5 pts)
(c) Determine the current flowing through R_3. (5 pts)
(d) What is the potential difference between points A and B? (5 pts)



$I_1 = I_2 + I_3$

① $9.0\ V - I_2 (4.0\Omega) - I_1 (5.0\Omega$
$- 6.0\ V - I_1 (5.0\Omega) = 0$

② $12.0\ V - I_3 (3.0\Omega) + I_2 (4.0\Omega$
$- 9.0\ V - I_3 (3.0\Omega) = 0$

(a) + (b) + (c)

① $3.0V - I_2 (4.0\Omega) - I_1 (10.0\Omega) = 0$

② $3.0V + I_2 (4.0\Omega) - I_3 (6.0\Omega) = 0$

① $3.0V - I_2 (4.0\Omega) - (I_2 + I_3)(10.0\Omega) = 0$

① $[3.0V - I_2 (14.0\Omega) - I_3 (10.0\Omega) = 0] \times 6$

① $18.0V - I_2 (84.0\Omega) - I_3 (60.0\Omega) = 0$

② $30.0V + I_2 (40.0\Omega) - I_3 (60.0\Omega) = 0$

$12.0V + I_2 (124\Omega) = 0$

(b)

$I_2 = \frac{-12.0V}{124\Omega} = \boxed{-0.097A}$

$3.0V - (-0.097A)(4.0\Omega$
$-I_1 (10.0\Omega) = 0$

(a)
$\boxed{I_1 = 0.34\ A}$

$I_3 = I_1 - I_2$

(c) $= 0.34A + 0.097$

$\boxed{I_3 = 0.44\ A}$

$$V_A - V_B = \mathcal{E}_2 - I_2 (4.0\Omega)$$

$$= 9.0V - (-0.097A)(4.0\Omega)$$

$$\boxed{V_A - V_B = 9.39 \ V}$$

6) A capacitor with an initial potential difference of 150. V is discharged through a resistor when a switch is closed at t = 0. At t = 10.0 s, the potential difference across the capacitor is 1.50 V.

a) Determine the time constant τ of the circuit. (4 pts)

$$V = V_o\, e^{-t/\tau}$$

$$1.50\,V = 150.\,V\, e^{-\frac{10.0s}{\tau}}$$

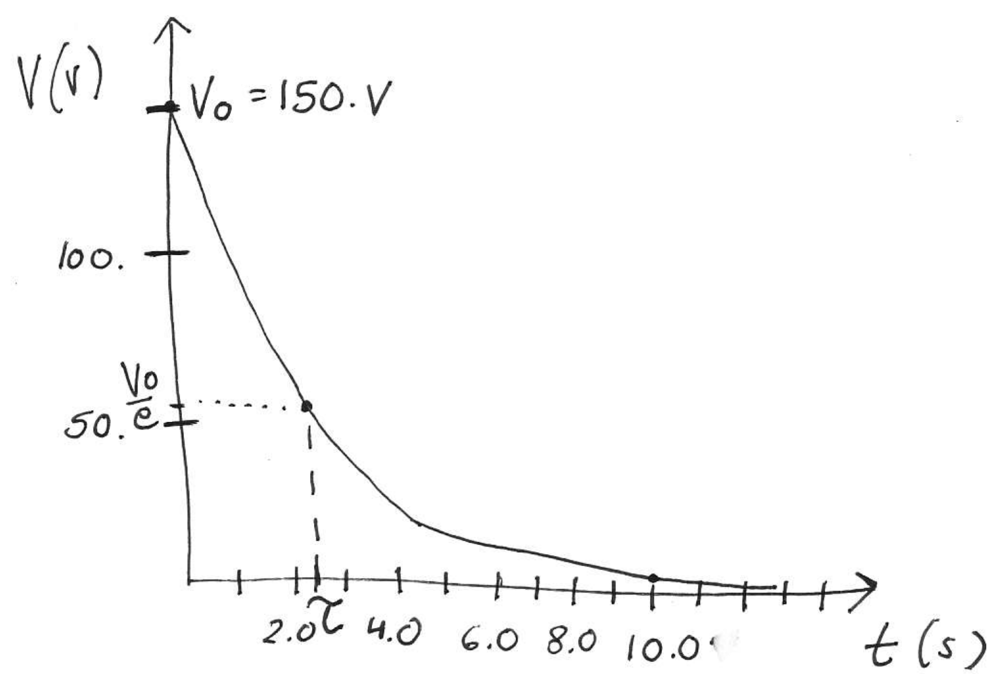
$$\frac{1.50}{150.} = e^{-10.0s/\tau}$$

$$\ln(0.01) = -10.0s/\tau$$

$$-4.605 = -10.0s/\tau$$

$$\boxed{\tau = 2.17\,s}$$

b) Draw a sketch of the graph of potential difference across the capacitor as a function of time for this circuit. Label your axes with proper units. On your graph, indicate the initial potential difference and the location of the time τ. (4 pts)



c) Determine the potential difference across the capacitor at t = 13.0 s. (4 pts)

$$V = V_0 e^{-t/\tau}$$

$$V = (150.\,V)\, e^{-13.0s/2.17s}$$

$$\boxed{V = 0.375\,V}$$

d) Determine the current through the resistor at t = 13.0 s. (4 pts)

$$I = \frac{dq}{dt} = \frac{d}{dt}\left(q_0\, e^{-t/\tau}\right) = q_0\left(-\frac{e^{-t/\tau}}{\tau}\right)$$

$$I = -\frac{C\left(V_0\, e^{-t/\tau}\right)}{RC} = -\frac{(0.375V)}{15.0\times10^{3}\,\Omega} = \boxed{-2.5\times10^{-5}\,A}$$

e) If the resistor is a 15.0 kΩ resistor, what is the capacitance of the capacitor? (4 pts)

$$\tau = RC$$

$$C = \frac{\tau}{R} = \frac{2.17s}{15.0\times10^{3}\,\Omega} = \boxed{1.45\times10^{-4}\,F}$$

7) A small circular coil with 1200 turns of radius 1.50 cm is placed in the plane of, and concentric with, a large circular coil with 200 turns of radius 65.0 cm.

a) If the current in the large coil is changed uniformly from 7.00 A to -7.00 A (a change in direction) in a time of 0.500 s beginning at t = 0 s, determine the magnetic flux in the smaller coil at t = 0 s, t = 0.250 s and t = 0.500 s. (Assume the magnetic field produced by the large coil is essentially constant over the area of the smaller coil at any point in time and has a value equal to the value at the centre of the large coil.) (6 pts)

$$\Phi^{mag} = \int \vec{B}\cdot d\vec{A} = \vec{B}\cdot\vec{A} = BA\cos\theta$$

Assume: $\vec{A}$ ⊙

I

@ $t=0$ $\quad \Phi^{mag} = \dfrac{N\mu_0 |I|}{2R} A\cos 180°$

$$= \frac{(200)(4\pi\times10^{-7}\,\frac{Tm}{A})(7.00A)(-1)\pi(0.015m)^2}{2(0.65m)}$$

$$\boxed{= -9.56\times10^{-7}\ Tm^2 \leftarrow Wb}$$

@ $t = 0.250$ s $\quad \boxed{\Phi^{mag} = 0}$

$\theta = 0°$

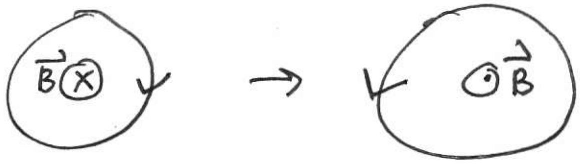
@ $t = 0.500$ s $\quad \boxed{\Phi^{mag} = 9.56\times10^{-7}\ Tm^2}$

b) What is the magnitude of the emf induced in the smaller coil at t = 0.250 s? (4 pts)

$$|\varepsilon| = \left| -N\frac{d\Phi^{mag}}{dt} \right|$$

$$|\varepsilon| = \frac{(1200)(2)(9.56\times10^{-7}\,Wb)}{(0.500s)} = \boxed{4.59\times10^{-3}\ V}$$

c) If the current direction in the large coil changes from clockwise to counter-clockwise as seen from above, what is the direction of the induced current in the smaller coil? Explain your reasoning. (2 pts)



$\vec{B}$ increases upwards so $\vec{B}$ ind is downwards

$\rightarrow$ I ind clockwise due to Lenz's Law the ind. $\vec{B}$ counters the change in $\vec{B}$.

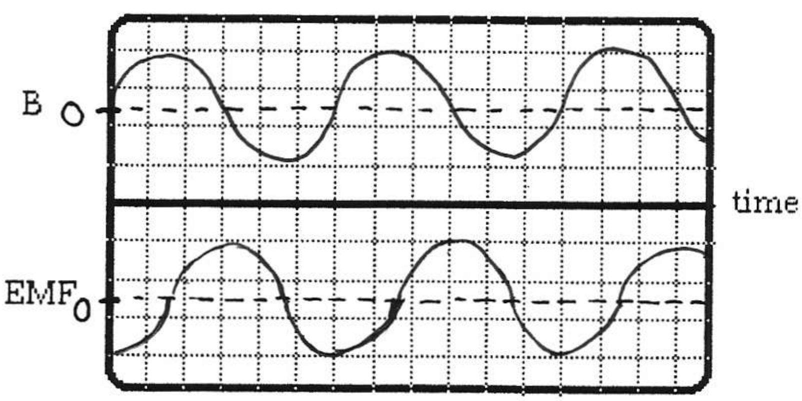
d) If the large coil instead carries an AC current $I = I_0 \sin(\omega t)$ where $I_0 = 4.60$ A and $\omega = 23.0$ rad/s, determine the induced *emf* in the smaller coil as a function of time. (5 pts)

$$\mathcal{E} = -N \frac{d\Phi_{mag}}{dt} = -N \frac{d}{dt}\left[\frac{N_f \mu_0 A \, I_0 \sin(\omega t)}{2R}\right]$$

$$\mathcal{E} = -\frac{(1200)(200)(4\pi \times 10^{-7} \frac{Tm}{A})\,\pi(0.015m)^2(4.60A)}{2(0.65m)} \times \left(23.0 \tfrac{rad}{s} \cos\omega t\right)$$

$$\boxed{\mathcal{E} = -0.0173 \cos\left(\underset{(rad/s)}{23.0}\, t\right) \text{ V}}$$

e) Qualitatively sketch the magnetic field at the centre of the large coil and the *emf* induced in the small coil as functions of time over at least two periods for the situation in d). (3 pts)



Extra Credit (2 pts): What is your favorite physics unit (i.e. Newton, Henry, slug…)?

Henry!