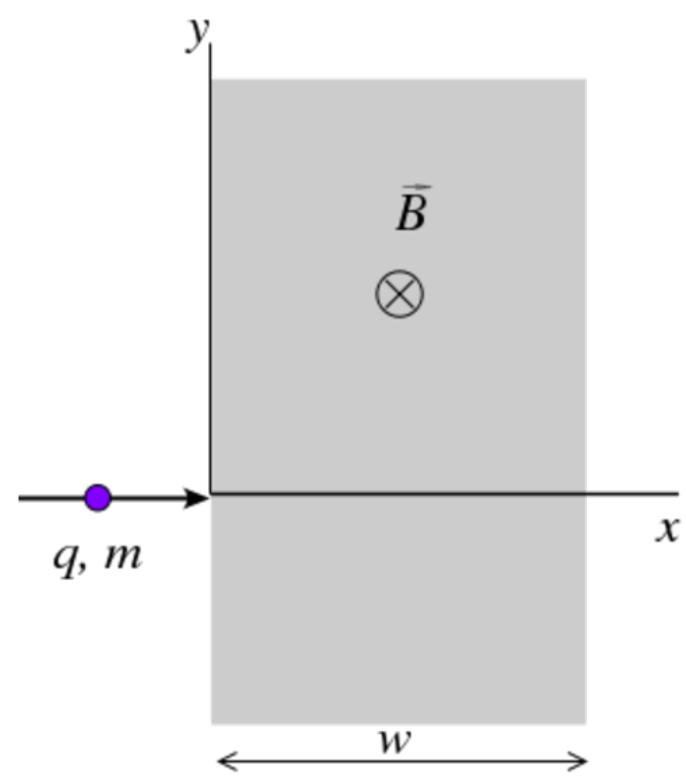
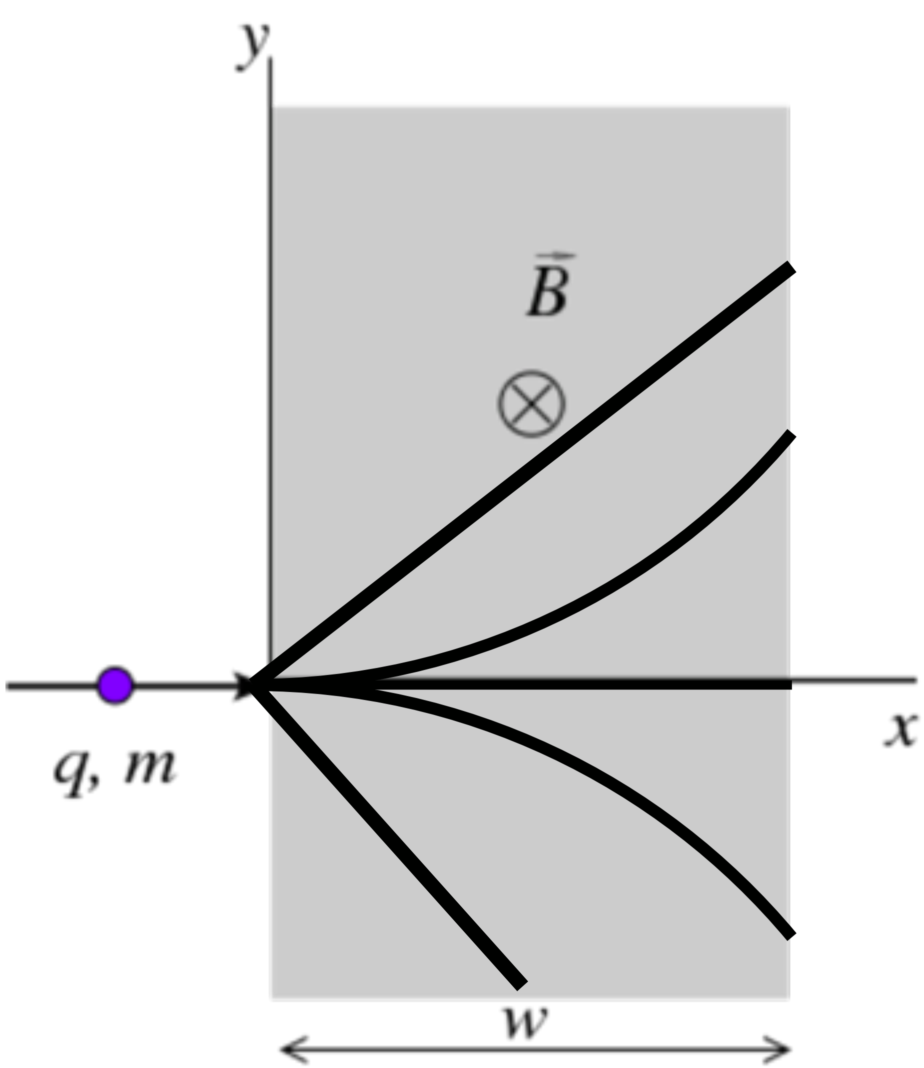
The Lorentz Force

A region of space has is a constant magnetic field, B = 1.2 T, directed along the negative z axis (into the page). The magnetic field region extends infinitely far in the positive and negative y directions, but is constrained to the region between x = 0 and x = 1 m. A proton travels along the x axis toward this region, with initial speed v_0.



What is the path of the proton?

Which path does the proton follow after it enters the magnetic field region?



A

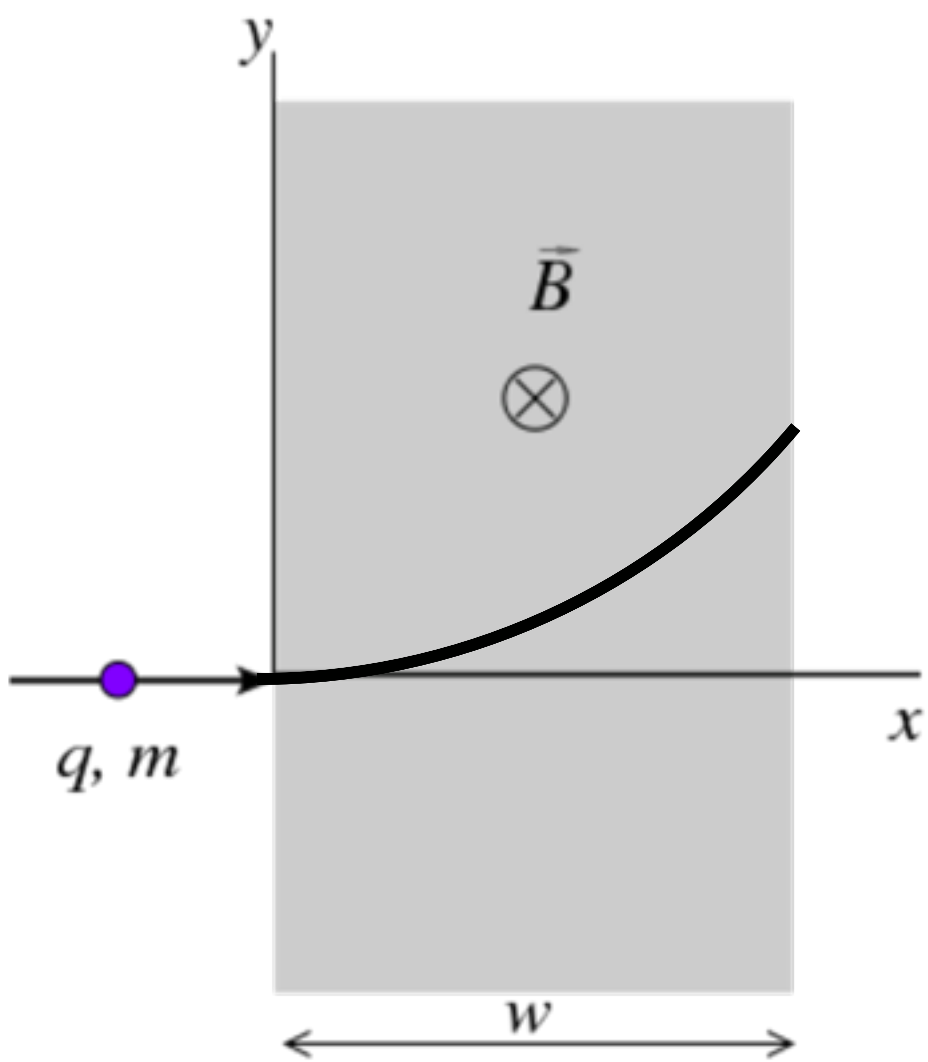
B

C

D

E

What is the radius of curvature of the proton's path after it enters the magnetic field region?



$$F_{\perp} = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

```
>>> 1.67e-27*2e8/(1.6e-19*1.2)
1.7395833333333335
```

$$r = \frac{mv}{qB} = 1.74 \text{ m}$$

m = 1.67 x 10^{-27} kg, q=e=1.6x10^{-19} C,
v_0 =2x10^8 m/s, w = 1 m, B=1.2T

m = 1.67 x 10^{-27} kg, q=e=1.6x10^{-19} C,
v_0 =2x10^8 m/s, w = 1 m, B=1.2T

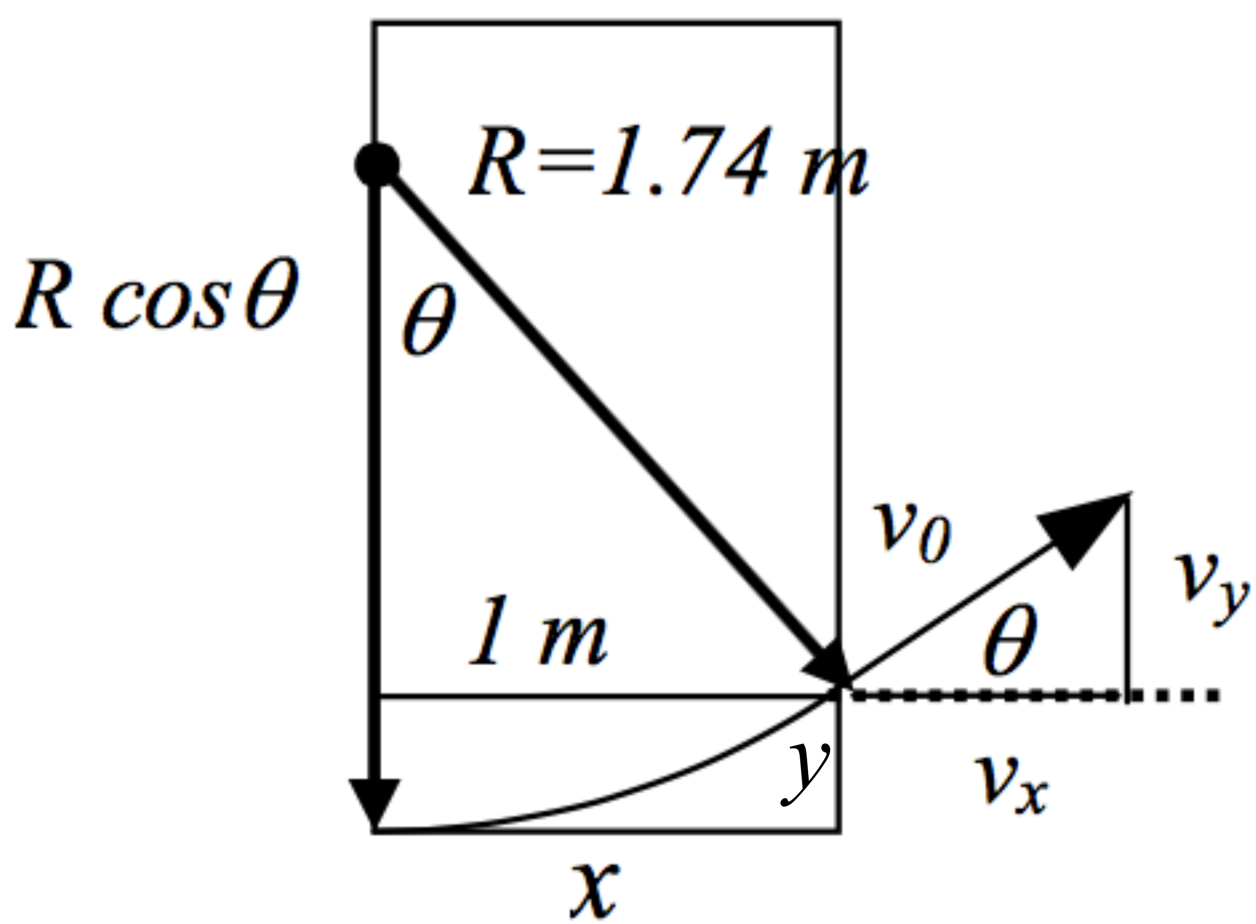
$$F_\perp = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB} = 1.74 \text{ m}$$

```
>>> 1.67e-27*2e8/(1.6e-19*1.2)
1.7395833333333335
```

At what x and y positions will the proton leave the magnetic field region?

Sketch the proton's path on the figure, and use your knowledge of the radius of curvature.



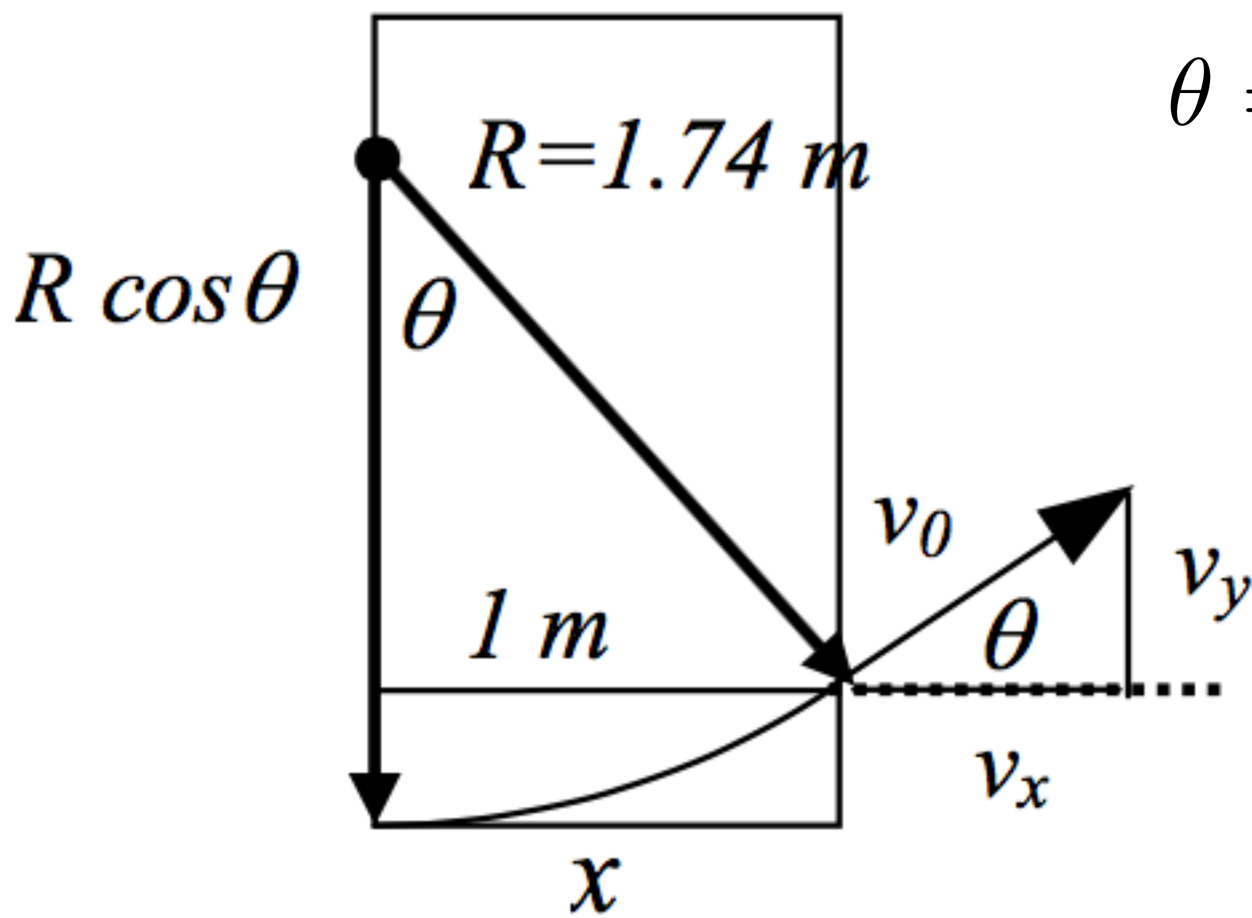
$$y = 1.74 - \sqrt{1.74^2 - 1}$$
$$= 1.74 - 1.424 = 0.316 \text{ m}$$

$$\theta = \sin^{-1}\left(\frac{1}{1.74}\right) = 0.612 \text{ rad}$$

m = 1.67 x 10^{-27} kg, q=e=1.6x10^{-19} C,
v_0 =2x10^8 m/s, w = 1 m, B=1.2T

At what x and y positions will the proton leave the magnetic field region?

Sketch the proton's path on the figure, and use your knowledge of the radius of curvature.



$$\theta = \sin^{-1}\left(\frac{1}{1.74}\right) = 0.612 \text{ rad}$$

What is the magnitude v of the proton's speed as it exits the magnetic field?

If the speed of the proton has changed, what
has happened to its kinetic energy?
Can the Lorentz force cause such an effect?

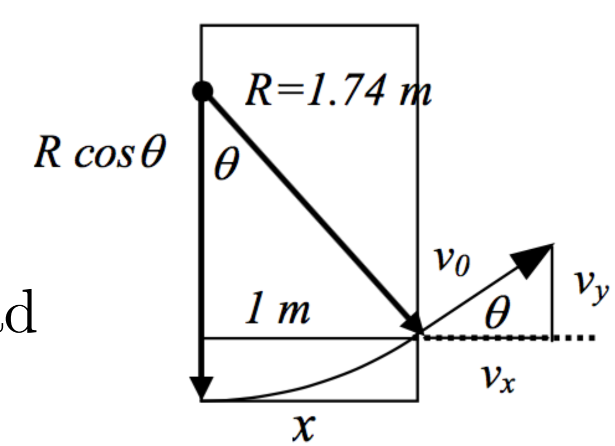
Because the Lorentz force is always $\perp$ to the proton path, it does no work
$\Rightarrow$ no change in speed thus $v_{exit}=v_0 =2\times10^8$ m/s

Suppose the proton had an initial component of velocity in the +z direction of $v_z = 1 \times 10^8$ m/s, and the same initial speed v_0 as before in the xy plane.
What then would be its final velocity v as it exits the magnetic field? This time, express your answer as a vector with three components.

The Lorentz force is in the x-y plane hence no z acceleration.
Thus v_z is unchanged $v_z = 1\times10^8$ m/s.
From diagram you can see

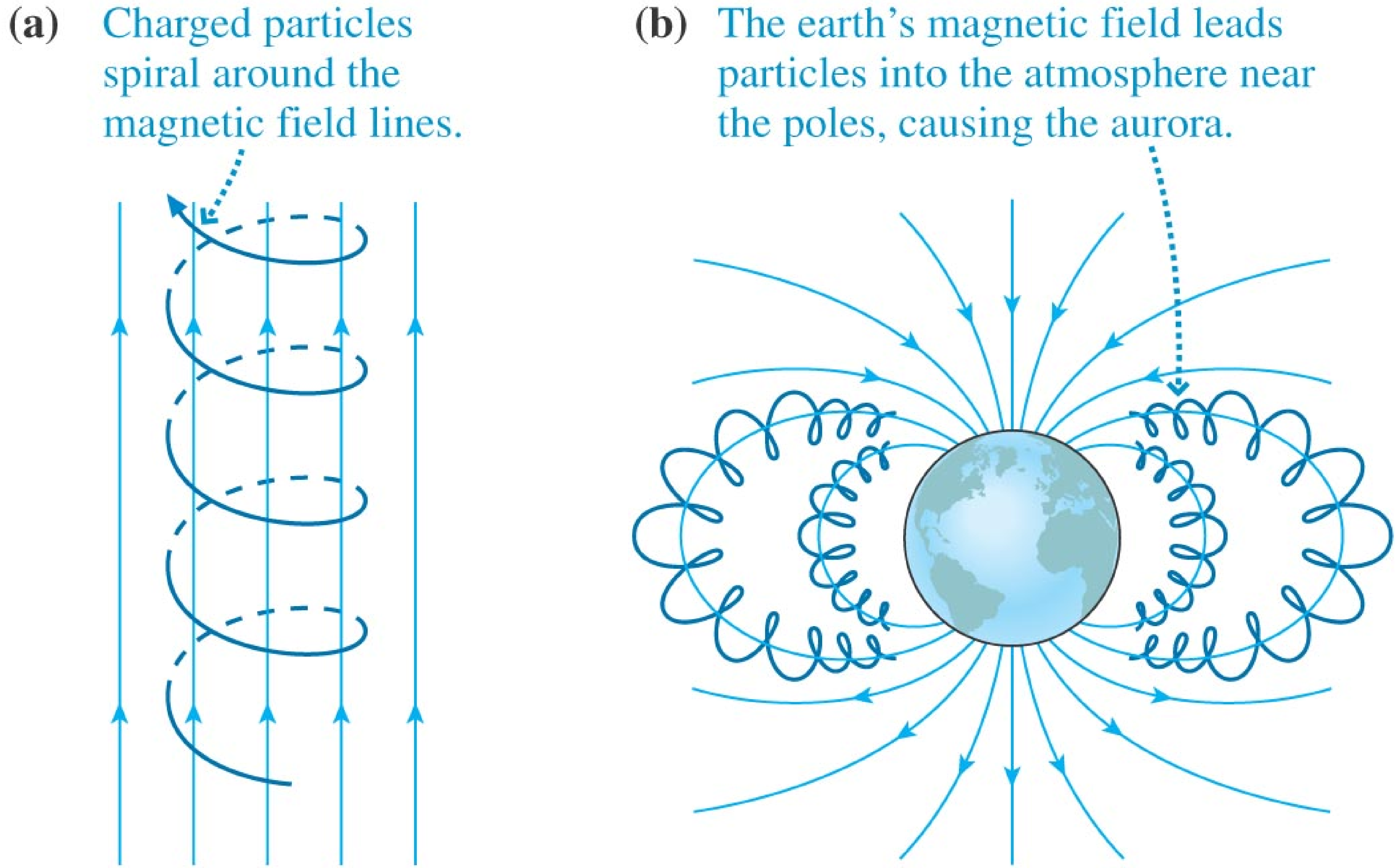
$v_x = v_0 \cos\theta = 2\times10^8 \cos(0.612) = 1.63\times10^8$ m/s
$v_y = v_0 \sin\theta = 2\times10^8 \sin(0.612) = 1.15\times10^8$ m/s

$$\theta = \sin^{-1}\left(\frac{1}{1.74}\right) = 0.612 \text{ rad}$$



$$\theta = \sin^{-1}\left(\frac{1}{1.74}\right) = 0.612 \text{ rad}$$

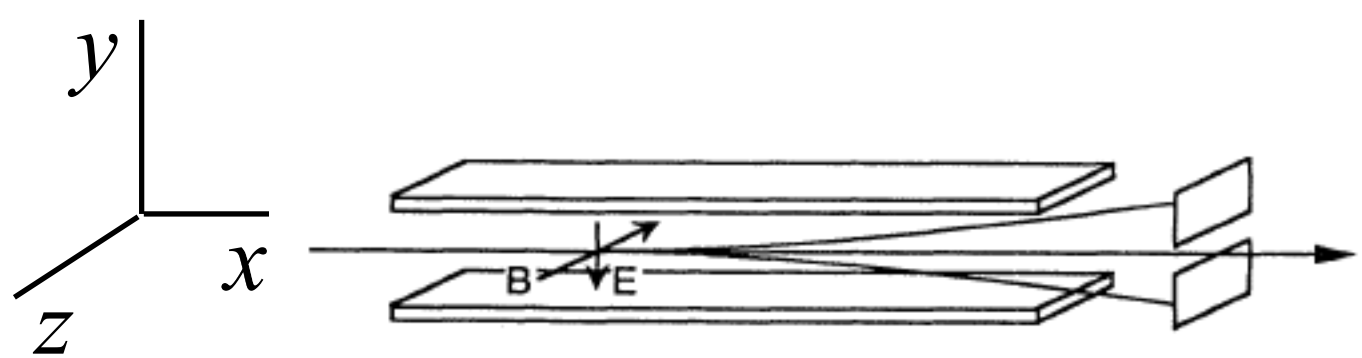
With a z component of velocity and B in the z direction the particle follows a spiral path.



Finally, a constant electric field E is added in the shaded region. The effect of this field is that all charged particles launched with initial velocity v_0 in the x direction travel straight through the field region without being deflected at all! What is the magnitude and direction of the field E?

$$F_B = qB_z v_0 = F_E = qE$$
$$\Rightarrow |E| = |B_z|\, v_0 = (1.2\ \mathrm{T})\ 2\times10^8\ \mathrm{m/s}$$

$$\vec{E} = -2.4 \times 10^8 \hat{\jmath}\ \mathrm{N/C}$$



Unit Check

$$v = E/B$$

$$[N/C/T]$$

$$E: [kg \cdot m \cdot s^{-2} \cdot A^{-1}\ s^{-1}]$$

$$B: [T = kg \cdot s^{-2} \cdot A^{-1}]$$

$$v: [m \cdot s^{-1}]$$