PHYSICS 121 MIDTERM 2 KEY

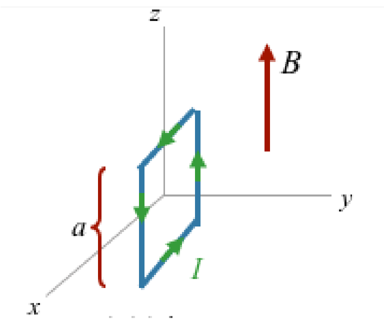
2019-07-05

Academic Honesty: Cheating in an examination includes the following:

1. the unauthorized sharing of material such as textbooks during an open book" examination;
2. concealing information pertaining to the examination in the examination room, or in washrooms or other places in the vicinity of the examination room;
3. using course notes or any other aids not approved by an Instructor during an examination; or,
4. the unauthorized possession or use of an examination question sheet, an examination answer book, or a completed examination or assignment.

There are 10 multiple choice questions. Select the correct answer for each one and mark it on the front page. Each question has only one correct answer. (2 marks each)

1. A square loop of side a lies in the xz plane with current I as shown. The loop can rotate about x axis without friction. A uniform field B points as shown. In which direction is the magnetic dipole moment?



   (a) $+x$
   (b) $-x$
   (c) $+y$ *
   (d) $-y$
   (e) $+z$

2. An uncharged capacitor of value $C = 1\ \mu$F is in series with resistor $R = 1\ \text{k}\Omega$. At $t = 0$ the capacitor and resistor are connected to a voltage source of 10 volts. After a time $t = RC$ the voltage on the capacitor is

   (a) 2.7 V
   (b) 3.7 V
   (c) 5.0 V
   (d) 6.3 V *
   (e) 10.0 V

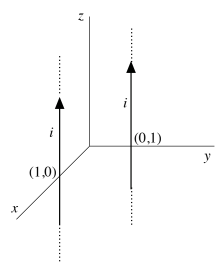
3. An electron is passing through a region with a nonzero, uniform electric and a nonzero uniform magnetic field, $\vec{E}$ and $\vec{B}$. The electron's velocity, v_0, is not changed by the fields. One can conclude

   (a) $\vec{E}$, $\vec{B}$ and $\vec{v}_0$ are in the same direction.
   (b) $\vec{B}$ is perpendicular to $\vec{E}$ and $\vec{E}$ is in the same direction as $\vec{v}_0$
   (c) $\vec{B}$ is parallel to $\vec{E}$ and $\vec{E}$ perpendicular to $\vec{v}_0$
   (d) $\vec{B}$ is perpendicular to $\vec{E}$ and both $\vec{E}$ and $\vec{B}$ are perpendicular to $\vec{v}_0$ *
   (e) none of the above

4. A current loop with magnetic moment $\vec{\mu}$ is in a magnetic field B. The maximum torque on the current loop is achieved when

   (a) $\vec{\mu}$ is perpendicular to $\vec{B}$ *
   (b) $\vec{\mu}$ is parallel to $\vec{B}$
   (c) $\vec{\mu}$ is antiparallel to $\vec{B}$

5. Two infinitely long wires parallel to the z axis both carry current i in the $+\hat{k}$ direction. One wire passes through the xy plane at (1,0) and the other at (0,1). What is the direction of the magnetic field at the origin produced by these two currents?



   (a) $\hat{k}$
   (b) $(\hat{\imath} + \hat{\jmath})/\sqrt{2}$
   (c) $(-\hat{\imath} + \hat{\jmath})/\sqrt{2}$
   (d) $(\hat{\imath} - \hat{\jmath})/\sqrt{2}$ *
   (e) None of the choices above or $\vec{B} = 0$.

6. Which of the following situations could NOT, in the general case, result in an induced emf in a loop of conducting wire:

   (a) Accelerating through a uniform, constant magnetic field, without changing the loop's orientation with respect to the magnetic field direction. (Assume $v << c$.) *

   (b) Rotating the loop with respect to a uniform, constant magnetic field

   (c) Remaining stationary in a magnetic field that changes with time.

   (d) Changing shape from a circular loop to an elliptical loop while maintaining the same length of conductor around the loop.

   (e) Moving at constant velocity through a non-uniform magnetic field without changing the loop's orientation

7. The magnetic field in an infinitely long, current-carrying solenoid of radius 5 cm is 8.0 mT. Another solenoid with radius of 10 cm is made, without changing the current or turn density, what is the magnitude of its magnetic field?

   (a) 4.0 mT

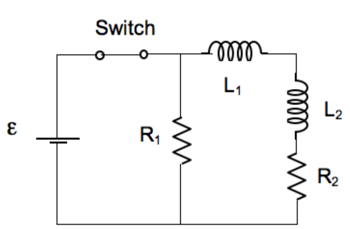
   (b) 5.6 mT

   (c) 8.0 mT *

   (d) 11.3 mT

   (e) 16.0 mT

8. In the circuit shown below, the switch has been closed for a long time.



ε = 9 V
R_1 = 900 Ω
R_2 = 100 Ω
L_1 = 1 x 10^{-3} H
L_2 = 1 x 10^{-3} H

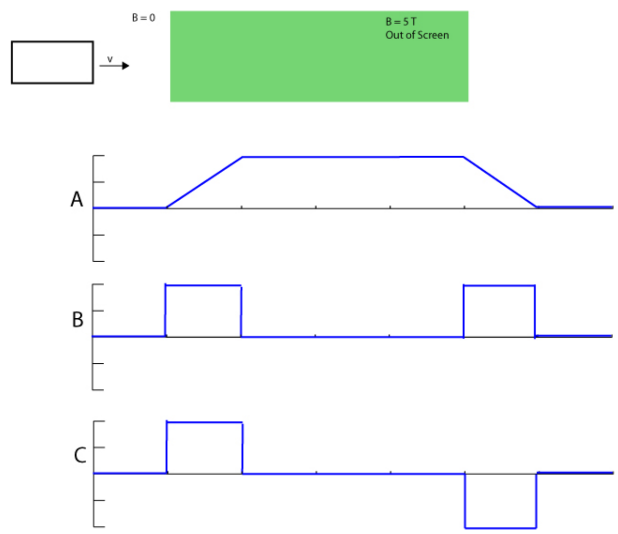
Compare the magnitude of the current through R_1 just before the switch is opened to the magnitude of the current through R_1 just after the switch is opened.

   (a) $|I_1(\text{before})| > |I_1(\text{after})|$

   (b) $|I_1(\text{before})| < |I_1(\text{after})|$ *

   (c) $|I_1(\text{before})| = |I_1(\text{after})|$

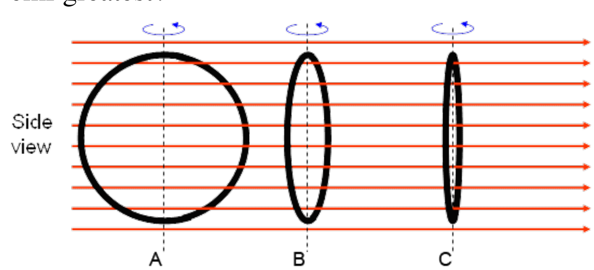
9. A wire loop travels to the right at a constant velocity. Which plot best represents the induced current in the loop as it travels from left of the region of magnetic field, through the magnetic field, and then entirely out of the field on the right side.



C

10. A circular wire loop is placed in a uniform magnetic field pointing to the right. The loop is rotated with constant angular velocity around a vertical axis (dashed line).

    At which of the three times shown is the induced emf greatest?



A

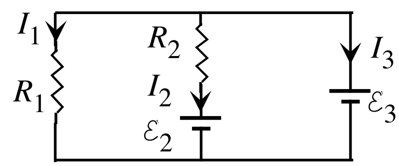
To get full credit for the *written problems* you must clearly show all your steps and explain well. 10 marks each.

---

11. Analyse the circuit below using Kirchhoff's Rules.

   (a) Write out the equations derived from applying Kirchhoff's rules to the circuit. (Your equations should use the directions of the currents as indicated by the arrows.)

   Outer Loop: $-I1R1 + E3 = 0$
   Left Loop: $-I1R1 + I2R2 + E2 = 0$
   Right Loop: $-E2 - I2R2 + E3 = 0$
   Node: $I1 + I2 + I3 = 0$



   (b) Find the equation for the currents I_1, I_2 and I_3 in terms of the $\mathcal{E}$s and Rs.

   $I1 = E3/R1$
   $I2 = (E3 - E2)/R2$
   $I3 = -I1 - I2 = (E2 - E3)/R2 - E3/R1$

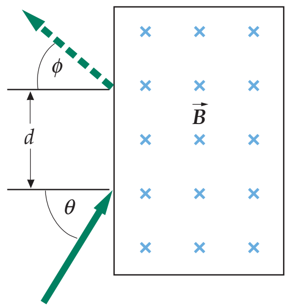
   (c) If $\mathcal{E}_2 = 3$ V and $\mathcal{E}_3 = 9$ V, $R_1 = 1000\ \Omega$ and $R_2 = 2200\ \Omega$, find all three currents.

   $I1 = E3/R1 = 9$ mA
   $I2 = (E3 - E2)/R2 = 6/2200 = 2.7$ mA
   $I3 = -I1 - I2 = (E2 - E3)/R2 - E3/R1 = -11.7$ mA

Use this page for extra work on the current problem.

12. A particle enters a rectangular region with a nonzero magnetic field $\vec{B}$ and exits as shown by the solid-arrow and dashed-arrow lines in the figure. (The arrows indicate angle of entry and exit only.) Suppose that in figure the magnetic field has a magnitude of 60 mT, the distance d is 40 cm, and θ is 45°.

(a) What is the radius of curvature of the trajectory of the particle?



As you can see from the figure, lines drawn perpendicular to the trajectory at entry and exit form an isosceles right triangle. The Trajectory is symmetric so $r^2 + r^2 = d^2$.
Therefore, $r = d/\sqrt{2}$.

(b) Find the speed v at which a particle enters the region if the particle is a proton ($1.67262158 \times 10^{-27}$ kilograms, $1.60217646 \times 10^{-19}$ coulombs).

The magnitude of the magnetic field must equal the centripetal force of the circular trajectory in the magnetic field.

$$qvB = \frac{mv^2}{r}$$

Solve for v

$$v = \frac{qBr}{m} = 1625582 \text{ m/s}$$

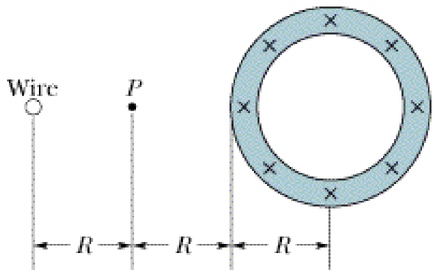
(3 significant figs ok.)

(c) Find the exit angle ϕ

As demonstrated in part (a) the trajectory is symmetric. Because the entry position and and exit position are directly aligned with each other, the angle of exit is 45°.

Use this page for extra work on the current problem.

13. A long circular pipe with outside radius R carries a (uniformly distributed) current i into the page as shown in the figure. A wire runs parallel to the pipe at a distance of $3R$ from centre to centre.



(a) Initially the wire carries no current. Find the magnetic field at point **P**.

By Ampere's law, consider a circle centred on the axis of the pipe passing through P. If there is no current in the wire then

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i_{\text{encl}}$$

The radius of the circle is $2\pi R$ therefore

$$B(2\pi(2R)) = \mu_0 i$$

$$B = \frac{\mu_0 i}{4\pi R}$$

(b) Find the magnetic field at the centre of the pipe with no current in the wire.

Draw a circle around the centre inside the pipe. B will have the same magnitude anywhere on the circle due to the symmetry. The circle does not enclose any current so B =0.

(c) Now let there be a current I in the wire, with the current in the pipe remaining the same, i. Find the amount and direction of the current in the wire such that the net magnetic field at point **P** has the same magnitude as the net magnetic field that is now at the centre of the pipe but is in the opposite direction.

$$B_{P-wire} + B_{P-pipe} = -B_{centre-wire}$$

Note that when there is current in the wire, there is magnetic field in the centre of the pipe, $\vec{B}$ is no longer zero there.

It's tricky keeping track of the signs because the B field from the wire is opposite that from the pipe at P when the currents are in the same direction. Note that if current going into the page is positive then the field from the wire is negative and the field from the pipe is positive at the points of interest.

$$-\frac{\mu_0}{2\pi R}I + \frac{\mu_0}{4\pi R}i = -\left(-\frac{\mu_0}{6\pi R}I\right)$$

Cancel the common factors

$$-I + \frac{1}{2}i = \frac{1}{3}I$$

$$I = \frac{3}{8}i$$

Use this page for extra work on the current problem.

14. You have a hollow paper tube and wire with which to make an inductor. The paper tube (similar to a toilet paper tube) has a radius r and length z and the volume inside is $V = 2\pi r^2 z$.

(a) For an air-core coil the inductance is given by $L = \mu_0 n^2 V$. Show that the number of turns, N, wrapped along the entire length of the tube, needed to achieve an inductance of L is given by

$$N = z\sqrt{\frac{L}{\mu_0 V}}$$

$$n = N/Z$$

$$L = \mu_0 \frac{N^2}{z^2} V$$

$$N^2 = z^2 \frac{L}{\mu_0 V}$$

$$N = z\sqrt{\frac{L}{\mu_0 V}}$$

(b) If the radius $r = 2$ cm and the length $z = 10$ cm how many turns of wire along the length would be needed for a 100 μH inductance? (Make sure your answer is reasonable.)

evaluatng: V=pi*r**2*z
z*sqrt(100e-6/(mu0*V))
79.577471545947674
Thus about 80 turns. It should fit on a length of 10 cm.

(c) If a current of 100 mA is passed through the inductor, how much magnetic flux is created through the tube?

magnetic flux is IL = 100e-3*100e-6 = 1e-5 weber or T m^2.

(d) What is the magnitude of the magnetic field in the tube with a current of 100 mA in the wire?

$B = \Phi_B/(\pi r^2 N)$= 10e-5 T = .00010 T

alternately

$B = \mu_0 \frac{N}{z} I$ gives the same result.

Use this page for extra work on the current problem.